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ENTANGLEMENT IN THE SCHRÖDINGER EXPERIMENT

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The mixed and entanglement states have been analyzed in the Schrödinger experiment. It is known that in an open system the «Schrödinger cat» paradox is explained by the decoherence phenomenon, but in a closed system it is explained by the Everett–Wheeler many-world interpretation of quantum mechanics. The quantum real world can be presented as a complex multispatial geometric figure and the classical world is one of the faces of this figure. In this paper it is shown that this figure is the simplex that is well known in the functional analysis. Such an interpretation of quantum mechanics enables one to obtain the nonuniform wave equation, and Schrödinger equation is the uniform equation of this one. Perhaps this equation is the equation of subquantum world about which Einstein has written.

В эксперименте «Шредингеровский кот» рассматриваются смешанное и запутанное состояния. Известно, что в открытой системе парадокс шредингеровского кота разрешается явлением декогеренции. В замкнутой системе одним из способов разрешения этого парадокса является многомировая интерпретация квантовой механики Эверетта–Уиллера. Реальный квантовый мир может быть наглядно представлен как некая сложная объемная фигура, и иллюзорный классический мир есть одна из граней этой фигуры. В данной работе показано, что эта фигура есть симплекс из функционального анализа. Такая интерпретация квантовой физики позволяет получить неоднородное волновое уравнение, и уравнение Шредингера есть его однородное уравнение. По-видимому, это неоднородное уравнение есть уравнение субквантового мира, о котором все время говорил Эйнштейн.

1. SUPERPOSITION IN QUANTUM MECHANICS

It is known that by using imagination experiment «Schrödinger cat» [1] it is proved that the superposition of two microstates (atom decayed and atom didn't decay) transforms to the superposition of two macrostates (cat is alive and cat is dead). In the open system this paradox is solved by the «decoherence» [2]. In this case the entanglement state $C = A_1B_2 + A_2B_1$, where C is the state of all the system (cat and atom), A is the state of cat (A_1 is alive, A_2 is dead cat), B is the state of atom (B_1 is normal and B_2 is decay atom), becomes the mixed state $C = A_1B_1 + A_2B_2$. This transform (from entanglement to mixed state) is result of decoherence; i.e., decoupling of wave functions of observer and surrounding world takes place. It is clear that in mixed state the probability theory can be used. However, in the entanglement state the probability theory cannot be used because how one can say

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about the cat or atom state if cat is alive and dead or atom is normal and decays at the same time. In the closed system the «Schrödinger cat» paradox is solved by the theory in which the consciousness is included. One of such theories is Everett–Wheeler many-world interpretation of quantum mechanics of [3, 4]. M. B. Menskii [5] has represented the quantum world symbolically as some complex multispatial figure and what we call «a classical reality» is only one of the projections of this figure. In this scheme the quantum world is objective and real because it does not depend on consciousness of the observer. In this case the system is closed; i.e., our consciousness is inside volumetric figure. The objective real world exists in the form of the parallel worlds, each of which is not realer than the rest. In scheme the classical world is illusion because it depends on consciousness of the observer. In this case the system is open; i.e., our consciousness is outside volumetric figure. Being outside the volumetric figure, our consciousness is interacting with the surrounding world and in consequence the decoherence takes place. The picture of the world seen by us is the result of the coupling of wave functions of our consciousness and the surrounding world [6]. We always see only one of the parallel worlds, but other worlds do not cease to exist. Therefore, the classical world is only one of many variants and it arises in our consciousness. From the beginning of existence of the quantum mechanics the famous scientists Pauli [7], Wigner [8], Schrödinger [1] said about the necessity of inclusion of the observer's consciousness in the quantum theory of measurements.

