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OPTICAL DEVICE ACCELERATING DYNAMIC PROGRAMMING

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The subject of this report is the comparison of the conventional deterministic computers versus analogue computer based on quantum optical system in resolving some NP-hard computational problems. We describe an optical machine which can be realized physically.

Доклад посвящен сравнению возможностей обычных детерминированных компьютеров и аналогового компьютера на основе квантово-оптической системы при решении некоторых NP-сложных задач. При этом мы описываем реализуемый вариант «оптической машины».

1. NP-HARD COMPUTATIONAL PROBLEMS

A problem of I instance lies within the class of NP-hard problems if:

a) there is a polynomial time $P(I)$ algorithm checking a solution (if this solution is provided),

b) the solution of this problem requires an exponential in I resource.

The lists of NP-hard problems can be found in [1] and [2].

1. Boolean knapsack, variant 1

Given positive integers c_j, $j = 1, 2, \ldots, n$ and K, is there a subset S of $\{1, 2, \ldots, n\}$ such that $\sum\limits_{j \in S} c_j = K$? In this case, the size $|I|$ can be estimated as $O(n \log K)$.

2. Boolean knapsack, variant 2

Given integers c_j and B_+, B_-, whenever there exist n boolean values $s_j \in \{0, 1\}$ such that $\sum\limits_{j=1}^{n} c_j s_j \in (B_-, B_+)$? Here the instance size is roughly $O(n \log B_+)$.

3. Optimization boolean knapsack

Given integers c_j and w_j, $j = 1, 2, \ldots, n$, and the number B_+, maximize the cost $\sum\limits_{j=1}^{n} c_j w_j$ defined by n boolean variables $s_j \in \{0, 1\}$ under condition that $\sum\limits_{j=1}^{n} c_j s_j < B_+$.

There is an important difference between the problems 1, 2 and 3. The output in 1 and 2 is «YES» or «NO», the output of 3 is a number, and one could try to approximate it.

2. DESCRIPTION OF THE OPTICAL MACHINES

Consider $n+1$ points $x_0 < x_1 < x_2 < \ldots < x_n$ in (x, y)-plane. At the first point we set a laser, which generates a narrow beam; its diameter $d_b \propto 2 \cdot 10^{-3}$ m, wave length $\lambda \propto 5 \cdot 10^{-7}$ m.

The possible scheme of an analogue optical device (OD) is presented in Fig. 1.

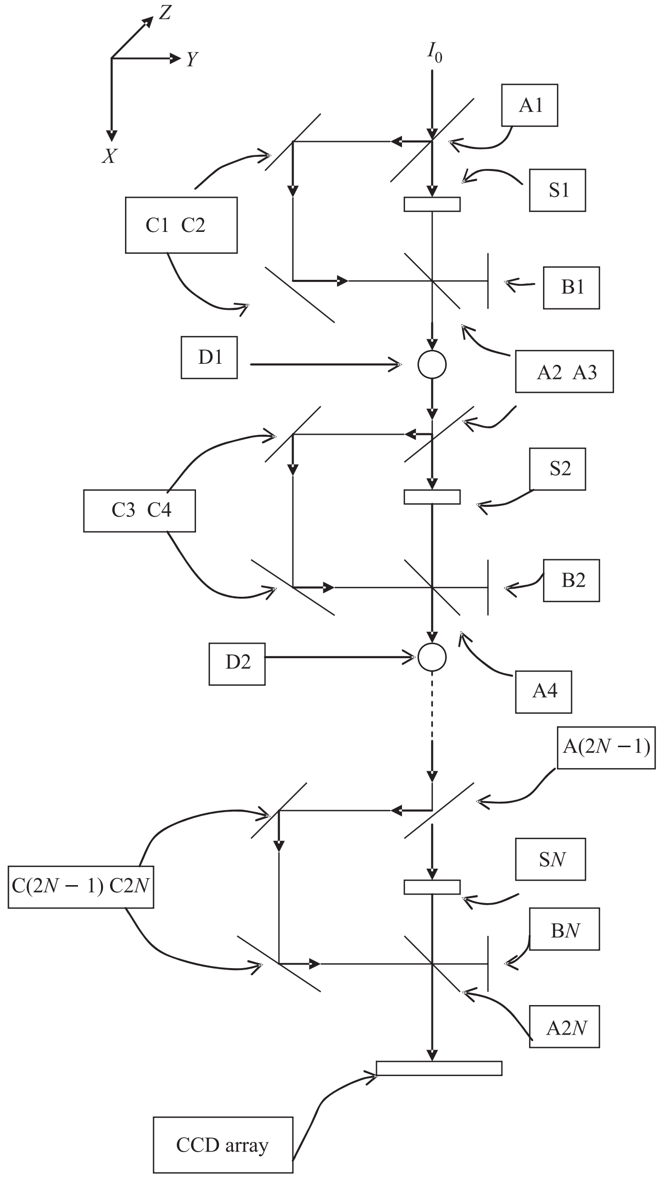

Fig. 1. Physical scheme of the optical device: I_0 — initial laser beam; AK — 50% mirrors; BK — absorbing boundaries; CK — reflecting mirrors; DK — amplifiers; SK — plane optical plates

Each optical plate is the corresponding beam on the value $c_s\kappa$ in (vertical) direction. We suppose that amplifiers have the characteristics shown in Fig. 2.

