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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

STABILITY OF YANG–MILLS FIELDS SYSTEM IN THE HOMOGENEOUS (ANTI-)SELF-DUAL BACKGROUND FIELD

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Stability of Yang–Mills fields system in the background field is investigated based on the Toda criterion, Poincare sections and the values of the maximal Lyapunov exponents. The existence of the region of regular motion at low densities of energy is demonstrated. Critical energy density of the order–chaos transition is analyzed for different values of the model parameter.

Исследована стабильность системы полей Янга–Миллса в фоновом поле на основании критерия Тода, сечений Пуанкаре и максимальных показателей Ляпунова. Установлено существование области регулярного движения при низких плотностях энергии. Проанализирована зависимость критической плотности энергии перехода от порядка к хаосу от величины модельного параметра.

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INTRODUCTION

In contrast to electrodynamics, the dynamics of Yang–Mills fields is inherently nonlinear and chaotic at any density of energy. This assumption was confirmed analytically and numerically [1–3]. Further analysis of spatially homogeneous field configurations [4] showed that inclusion of Higgs field leads to order–chaos transition at some density of energy of classical gauge fields [5–7]. Classical Higgs field regularizes chaotic dynamics of classical gauge fields below critical energy density and leads to the emergence of order–chaos transition.

Chaos in Yang–Mills fields [8] and vacuum state instability in nonperturbative QCD models [9–11] are also considered in connection with confinement. It has also been shown recently that interaction of the constant chromomagnetic field with axial field could generate confinement [12]. These results indicate the importance of nonperturbative background fields.

In our previous paper [13] we have investigated the stability of Yang–Mills–Higgs fields and described analytically the regions of chaotic and stable motion. In this work Yang–Mills fields are considered on the background of the homogeneous (anti-)self-dual field [14]. As the dynamics of arbitrary field configurations is too complicated, we follow [15] and reduce our model to spatially homogeneous fields which depend on time. After that we are left with

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only finite number of degrees of freedom (two in our case) which allows us to investigate the dynamics of the system using conventional methods developed for mechanical systems.

In this work one more mechanism of the chaos suppression in the Yang–Mills fields models is proposed. Homogeneous (anti-)self-dual field eliminates chaoticity of Yang–Mills dynamics below critical energy density.

1. HOMOGENEOUS (ANTI-)SELF-DUAL FIELD

In this paper the classical dynamics of *SU*(2) model gauge fields system is considered on the background of the homogeneous (anti-)self-dual field. Various properties of this solution of the Yang–Mills equations in *SU*(2) theory were investigated originally by other authors [9, 16–18]. It was demonstrated that self-dual homogeneous field provides the Wilson confinement criterion [19]. Therefore, this field is at least a possible source of confinement in QCD if it is a dominant configuration in the QCD functional integral.

Homogeneous self-dual field is defined by the following expressions [14]:

$$B^a_\mu = Bn^a b_{\mu\nu} x_\nu,$$

$$F^a_{\mu\nu} = -2Bn^a b_{\mu\nu},$$

where B is value of the field strength, vector n^a and tensor $b_{\mu\nu}$ characterize the direction of the field, respectively, in color space and in space-time. The latter has the following properties:

$$b_{\mu\nu} = -b_{\nu\mu}, \quad b_{\mu\nu} b_{\mu\rho} = \delta_{\nu\rho}, \quad \widetilde{b}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta} b_{\alpha\beta} = \pm b_{\mu\nu},$$

where positive and negative signs in the last expression correspond, respectively, to self-dual and anti-self-dual cases.

As the directions of the background field in color space and in space-time can be chosen arbitrarily, we will assume that the gauge field has color components $(n^1, n^2, n^3) = (0, 0, 1)$ and space-time components $\mathbf{B} = (B_1, B_2, B_3) = (0, 0, B)$.

2. MODEL POTENTIAL OF THE SYSTEM

The Lagrangian of *SU*(2) gauge theory in Euclidean metrics is

$$L = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu},$$

where $G^a_{\mu\nu}$ is a field tensor which is of the following form:

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc} A^b_\mu A^c_\nu.$$

In the last expression $A^a_\mu, a = 1, 2, 3$ are the three non-Abelian Yang–Mills fields and g denotes the coupling constant of these fields.

