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КОМПЬЮТЕРНЫЕ ТЕХНОЛОГИИ ФИЗИКИ

A GENERALIZATION OF QUANTUM TELEPORTATION AND SPLITTING OF ENTANGLEMENT VIA LOCAL CLONING

I. Ghiu[1]

Department of Physics, University of Bucharest, Bucharest-Măgurele, Romania

In the original one-to-one teleportation protocol of Bennett et al. [1], an observer Alice transmits the information of a d-level system to another observer Bob with perfect fidelity, by using a maximally entangled state. We introduce a generalization called the many-to-many teleportation, where the information is sent from N observers to M receivers situated at different locations. One of the most interesting applications of quantum cloning is the symmetric broadcasting of entanglement proposed by Buzek et al. [2]. We propose the splitting of entanglement based on local optimal universal asymmetric cloning machine, and then, by applying the Peres–Horodecki criterion, we analyze the inseparability of the final states.

В оригинальном протоколе телепортации один-на-один (Бенетт и др. [1]) наблюдатель Алиса передает информацию о d-уровневой системе другому наблюдателю Бобу с идеальной точностью, используя механизм запутанных состояний. Мы вводим обобщение, называемое телепортацией многих-на-многие, где информация передается от N наблюдателей к M получателям, находящимся в разных местах. Одним из самых интересных применений квантового клонирования является симметрическая передача запутанных состояний, предложенная Бузеком и др. [2]. Мы предлагаем разделение запутанных состояний, базирующееся на оптимальной универсальной асимметричной клонирующей машине, и затем, применяя критерии Переса–Городецкого, анализируем несепарабельность конечных состояний.

INTRODUCTION

Quantum entanglement is the basic resource in many quantum information processes such as quantum teleportation, quantum cryptography, telecloning or superdense coding [3]. Much work has been devoted to the analysis of quantum teleportation and its possible applications. Murao et al. have found a generalization of quantum teleportation, namely, one-to-many teleportation of a d-level system [4].

In Sec. 2 we present a new generalization, many-to-many teleportation, where the information of a d-level particle is transmitted from N senders to M receivers with the help of a maximally entangled $(N+M)$-party state. In Sec. 3 we show how one can copy the inseparability of two-qubit system by using local optimal universal asymmetric cloning machines.

[1]E-mail: iughiu@barutu.fizica.unibuc.ro

1. ONE-TO-MANY TELEPORTATION

In the standard teleportation scheme proposed by Bennett et al. [1] an unknown qubit (a state of a d-level system) is faithfully transmitted from one observer, Alice, to another observer, Bob, while the initial Alice's state is destroyed. A more general scheme called one-to-many teleportation was introduced by Murao et al. [4] where the information of a d-level particle is sent from one sender to M spatially separated receivers denoted by $B_1, \ldots, B_M$.

Let the initial unknown Alice's state we wish to teleport be

$$|\,\psi\,\rangle = \sum_{k=0}^{d-1} \alpha_k |\,\psi_k\,\rangle_A, \tag{1}$$

with $\sum_{k=0}^{d-1} |\alpha_k|^2 = 1$, and $\{|\,\psi_k\,\rangle_A\}$ representing a basis in the d-dimensional space. In order to perform the teleportation, Alice and M receivers must share a quantum channel, which is a maximally entangled $M+1$-particle state:

$$|\,\xi\,\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} |\,\pi_j\,\rangle_A |\,\phi_j\,\rangle_{B_1 B_2 \cdots B_M}, \tag{2}$$

where $\{|\,\pi_j\,\rangle\}$ and $\{|\,\phi_j\,\rangle\}$ are bases in the d-level spaces of Alice's and receivers' particles, respectively.

With the help of the generalized Bell basis [1]

$$|\,\Phi_{m,n}\,\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} \exp\left(\frac{2\pi i k n}{d}\right) |\,\psi_k\,\rangle |\,\pi_{k+m}\,\rangle, \tag{3}$$

we can write the state of the whole system as

$$|\,\psi\,\rangle |\,\xi\,\rangle = \frac{1}{d} \sum_{m=0}^{d-1} \sum_{n=0}^{d-1} |\,\Phi_{m,n}\,\rangle \sum_{k=0}^{d-1} \exp\left(-\frac{2\pi i k n}{d}\right) \alpha_k |\,\phi_{k+m}\,\rangle. \tag{4}$$