Simplex. From the functional analysis [9] it is known that the sequence of points $\{x_{n+1}\}$ are in general provisions when these points are not in $(n-1)$-dimensional space. If these points are connected with each other, they form n-dimensional simplex. For example, one point — zero-dimensional simplex, the piece — one-dimensional one, the triangle — two-dimensional one, the tetrahedral — three-dimensional simplex, etc. It is known that if $x_1, x_2, \ldots, x_n$ points are in the general provisions, any $(k+1)$ points of them, where $k < n$, are also in the general provisions and form k-dimensional simplex named k-dimensional face of the given simplex. The number of k-dimensional faces of n-dimensional simplex is calculated by the combinatory formula $C_n^K = \dfrac{n!}{K!\,(n-K)!}$. For example, the three-dimensional simplex — the tetrahedral — has 4 two-dimensional faces (triangles), 6 one-dimensional faces (pieces) and 4 zero-dimensional faces (points). In total the sum of the faces equals 14. Let us consider the four-dimensional simplex. Here the number of the points are 5. All of them should not be located in the three-dimensional space. It is impossible to imagine such a figure. This four-dimensional simplex has 30 faces: 5 three-dimensional faces (tetrahedron), 10 two-dimensional faces (triangles), 10 one-dimensional faces (pieces) and 5 zero-dimensional faces (points). Thus, the simplex formed from more than four points cannot be presented in our three-dimensional space. It is the complex multispatial figure. The simplex in n-dimensional space is the minimal convex set; i.e., all points of a kind $\Sigma a_n x_n$, where $\Sigma a_n = 1$, belong to this simplex. From the theory of probabilities [10] it is known that the probability of event is closely connected to random and average value. The points belonging to a simplex are the set of all average values if we take that the tops of the simplex $\{x_n\}$ are the random value and a_n are probabilities of x_n. Thus, the physical interpretation of simplex is the following. The simplex is minimum corps which can embrace all events. From this point of view the consideration of a simplex is expedient.

2. CONSTRUCTION OF THE SIMPLEX

Apparently the above-mentioned complex volumetric figure is the simplex. Let us imagine that on two-coordinate plane $x0y$, on an axis x the number of dead «Schrödinger cats» and on an axis y the number of alive cats is marked. Let us suppose in experiment with 100 «Schrödinger cats» 80 cats are alive and 20 cats are dead. There may be different numbers of alive and dead cats from the total number of all cats. But we consider numbers 100, 80, 20. In this case the probabilities are approximately equal to 0.8 (alive cat) and 0.2 (dead cat). The points 20 and 80 are two tops of the simplex. In another case or at another moment of time there are 60 alive cats and 40 dead cats (100 «Schrödinger cats» are always considered). We put these points in another system of coordinates $x^1 0 y^1$ in three-dimensional space. If we connect given four points, then we obtain three-dimensional simplex — tetrahedral. If we consider a lot of points, we obtain the complex volumetric figure — n-dimensional simplex. Tetrahedral is the final simplex that we can represent in our three-dimensional space. The simplex of higher order has faces taken from this tetrahedron. The ribs of the tetrahedral indicate various probabilities. For example, the rib linking the points of 80 alive cats and 40 dead cats points to $80/120 = 2/3$ of probability of the case that cat is alive. In the case of 60 alive and 20 dead cats the rib of the simplex shows the probability that is equal to $60/80 = 3/4$ and so forth. The rib linking the points of 20 dead and 40 dead cats and the rib linking the points of 80 alive and 60 alive cats point out the probability that equals 1. Let us consider the faces of the simplex. In the case of alive cat on one of them the probability changes from 2/3 to 0.8; on another face — from 3/4 to 0.6; on the third — from 2/3 up to 0.6; on the fourth — from 3/4 to 0.8, etc. The simplex constructed by us differs from the simplex discussed above in Sec. 1 (let us discriminate between simplex and constructed simplex). The tops of the above-mentioned simplex were random numbers of the events that happened solely, whereas the tops of the constructed simplex are random numbers of both the events that happened (dead cat) and those that did not happen (alive cat) (the total number of viewed events is constant). It is obvious that the simplex is the face of the constructed simplex. One can say that the simplex is the simplex of classical world but the constructed simplex is the simplex of real quantum world. For example, points of 20 and 40 dead cats (Fig. 1) of classical world are points realized in experiment, in our life and, therefore, they are hard points. In the classical probability theory there is an indicator I of one event (called elementary event) that can be 0 (event is not realized) or 1 (event occurred) [10]. The points (20, 40) are hard because $I = 1$. The classical world is the world of realized events and, therefore, the statistics of such events takes place. The quantum world differs from the classical world. This world is the world of events which are not realized yet and it is not known how these events will be realized. Therefore, in the quantum mechanics there is an indicator too, but called wave function, that equals not only 0 or 1 but any value from interval [0,1].