We consider the fluctuations around background homogeneous self-dual field. Self-dual field is regarded as external one and it is taken into account by substituting modified vector potential in the Yang–Mills Lagrangian:

$$A^a_\mu \to A^a_\mu + B^a_\mu,$$

where A^a_μ is the fluctuation to the background field B^a_μ.

We use the gauge

$$A^a_4 = 0,$$

and consider spatially homogeneous field configurations [15]

$$\partial_i A^a_\mu = 0, \quad i = 1, \ldots, 3.$$

Our model of Yang–Mills fields in (anti-)self-dual field is constructed in Euclidean space. In order to analyze the model by using analytical and numerical methods, we should switch to Minkowski space. We consider chromomagnetic model. Thus, we put chromoelectric field is equal to zero. If $A^1_1 = q_1$, $A^2_2 = q_2$ and the other components of the perturbative Yang–Mills fields are equal to zero, the potential of the model is

$$V = \frac{1}{2} g^2 q_1^2 q_2^2 + \frac{1}{2} H^2 - gH q_1 q_2 + \frac{1}{8} g^2 H^2 (x^2 q_1^2 + y^2 q_2^2), \tag{1}$$

where H is chromomagnetic background field strength, x and y are coordinates which play the role of the parameters, q_1 and q_2 are field variables.

3. STABILITY OF THE MODEL

3.1. Toda Criterion. At first, stability of the model is investigated using well-known technique based on the Toda criterion of local instability [20, 21] which allows us to obtain the value of the critical energy density of order–chaos transition in the system. This energy and minimum of the energy as functions of the model parameter $s = gHxy$ are shown in Fig. 1.

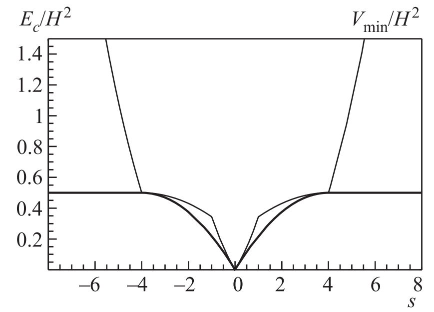

Fig. 1. Critical energy density of order–chaos transition (thin line) and minimum of the energy (thick line) as functions of the model parameter $s = gHxy$

Critical and minimal energies are close to each other for $s \in (-4, 4)$. This behavior indicates the absence of the region of regular motion in the system. In other case ($s \in (-\infty, -4)$ or $s \in (4, \infty)$), the critical energy density is much larger than the minimal one and the system is regular up to this energy. These results will be checked using numerical methods in the next subsection.

3.2. Numerical Calculations. The system is investigated using Poincare sections and Lyapunov exponents for a wide range of model parameter values. These numerical methods could indicate global regular regimes of motion, whereas the Toda criterion reveals only the local chaotic properties of the trajectories [22]. Thus, numerical methods are more precise for stability analysis.

Results of the numerical calculations for the system with model parameter $s = 0$ are shown in Figs. 2 and 3. Contrary to the Toda criterion, the system is regular at small energies below the energy of the background field $E_c = E_{\text{vac}} = \dfrac{1}{2}H^2$. There are only regular regimes of motion for small energies (e.g., Figs. 2, a and 3, a) and two types of motion for energies above E_c (Figs. 2, b and 3, b).

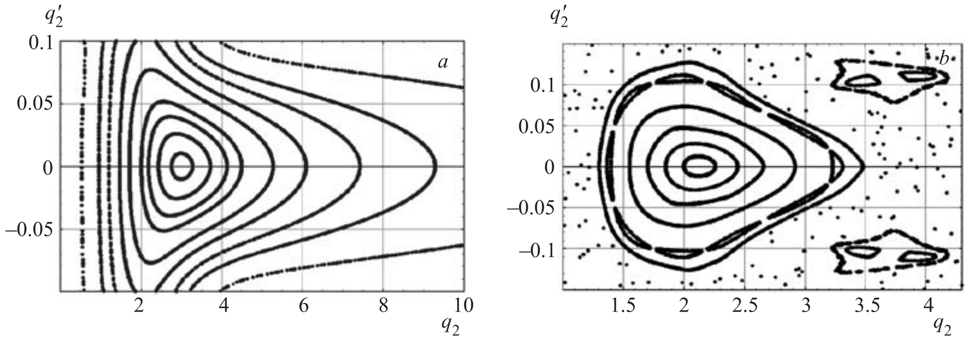

Fig. 2. Poincare sections for two-dimensional Yang–Mills system in the background field for $s = 0$, $H = 1$, $E = 0.005$ (a) and $E = 0.15$ (b)