Alice performs a Bell-type measurement and sends the result to M receivers. If the outcome is $|\,\Phi_{m,n}\,\rangle$, then the receivers apply a local unitary operation, which satisfies

$$V_{m;n} = \mathcal{V}_{B_1} \otimes \mathcal{V}_{B_2} \otimes \cdots \otimes \mathcal{V}_{B_M} = \sum_{j=0}^{d-1} \exp\left(\frac{2\pi i j n}{d}\right) |\,\phi_j\,\rangle\langle\,\phi_{j+m}\,|. \tag{5}$$

Therefore, the information of the initial state contained in the coefficients α_k has been transmitted to M distant parties:

$$|\,\phi\,\rangle = \sum_{j=0}^{d-1} \alpha_j |\,\phi_j\,\rangle_{B_1 B_2 \cdots B_M}. \tag{6}$$

2. MANY-TO-MANY TELEPORTATION

Let us analyze a more general scenario, when the initial information is distributed between N spatially separated senders $A_1, A_2, \dots, A_N$. We would like to distribute this information to M receivers located at different places [5]. The initial entangled state is

$$|\,\psi\,\rangle = \sum_{k=0}^{d-1} \alpha_k |\,\psi_k\,\rangle_{A_1} |\,\psi_k\,\rangle_{A_2} \cdots |\,\psi_k\,\rangle_{A_N}, \tag{7}$$

with $\sum_{k=0}^{d-1} |\alpha_k|^2 = 1$, and $\{|\,\psi_k\,\rangle_{A_j}\}$ representing a basis in the d-dimensional space of the jth sender.

The quantum channel is defined as a maximally entangled $(N+M)$-particle state shared between senders and receivers:

$$|\,\xi\,\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} |\,\pi_j\,\rangle_{A'_1} |\,\pi_j\,\rangle_{A'_2} \cdots |\,\pi_j\,\rangle_{A'_N} |\,\phi_j\,\rangle_{B_1 B_2 \cdots B_M}, \tag{8}$$

where we have the particles denoted by «B» that belong to the receivers. The states $\{|\,\pi_j\,\rangle_{A'_i}\}$ represent a d-dimensional basis for the ith sender.

The joint state of the initial system and the channel is

$$\begin{aligned} |\,\psi\,\rangle |\,\xi\,\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} \alpha_k \sum_{j=0}^{d-1} |\,\psi_k\,\rangle_{A_1} |\,\pi_j\,\rangle_{A'_1} |\,\psi_k\,\rangle_{A_2} |\,\pi_j\,\rangle_{A'_2} \cdots |\,\psi_k\,\rangle_{A_N} |\,\pi_j\,\rangle_{A'_N} \otimes \\ \otimes |\,\phi_j\,\rangle_{B_1 \cdots B_M} = \frac{1}{d^{\frac{N+1}{2}}} \sum_{m} \sum_{n_1, n_2, \dots, n_N} |\,\Phi_{m,n_1}\,\rangle |\,\Phi_{m,n_2}\,\rangle \cdots |\,\Phi_{m,n_N}\,\rangle \times \\ \times \sum_{k} \exp\left[-\frac{2\pi i k}{d}(n_1 + n_2 + \ldots + n_N)\right] \alpha_k |\,\phi_{k+m}\,\rangle. \end{aligned} \tag{9}$$

The protocol for many-to-many teleportation is the following:

A) Each sender performs a measurement of his particles in the generalized Bell basis.

B) The senders communicate the result of the measurement to M receivers.

Let us analyze the case when the outcome of the senders' Bell measurement is:

$$|\,\Phi_{m,n_1}\,\rangle |\,\Phi_{m,n_2}\,\rangle \cdots |\,\Phi_{m,n_N}\,\rangle. \tag{10}$$

Then, the receivers have to apply a local recovery unitary operation that fulfills:

$$V_{m;n_1,n_2,\dots,n_N} |\,\phi_k\,\rangle = \exp\left[\frac{2\pi i k}{d}(n_1 + n_2 + \ldots + n_N)\right] |\,\phi_{k-m}\,\rangle. \tag{11}$$

Therefore, the many-to-many teleportation distributes the information of the initial N-particle state (7) into the M-particle state:

$$|\,\psi\,\rangle = \sum_{j=0}^{d-1} \alpha_j |\,\psi_j\,\rangle_{A_1} |\,\psi_j\,\rangle_{A_2} \cdots |\,\psi_j\,\rangle_{A_N} \to |\,\phi\,\rangle = \sum_{j=0}^{d-1} \alpha_j |\,\phi_j\,\rangle_{B_1 B_2 \cdots B_M}. \tag{12}$$