3. THE DIRECTING COSINES AND THE PROBABILITIES

From the quantum mechanics [11] the following is known. Let us assume that e is the vector of statement system and vectors e_k are eigenvectors (the basis). Then $(e_k, e) = |e_k| \cdot |e| \cdot \cos\beta$, where $\cos^2\beta$ is the part of e corresponding to e_k. From the physical point of

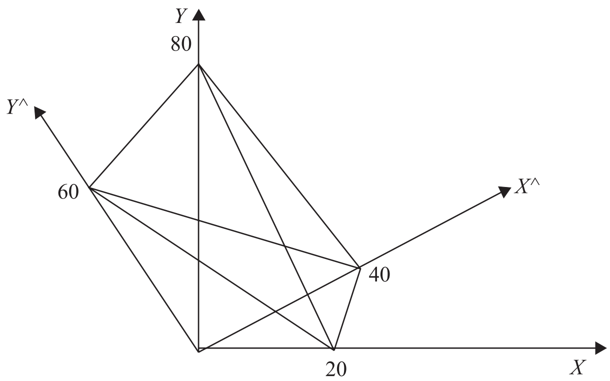

Fig. 1

view this means that if we measure the energy of the identical systems that are in the e state, then the part of the number of E_k energy measurements equals $\cos\beta$ (more exactly, it equals $|\cos\beta|$). Therefore, $|\cos\beta|$ is called directing cosine and it is connected with the probability that the system has E_k energy in the state e. It is clear that $\cos\beta$ is wave function ψ of system. It is known that for many operators $\cos\beta$ is the complex number $\cos\beta = x + iy = A\,e^{i\alpha}$, where $A = \text{const}$ is the length of the vector $\cos\beta$, α is any real number. It is known that wave function $\psi = A\,e^{i\alpha}$. One can see that ψ is determined to within constant phase factor $e^{i\alpha}$. This ambiguity is of principle and cannot be removed by us. But in quantum mechanics it is considered that it is immaterial because it does not influence any physical result. This is true because not the wave function but the square of the wave function amplitude $|\psi|^2$ is the probability p of result, i.e., $p = |\psi|^2 = \psi\psi^* = \cos^2\beta = x^2 + y^2 = A^2$. Thus, in quantum mechanics the angle α is not examined. Here it is possible to see the principal moments of quantum mechanics. In quantum mechanics, since the angle α is not examined, only one state U of physical system is considered. However, it is possible only in classical world because there is only one state of system that was realized as a result of decoherence. In quantum world there is Heisenberg's uncertainty principle. Therefore, it is necessary to give not only one state U, but many states $U_1, U_2, \ldots$ For this purpose in quantum mechanics the linear operator H is introduced [11] and any state of physical system is presented as $U_k = HU$. Therefore, in the quantum mechanics not only one state U presented by one vector, as in classical physics, is considered but many states $U_1, U_2, \ldots$ presented by spheres of different radius. In this case not the angle α, but the length of radius ($A^2 = \cos^2\beta = p$) of sphere seems interesting. In this paper the angle α and its meaning will be discussed.

4. ANGLE α IN QUANTUM MECHANICS

Using the mentioned simplex, the above can be imagined and presented as follows.

In Fig. 2 let us assume that e_1 and e_2 are vectors on which the states of atom (normal and decayed) and of course of cat (alive and dead) are presented. It is significant we are not viewing the energy value corresponding to these states (E_1 is the energy of decayed atom, E_2 is the energy of normal atom). Thus, on the axes e_1, e_2 there are states of atom and cat (normal and decayed, respectively). The statement of physical system and uncertainty of information are presented by vector e and angle α that is formed by rotation of e on the plane $e_1 0 e_2$, respectively (Fig. 2).