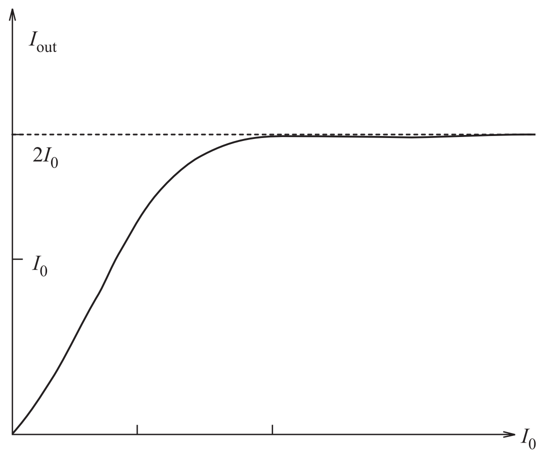

Fig. 2. Gain characteristics of the amplifiers

After the passage of m mirrors, the propagating light contains beams shifted in Z direction at all possible distances $\sum_{j=1}^{m} c_j s_j \kappa$. Then, we have physical implementation of problems 1 and 2. For problem 3 we use the modification of our machine presented in Fig. 3.

After the passage of m mirrors, one obtains the set of beams whose z- and y-shifts are different sums

$$\sum_{i=1}^{n} c_i s_i, \quad \sum_{i=1}^{n} w_i s_i.$$

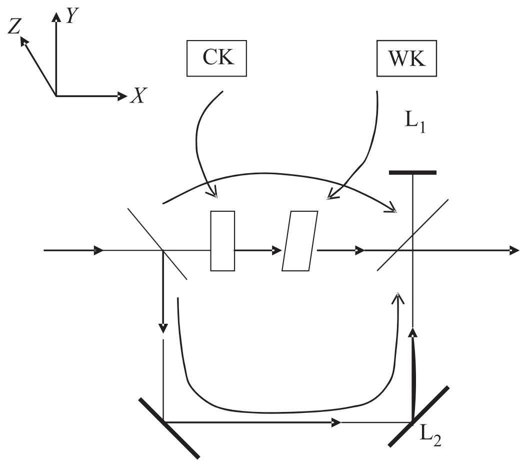

Fig. 3. Plane plate CK shifts beam in vertical Z direction on c_k, plane plate WK shifts beam in horizontal Y direction on w_k

Solving the dynamic programming by the optical device

a) the implementation cost CI;

b) the energy cost CE;

c) the running time Time when we solve M times the same problem with different inputs.

For problems 1 and 2 parameter K describes the value of the given sums.

RESULTS

I. For problems 1 and 2

$$CI_{\mathrm{quant}} = O(Kn), \quad CE_{\mathrm{quant}} = O(Kn(n+K)M), \quad CE_{\mathrm{det}} = O(KnM),$$

$$\mathrm{Time}_{\mathrm{quant}} = O(M(n+K)), \quad \mathrm{Time}_{\mathrm{quant}} = O(KnM).$$

II. For problem 3

$$CI_{\mathrm{quant}} = O(K^2 n), \quad CE_{\mathrm{quant}} = O(K^2 n(n+K)M),$$

$$\mathrm{Time}_{\mathrm{quant}} = O(M(n+K)).$$

III. For the approximating solution of problem 3 (with precision ε).

$$\mathrm{Time}_{\mathrm{quant}} = O(M(n+\delta/\varepsilon\kappa)), \quad \mathrm{Time}_{\mathrm{det}} = O(Mn^4\varepsilon^{-1}),$$

$$CE_{\mathrm{quant}} = O(M(\delta/\varepsilon\kappa)^2 n(n+\delta/\varepsilon\kappa)), \quad CE_{\mathrm{det}} = O(n^4 M/\varepsilon),$$

where δ is a pixel size and $\kappa \propto 0.3d_b$.

REFERENCES

1. *Garey M. R., Johnson D. S.* Computers and Intractability. 1979.

2. *Papadimitriou C., Steiglitz K.* Combinatorial Optimization. 1982.