3. SPLITTING OF INSEPARABILITY VIA LOCAL CLONING

Wootters and Zurek have considered a cloning machine that is supposed to copy an arbitrary qubit and have shown that this is impossible without introducing errors [6]. This result is known as the no-cloning theorem. Therefore, some approximate methods for cloning were proposed, where the fidelity between the final identical states and the initial one is less than the unity [7, 8]. In the case of asymmetric cloning (when the two final clones are not identical), it is interesting when the universal cloning machine is optimal, that means a machine that creates the second clone with maximal fidelity for the given fidelity of the first one [9]. Cerf has found the expression of the optimal universal asymmetric cloning machine of d-level states using a reference state [9]. We have also obtained an equivalent expression of this cloning machine, by eliminating the reference state [5]:

$$U|\,j\,\rangle|\,00\,\rangle = \frac{1}{\sqrt{1+(d-1)(p^2+q^2)}}\Bigg(|\,j\,\rangle|\,j\,\rangle|\,j\,\rangle + \\ + p\sum_{r=1}^{d-1}|\,j\,\rangle|\,j+r\,\rangle|\,j+r\,\rangle + q\sum_{r=1}^{d-1}|\,j+r\,\rangle|\,j\,\rangle|\,j+r\,\rangle\Bigg), \quad (13)$$

where $p+q=1$.

An interesting application of quantum cloning is the broadcasting of entanglement proposed by Bužek et al. [2]. In this process, the entanglement originally shared by two observers is broadcast into two identical entangled states by using local $1 \to 2$ optimal universal symmetric cloning machine.

Suppose now that we want to copy an entangled state asymmetrically, this means that the two output states are different. How can we implement this? In what follows, we show that one can split the entanglement using the optimal universal *asymmetric* cloning machine by employing the formula (13) for $d=2$:

$$\begin{aligned} U(p)|\,0\,\rangle|\,00\,\rangle &= \frac{1}{\sqrt{1+p^2+q^2}}(|\,000\,\rangle + p|\,011\,\rangle + q|\,101\,\rangle), \\ U(p)|\,1\,\rangle|\,00\,\rangle &= \frac{1}{\sqrt{1+p^2+q^2}}(|\,111\,\rangle + p|\,100\,\rangle + q|\,010\,\rangle), \end{aligned} \quad (14)$$

with $p+q=1$, where the first two qubits represent the clones and the last one is the ancilla [5]. The initial entanglement shared by two observers, Alice and Bob, is:

$$|\,\psi\,\rangle_{12} = \alpha|\,00\,\rangle + \beta|\,11\,\rangle. \quad (15)$$

Therefore, the state of the total system, consisting of the two particles 1 and 2, and another four particles, the blank states 3 and 4, and the ancillas 5 and 6, after applying the cloning transformation (14) by Alice and Bob, is [5]:

$$
\begin{aligned}
|\,\eta\,\rangle = U(p)\otimes U(p)|\,\psi\,\rangle_{12}|\,00\,\rangle_{35}|\,00\,\rangle_{46} = \frac{1}{\sqrt{1+p^2+q^2}}\{|\,00\,\rangle_{56}[\alpha|\,00\,\rangle_{13}|\,00\,\rangle_{24}+ \\
+\beta p^2|\,10\,\rangle_{13}|\,10\,\rangle_{24}+\beta pq|\,10\,\rangle_{13}|\,01\,\rangle_{24}+\beta pq|\,01\,\rangle_{13}|\,10\,\rangle_{24}+\beta q^2|\,01\,\rangle_{13}|\,01\,\rangle_{24}]+ \\
+|\,01\,\rangle_{56}[\alpha p|\,00\,\rangle_{13}|\,01\,\rangle_{24}+\alpha q|\,00\,\rangle_{13}|\,10\,\rangle_{24}+\beta p|\,10\,\rangle_{13}|\,11\,\rangle_{24}+\beta q|\,01\,\rangle_{13}|\,11\,\rangle_{24}]+ \\
+|\,10\,\rangle_{56}[\alpha q|\,10\,\rangle_{13}|\,00\,\rangle_{24}+\alpha p|\,01\,\rangle_{13}|\,00\,\rangle_{24}+\beta p|\,11\,\rangle_{13}|\,10\,\rangle_{24}+\beta q|\,11\,\rangle_{13}|\,01\,\rangle_{24}]+ \\
+|\,11\,\rangle_{56}[\alpha p^2|\,01\,\rangle_{13}|\,01\,\rangle_{24}+\alpha pq|\,01\,\rangle_{13}|\,10\,\rangle_{24}+ \\
+\alpha pq|\,10\,\rangle_{13}|\,01\,\rangle_{24}+\alpha q^2|\,10\,\rangle_{13}|\,10\,\rangle_{24}+\beta|\,11\,\rangle_{13}|\,11\,\rangle_{24}]\}, \quad (16)
\end{aligned}
$$

where the particles denoted by odd numbers belong to Alice, while the even particles belong to Bob.