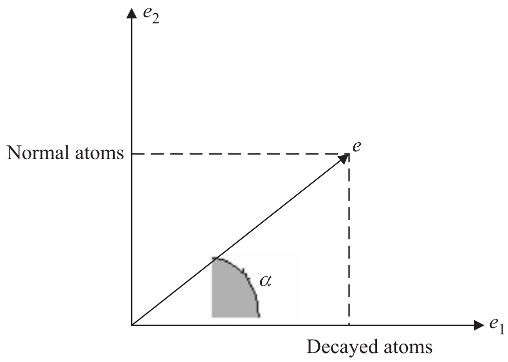

Fig. 2

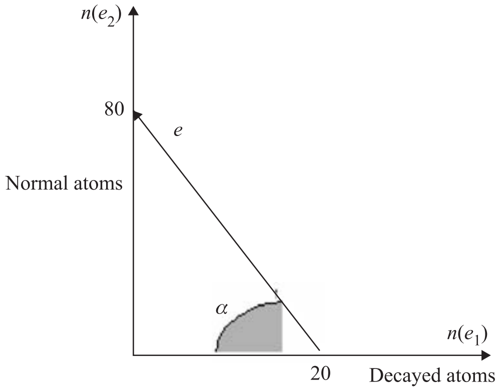

Fig. 3

Note that in paper [12] the statements of physical system are pictured by unit rays. To one state there corresponds one ray. The rotation of unit vector was considered by Y. F. Orlov [13] and called by him «the intention» of the quantum system. Thus vector e is the ray and angle α is the measure of uncertainty. The vector e is non-eigenvector and it may be written in the form of superposition of vectors e_1 and e_2, i.e., $\mathbf{e} = C_1\mathbf{e}_1 + C_2\mathbf{e}_2$. The projections of e on e_1 and e_2 axes are the numbers of decayed and normal atoms, respectively, and $\cos\alpha$ is the part of decayed atoms and $\sin\alpha$ ($\sin\alpha = \cos(90 - \alpha)$) is part of normal atoms of total number of all atoms. The exact information about the amounts of normal or decayed atoms is presented on the $n(e_1)$ and $n(e_2)$ axes in Fig. 3. In this figure 20 and 80 numbers are taken from constructed simplex (Fig. 1). In Fig. 3 let us assume that e_1 is a real axis and e_2 is an imaginary axis of a complex number. It is clear that probabilities are equal to $p_1 = \dfrac{a}{a+b}$ (cat is alive) and $p_2 = \dfrac{b}{a+b}$ (cat is dead). It is the main point that $p_1 + p_2 = 1$.

But it is possible to take also another measure p^* for which the equality $\Sigma p^* = 1$ will be held true. It is known that $\sin^2\alpha + \cos^2\alpha = 1$. Assume $\cos^2\alpha = p_1^*$ and $\sin^2\alpha = p_2^*$. Here p_1^* and p_2^* are other measures which we will call «the new probabilities». From Fig. 3 it is clear that

$$p_1^* = \frac{a^2}{a^2 + b^2} = \frac{a}{a+ib}\frac{a}{a-ib} \text{ (cat is alive)},$$

and

$$p_2^* = \frac{b^2}{a^2 + b^2} = \frac{ib}{a+ib}\frac{-ib}{a-ib} \text{ (cat is dead)}.$$

In quantum mechanics the wave function ψ is interpreted as follows. The square of wave function amplitude $|\psi|^2$ is the probability p that the particle is in state E. We suppose that in our consideration $|\psi|^2$ is not the probability p, but that is «the new probability p^*». Thus,

$$|\psi|_1^2 = \frac{a}{a+ib}\frac{a}{a-ib}. \text{ Then } \psi_1 = \frac{a}{a+ib} = \cos\alpha\, \mathrm{e}^{-i\alpha} \text{ and } \psi_1^* = \frac{a}{a-ib} = \cos\alpha\, \mathrm{e}^{i\alpha};$$

$|\psi|_2^2 = \dfrac{bi}{a+bi}\dfrac{-bi}{a-bi}$. Then $\psi_2 = \dfrac{bi}{a+bi} = \sin\alpha\,\mathrm{e}^{-i\alpha}$ and $\psi_2^* = \dfrac{-bi}{a-bi} = -\sin\alpha\,\mathrm{e}^{i\alpha}$.