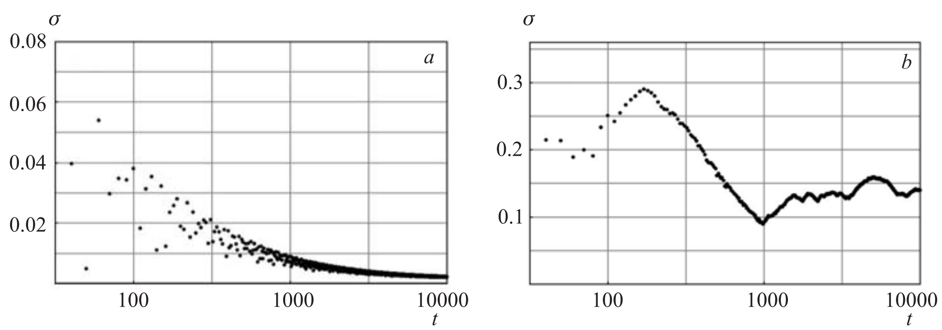

Fig. 3. Maximal Lyapunov exponents for two-dimensional Yang–Mills system in the background field for $s = 0$, $H = 1$, $E = 0.15$ (a) and $E = 0.68$ (b)

We have the following Poincare sections (Fig. 4) for large model parameter values. It is seen that the system is fully regular (Fig. 4, a) for high values of energy ($E_{\rm vac} \ll E < E_c$) as it was revealed by the Toda criterion. All trajectories have zero maximal Lyapunov exponents for this case. Two types of trajectories (chaotic and regular) are present in the system with energies above the critical one (Fig. 4, b).

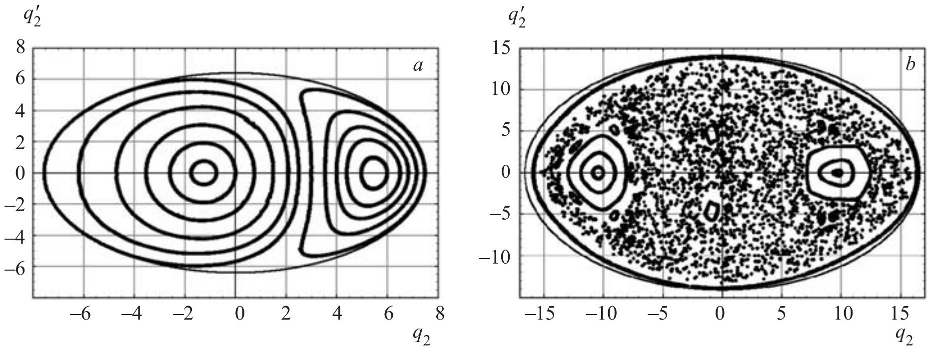

Fig. 4. Poincare sections for two-dimensional Yang–Mills system in the background field for $s = 25.5$, $g = 0.1$, $H = 1$, $x = 15$, $y = 17$, $E = 21$ (a) and $E = 100$ (b). Thin line is the border of the phase space

It is seen that the Toda criterion describes rather well the region of the large values of model parameter s and fails for $s \in (-4, 4)$.

Numerical calculations have shown that there is a region of regular motion at low densities of energy in our system at any value of the model parameter. Therefore, homogeneous (anti-)self-dual field regularizes chaotic dynamics of the Yang–Mills fields system.

CONCLUSIONS

The existence of the nonperturbative component of the Yang–Mills field is crucial for the confinement phenomenon. On the other hand, the classical dynamics of the Yang–Mills field is chaotic at any density of energy in the absence of background fields and can be regularized only if some other fields, for example, the Higgs field, are included in the model.

In this work we have demonstrated that the homogeneous (anti-)self-dual background field has similar properties. The Yang–Mills field on such a background has the region of regular motion at low densities of energy. There is an order–chaos transition in the system at any values of model parameters. The critical density of energy of this transition is equal to the background field energy for small parameters and is much larger for large parameter values.

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