The input state $|\,\psi\,\rangle_{12}$ is splitted if the following two necessary conditions are satisfied: (i) the local reduced density operators ρ_{13} and ρ_{24} are separable, and (ii) the nonlocal states ρ_{14} and ρ_{23} are inseparable. The expression of the reduced density operators of the local states is:

$$
\begin{aligned}
\rho_{13}=\rho_{24} \quad = \quad & \frac{1}{(1+p^2+q^2)^2}[\alpha^2(1+p^2+q^2)|\,00\,\rangle\langle\,00\,| \; +\beta^2(1+p^2+q^2)|\,11\,\rangle\langle\,11\,|+ \\
& +(p^2q^2+\beta^2q^4+\beta^2q^2+\alpha^2p^4+\alpha^2p^2)|\,01\,\rangle\langle\,01\,|+ \\
& +(p^2q^2+\beta^2p^4+\beta^2p^2+\alpha^2q^4+\alpha^2q^2)|\,10\,\rangle\langle\,10\,|+ \\
& +(pq+p^3q+pq^3)(|\,01\,\rangle\langle\,10\,|+|\,10\,\rangle\langle\,01\,|)]. \quad (17)
\end{aligned}
$$

Applying the Peres–Horodecki theorem [10, 11], we get the condition for the separability of the local states

$$
\frac{1}{2}\left[1-\sqrt{1-4p^2(1-p)^2}\right]\leqslant\alpha^2\leqslant\frac{1}{2}\left[1+\sqrt{1-4p^2(1-p)^2}\right]. \quad (18)
$$

The nonlocal pairs of particles are described by the following density operators:

$$
\begin{aligned}
\rho_{14} \quad = \quad & \frac{1}{(1+p^2+q^2)^2}\{[p^2q^2+\alpha^2(1+p^2+q^2)]|\,00\,\rangle\langle\,00\,|+[p^2q^2+ \\
& +\beta^2(1+p^2+q^2)]|\,11\,\rangle\langle\,11\,|+4pq\alpha\beta(|\,00\,\rangle\langle\,11\,|+|\,11\,\rangle\langle\,00\,|)+ \\
& +(\beta^2q^4+\beta^2q^2+\alpha^2p^4+\alpha^2p^2)|\,01\,\rangle\langle\,01\,|+ \\
& +(\beta^2p^4+\beta^2p^2+\alpha^2q^4+\alpha^2q^2)|\,10\,\rangle\langle\,10\,|\} \quad (19)
\end{aligned}
$$

and

$$
\begin{aligned}
\rho_{23} \quad = \quad & \frac{1}{(1+p^2+q^2)^2}\{[p^2q^2+\alpha^2(1+p^2+q^2)]|\,00\,\rangle\langle\,00\,|+[p^2q^2+ \\
& +\beta^2(1+p^2+q^2)]|\,11\,\rangle\langle\,11\,|+4pq\alpha\beta(|\,00\,\rangle\langle\,11\,|+|\,11\,\rangle\langle\,00\,|)+ \\
& +(\beta^2p^4+\beta^2p^2+\alpha^2q^4+\alpha^2q^2)|\,01\,\rangle\langle\,01\,|+ \\
& +(\beta^2q^4+\beta^2q^2+\alpha^2p^4+\alpha^2p^2)|\,10\,\rangle\langle\,10\,|\}. \quad (20)
\end{aligned}
$$

Then, the two nonlocal states are inseparable if

$$\frac{1}{2}\left(1-\sqrt{1-4\lambda}\right) \leqslant \alpha^2 \leqslant \frac{1}{2}\left(1+\sqrt{1-4\lambda}\right), \tag{21}$$

where

$$\lambda = \frac{p^4q^4+p^2q^4+p^4q^2+p^2q^2}{2p^4q^4+2p^4q^2+2p^2q^4-q^8-2q^6-q^4-p^8-2p^6-p^4+18p^2q^2}. \tag{22}$$

The requirements that $1-4\lambda$ has to be positive and that the local states are separable when the nonlocal ones are inseparable lead to

$$\frac{1}{2}\left(1-\sqrt{-9+2\sqrt{21}}\right) \leqslant p \leqslant \frac{1}{2}\left(1+\sqrt{-9+2\sqrt{21}}\right). \tag{23}$$

Therefore, we have proved that by applying the local optimal universal asymmetric cloners on an arbitrary entangled state, one can split the inseparability only in the case when the parameter p which characterizes the cloning machine satisfies Eq. (23).

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