So we suppose that in quantum mechanics the directional cosines can be presented not in the form $\cos\alpha = a+ib$, but as $\cos\alpha = \dfrac{a}{a+ib}$. Taking $\cos\alpha = a+ib$ and $\cos^2\alpha = a^2+b^2$, we cannot see that $\cos^2\alpha$ is the part of the total amount; on the contrary, taking $\cos\alpha = \dfrac{a}{a+ib}$ we can see that $\cos^2\alpha$ $\left(\cos^2\alpha = \dfrac{a^2}{a^2+b^2}\right)$ is the part of the total amount. So the rubs of our simplex accordingly point to the different statement of physical system and probabilities. In this paper the original hypothesis can seem wrong because, as some physicists consider wrongly, in quantum mechanics states form a sphere, but not a simplex as in the present paper. As is well known, the quantum mechanics is physics of one state, of one measure because the statistics for this state and then, as stated above, by means of the linear operator, the statistics of other states have been established. In quantum mechanics this one state can be presented as sphere. Note that the state in classical physics is presented as point. In this paper it has been shown that many states of physical system after action of the operator form simplex. Of course, it will be better to consider the simplex with unit rib inside unit sphere. Of most importance for us are the angles of inclination of ribs of the simplex.

5. THE SCHRÖDINGER EQUATION AND NEW WAVE EQUATION

It is known that Schrödinger equation $\dfrac{h^2}{2m}\Delta\psi - E\psi = 0$ (1) (whose solution $\psi = A\exp\left(-\dfrac{i}{h}(Et-px)\right)$) cannot be derived and it was obtained intuitively in order to explain the strange properties of the microscopic world. In this paper it has been shown that Schrödinger equation has been derived by using the above-mentioned geometrical interpretation of quantum mechanics. As was shown in Sec. 3, $\psi = \cos\alpha\,\mathrm{e}^{-i\alpha}$ (2). Then it is easy to derive the following equation: $\Delta'\psi + 4\psi = 2$. Here $\Delta' = \dfrac{\partial^2}{\partial^2\alpha}$. However, comparing $\psi = A\exp\left(-\dfrac{i}{h}(Et-px)\right)$ with $\psi = \cos\alpha\,\mathrm{e}^{-i\alpha}$, we can write $\alpha = \dfrac{Et-px}{h}$ (3). In the stationary case $\alpha = \alpha(x)$. Then $\Delta = \dfrac{\partial^2}{\partial\alpha^2} = \dfrac{h^2}{p^2}\dfrac{\partial^2}{\partial x^2}$. But $p^2 = m^2v^2 = 2m\dfrac{mv^2}{2} = 2mE$. Therefore, $\Delta = \dfrac{\partial^2}{\partial\alpha^2} = \dfrac{h^2}{2mE}\dfrac{\partial^2}{\partial x^2}$. Thus, $\dfrac{h^2}{2mE}\dfrac{\partial^2}{\partial x^2}\psi + 4\psi = 2$ or $\dfrac{h^2}{2m}\Delta\psi + 4E\psi = 2$ (4). In order to solve this nonuniform differential equation, we should solve the corresponding uniform equation $\dfrac{h^2}{2m}\Delta\psi + 4E\psi = 0$ (5), which is Schrödinger equation (Eq. (1)), if the factor 4 substitutes for -1. From the theory of differential equations it is known that the general solution of Eq. (4) is equal to the solution of Eq. (5) plus one partial solution of Eq. (4) which, taking into account the expression (3), is in the given case $\cos\alpha\,\mathrm{e}^{-i\alpha} = \cos\dfrac{Et-px}{h}\exp\left(-i\dfrac{Et-px}{h}\right)$. Thus, the general solution of Eq. (4) has the form $\psi = A\exp\left(-i\dfrac{Et-px}{h}\right) + \cos\dfrac{Et-px}{h}\exp\left(-i\dfrac{Et-px}{h}\right)$. One can see that this equation consists of two parts, i.e., Schrödinger equation plus uncertainty. In his works Planck often wrote about different worlds: real world, classic world and world of physical science [14]. One can say that if Schrödinger equation deals with the world of physical science, then this equation deals with the real world. Most likely this equation deals with the latent (secret) parameters of Einstein subquantum world [15, 16].

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