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THE LOGICAL GATES FROM BIPHOTONS

Yu. A. Asikritova [a], *I. D. Balatsky* [a], *V. N. Gorbachev* [a,b,1]

[a] Nor'Westerly Institute of Printing of St. Petersburg State University of Technology and Design, St. Petersburg, Russia

[b] Laboratory for Quantum Information & Computing, St. Petersburg State University of Aerospace Instrumentation, St. Petersburg, Russia

A set of deterministic gates based on the quantum measurement is considered. These gates exploit bipartite entanglement, allow performing any operations on the given qubit states and can be implemented from biphotons.

Рассматривается набор детерминистских ячеек на основе квантового измерения. Ячейки используют двухчастичную перепутанность, позволяют выполнять любые операции над заданными состояниями кубитов и могут быть реализованы на бифотонах.

INTRODUCTION

Along with unitary evolution, quantum measurement is a way of changing state of physical system and can be used as primitive for computation in two measurement-based models. First of the models is teleportation quantum computation (TQC) [1–3] with gates process on teleportation. Second is one-way quantum computer (1WQC) introduced by H. J. Briegel [4, 5], in which computation proceeds via local single-qubit measurements on the multiparticle entangled states, known as cluster or graph states. Relationship between these models has been studied by many authors, for example, by R. Jozsa [6]. Experimental implementations of 1WQC using four-qubit optical cluster states have been demonstrated by A. Zeilinger [7] and experimental analysis of cluster states has been made by H. Weinfurter [8].

In this work we consider a model of measurement-based gates from biphotons. Biphotons are well known in quantum optics and there is a significant progress in their generating and manipulating [9]. We focus on the next questions: 1) which is a structure of the gates from biphotons, 2) which of operations can be performed. We found that deterministic gates have to include a set of retrieval operators and they can perform any operation on given input states. The presented gates are scalable and differ from gates of the TQC and 1WQC models.

[1] E-mail: vn@vg3025.spb.edu

1. RESOURCES

1.1. Biphotons. Biphotons or pair of highly correlated photons are the main resource of our schemes. They can be generated in down conversion processes in which a photon of pump transforms into two photons according to the laws of conservation of energy and momentum. For our purposes we assume that the state of biphoton has the form of the maximally entangled Einstein–Podolsky–Rosen (EPR) pair:

$$\varphi = (1/\sqrt{2})(|00\rangle + |11\rangle), \tag{1}$$

where logical variables 0 and 1 can be associated with physical states of polarized light, say, vertically and horizontally polarized photon $|V\rangle, |H\rangle$, or with the Fock states with 0 and 1 photon. In experiment the state of biphoton is more complicated and for some regimes of generation it has the form $\phi = |\mathrm{vac}\rangle(1-\epsilon^2) + \epsilon(|HV\rangle + |VH\rangle + \dots)$, where ϵ^2 is efficiency of generation of biphoton. Here we see a large contribution of vacuum and a small portion of the desired correlated photons. The ideal state (1) we will consider is achieved after post selection. It means that biphoton is prepared with some probability given by ϵ^2. However, the rate of generation of biphotons may be high. In fact, let there be a pulsed-pump laser with pulse of 100 fs, repetition rate of 100 MHz and the average power 200 mW. Then probability of generation of pair ϵ^2 is about 10^{-4} or one pair per 10^4 pulses and the rate of generation of bihotons is 10^4 per second. This is a high rate and it is attractive for real quantum communications. In experimental implementation of 1QWC a four-photon cluster state $\psi = (1/2)(|HHHH\rangle + |HHVV\rangle + |VVHH\rangle + |VVVV\rangle)$ has been generated to proceed with the quantum search algorithm [7]. From the presented estimations it follows that the rate of generation of the cluster is about one state per second.

1.2. Measurement. The basic property of biphoton is a strong correlation between photons. Let us perform a measurement of observable $Z = \sigma_z$, where σ_z is Pauli operator. Operator Z has eigenstates $|0\rangle, |1\rangle$ known as computation basis and two eigenvalues ± 1 to be the measurement outcomes we will denote by 0 and 1. Considering the measurement of Z on a photon of biphoton (1), we find two outcomes 0 and 1 with equal probability 1/2. If outcome 0 or 1 arises, then the state of the remaining photon is $|0\rangle$ or $|1\rangle$. Consider two independent biphotons and the measurements of Z of a photon from each biphotons. We shall name the photons involved in measurement the working photons and we shall name the remaining two photons the input and output ones. So, there are two working photons. Four outcomes $00, 01, 10, 11$ result in the four states of the input I and output O photons $|00\rangle_{IO}, |01\rangle_{IO}, |10\rangle_{IO}, |11\rangle_{IO}$. We can ask a question of which of operations connects the states of I and O photons? It is clear that if outcome 00 or 11 arises, then the states of I and O photons are equal; therefore, they are connected by the unit operation 1, otherwise there is a NOT operation. The measurement can be considered as a way of transformation of state I photon into O photon and we find a gate. But this gate is not interesting for computation because of its probabilistic nature due from measurement. In fact, it performs the desired operation, say NOT with probability of 50%, otherwise there are unwanted outcomes. However, unwanted outcomes can be exploited if we will correct the state of output photon by a retrieval operator. For example, to perform NOT operation one needs a retrieval operator to exploit two unwanted outcomes $00, 11$. If we choose this operator as $X = \sigma_x$ the output state can be flipped, when unwanted outcomes arise. Then the desired operation is achieved for any outcomes and one finds a deterministic gate that performs NOT operation. As a result, for deterministic gate one needs retrieval operators. These operators are in QTC and 1WQC models.

2. GATES

2.1. Scheme of Gate. A one-qubit gate consists of two biphotons. There are two working photons W measured in Z basis and there are I and O photons, which are input and output of the gate, retrieval operator R manipulates the state of the output photon. The state of the gate and notation of R read

$$\begin{gathered}\varphi^{\otimes 2} = (1/2) \sum_{n,m=0,1} |n\rangle_I |m\rangle_O |nm\rangle_W, \\ R : R(n,m)|m\rangle = U_G|n\rangle,\end{gathered} \tag{2}$$

where U_G is a gate operation; n, m are variables of the first and second biphoton. The gate processes as follows. If outcome nm arises, then biphotons are projected into the state $|n\rangle_I |m\rangle_O = |n\rangle_I R^\dagger U_G |n\rangle_O$. When the retrieval operator R acts on the output photon, the state has the form $|n\rangle_I U_G |n\rangle_O$. It means that the gate performs operation $U_G : |n\rangle \to U_G|n\rangle$. Two points may be made about it: 1) the input and output of the gate can be spatially separated, 2) if $U_G = 1$, one finds teleportation.

2.2. Retrieval Operators. Retrieval operator R allows exploiting unwanted outcomes of the quantum measurement and results in deterministic gate. We found that R is a product of the form

$$R(n,m) = U_G X^{n+m}, \tag{3}$$

where the first operator X^{n+m} carries out teleportation of input state to output state and the second operator U_G performs the gate operation on the teleported state. The retrieval operator is a conditional unitary transformation because it depends on the label of the states n and m. It has the form similar to operator from QTC and 1WQC models. There is a simple optical implementation of the conditional X^p operation. This is a well-known Pockels cell. If a voltage is applied the Pockels cell transforms polarized photon H to V and vice versa.

2.3. Gates of QTC and 1WQC Models. There are two protocols for teleporting an unknown state of qubit. The first of them has been proposed by C. Bennet et al. [10]. This protocol accomplishes the task by the EPR channel and two bits of classical information gained from the Bell-state measurement. In the other teleportation protocol, proposed by D. Gottesman [1], unknown state of qubit is entangled with ancilla and teleportation is achieved by one bit of classical information gained from a single-particle measurement. Both protocols are exploited in the QTC model for the gates that perform operation U_G included in their retrieval operators, which have the form of (3).

A one-way gate, the basis of 1WQC introduced by Briegel [4], consists of three parts: input, working and output. At first a cluster state is prepared by entangling all qubits, then single-particle measurements perform on input and working qubits and the result of operation is stored in output qubits.

In contrast to the QTC and 1WQC models, our gates have no entanglement between input and output state. It results in the fact that our gates can perform operation U_G on a given input state only, which however is not destroyed.

2.4. Two-Qubit Conditional Gates. Due from linearity, the scheme given by (2) can be directly generalized into multiqubit gates. For example, consider two-qubit gates that perform two important conditional operations CNOT (controlled NOT) and C-Phase (controlled phase shift). They belong to a universal set of quantum logical operations. The gates consist of two output and two input biphotons in the state $\varphi^{\otimes 4}$. For measuring a single photon from each biphoton in Z basis we find the retrieval operators which allow deterministic operations in computational basis $|0\rangle, |1\rangle$. Let variables of input and output biphotons be n_1, n_2 and m_1, m_2, then the retrieval operators have the form

$$\begin{gathered}\text{CNOT:}|n_1 n_2\rangle \to |n_1, n_2 \oplus n_1\rangle,\ R = [1 \otimes X^{n_1}][X^{n_1+m_1} \otimes X^{n_2+m_2}],\\ \text{C-Phase:}|n_1 n_2\rangle \to (-1)^{n_1 n_2}|n_1, n_2\rangle,\ R = [1 \otimes Z^{n_1 n_2}][1 \otimes X^{n_1+m_1} \otimes X^{n_2+m_2}].\end{gathered} \tag{4}$$

From these equations it follows that the gate operations $U_G = X^{n_1}, Z^{n_1 n_2}$ are included in the retrieval operators in accordance with (3). Such a structure of R results in the fact that the measurement is in Z basis only.

2.5. Different Measurement Patterns. Several logical operations can be achieved by measuring in different bases. Measurement of Z of the working photon from biphoton projects the reminder photon into one of the eigenstates of Z, which belongs to computational basis. We can choose a new observable obtained from Z by a unitary transformation $S : A = SZS^\dagger$. Then the measurement of A on the working photon projects the reminder photon into a state which can be transformed from computational basis by a unitary operator K dependent on S. As a result, one can find that biphoton is an eigenvector of product K and S: $(K \otimes S)\varphi = \varphi$. Unitary operation S and K can be considered as a rotation of wave vector. Then any rotation of one of the photons from biphoton results in rotation of another photon because of its correlation. This is a key property to achieve the gates on different measurement patterns. For example, consider two biphotons, whose working photons are measured in Z and in $SZS^\dagger$ basis. Then we find that the gate performs operation $U_G = K : |n\rangle \to K|n\rangle$ and the retrieval operator has the form $KRK^\dagger$, where $R = X^{n+m}$ makes teleportation.

Note the main features of this gate. Because of the different measurement patterns the input state of photon is teleported into the output state up to unitary transformation U_G due from the measurement. Then for any given input states and for any desired gate operation, there are a set of measurements and retrieval operators that accomplish the task. This result can be directly generalized to multiqubit gates.

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