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THE CRITICAL PROPERTIES OF THE AGENT-BASED MODEL WITH ENVIRONMENTAL-ECONOMIC INTERACTIONS

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The steady-state and nonequilibrium properties of the model of environmental-economic interactions are studied. The interacting heterogeneous agents are simulated on the platform of the emission dynamics of cellular automaton. The diffusive emissions are produced by the factory agents and the local pollution is monitored by the randomly walking (mobile) sensors. When the threshold concentration is exceeded, a feedback signal is transmitted from the sensor to the nearest factory that affects its actual production rate. The model predicts the discontinuous phase transition between safe and catastrophic ecology. Right at the critical line, the broad-scale power-law distributions of emission rates have been identified. The power-law fluctuations are triggered by the screening effect of factories and by the time delay between the environment contamination and its detection. The system shows the typical signs of the self-organized critical systems, such as power-law distributions and scaling laws.

Исследуются установившиеся и неравновесные свойства модели взаимосвязи экономики и окружающей среды. Взаимодействие гетерогенных агентов моделируется на основе динамики выбросов в клеточных автоматах. Диффузные выбросы производятся фабриками, и локальное загрязнение мониторируется случайно перемещающимися (мобильными) сенсорами. Когда пороговая концентрация превышена, сигнал от сенсора передается на ближайшую фабрику, что влияет на ее текущую производительность. Модель предсказывает дискретный фазовый переход от безопасного к катастрофическому состоянию экологии. Непосредственно на критической линии обнаруживаются широкомасштабные степенные распределения мощности выбросов. Степенные флуктуации обусловлены эффектом экранирования фабрик и временными задержками между моментом загрязнения среды и его регистрацией. Система демонстрирует типичные признаки самоорганизующихся систем, а именно степенные распределения и законы скейлинга.

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INTRODUCTION

On the onset of global ecological crisis questions related to regulation technologies and safe handling of environmental resources become urgent. Governments are exposed to cross-fire of economic interests and ecological goals of society. In the present paper this problem is discussed from the viewpoint of complex feedback architecture. In general, the nonlinear feedback is one of the up-bottom mechanisms that enable to keep control of complex systems. The mechanism has been exploited in many different systems. Stabilization of dynamical regimes via feedback appears in the simulations of investments Ref. [1]. According to Ref. [2] the feedback principle plays a crucial role in the real-time control of the semiconductor laser systems. As stated in Ref. [3] the steady *self-organized critical* (SOC) regime Ref. [4] implies the operation of inherent feedback mechanism that ensures the steady state is marginally stable against disturbances. According to Ref. [5] the nonlinear feedback allows one to transform the models with «unstable» phase transitions (i.e., classic models) to SOC models. It is reasonable to propound a question whether the contamination control drives the environment to a critical regime or it is able to stabilize the global pollution rate. Certainly, there are many ways how the feedback paradigm can be incorporated into ecological models. As an example, which is related to our present work, can serve the multi-country feedback model of the international emission control Ref. [6]. The cited paper demonstrates a paradox how the cooperative optimization of the global concentration of pollutants may lead to a more intensive environmental contamination than the individualistic strategies of firms or countries. Locale concentration threshold of pollution emphasized in our model is analogous to that in Ref. [7], where the feedback operation is mediated by chemical sensors with specific thresholds for different pollutants. In this work we study a model where the feedback is distributed over the population of autonomous sensing mobile agents. As usual, the thinking about distributed control is motivated by the expected robustness against accidental failures. The idea of walking sensorial agents in our model is analogous to the distributed pollution sensing by pigeon bloggers proposed by de Costa Ref. [8], where the pigeon agents are equipped by communication chips and sensors for carbon monoxide.

THE MODEL

To emphasize the frustration effects of ecological policies and economical development it is indispensable to adapt some simplifications. In this section we present a model consisting of a population of heterogeneous agents. The first point we consider is the dynamics of diffusive emissions emitted by sources positioned on different lattice sites. To accumulate large statistics we faced the problem of sufficiently fast simulation. This problem implies the implementation of cellular automation (CA) substrate based on integer rules. This suggestion is partially inspired by the lattice gas models of pollutants Ref. [9]. However, we preferred a more macroscopic formulation where concentration field varies via the recursive rule

$$m(t+1,\mathbf{R}) = m(t,\mathbf{R}) + I_{\rm nn}(t,\mathbf{R}) + I_F(t,\mathbf{R}) \tag{1}$$

that replaces in one time step t the $m(t,\mathbf{R})$ «integer pieces» of local concentration of the emissions at the position $\mathbf{R} \equiv [R_x, R_y]$ on the square sample $L \times L$. The emission current caused by the factory agents $I_{\rm F}$ is discussed in below. The current between the nearest neighbor cells is described by the equation

$$I_{\rm nn}(t,\mathbf{R}) \equiv -4\left[\frac{m(t,\mathbf{R})}{5}\right]_{\rm int} + \sum_{\mathbf{r}\in{\rm nn}(\mathbf{R})} \left[\frac{m(t,\mathbf{r})}{5}\right]_{\rm int}, \tag{2}$$

where the brackets $[\ldots]_{\rm int}$ denote an integer part of argument and $\sum_{{\rm nn}(\mathbf{R})}$ refers to the four nearest neighbor sites of $\mathbf{R}$ on the square lattice. In the case $I_{\rm nn} = 0$ the inflow from the four neighboring cells $\sum_{\mathbf{r}\in{\rm nn}(\mathbf{R})} [m(t,\mathbf{r})/5]_{\rm int}$ is balanced by the cell outflow $-4[m(t,\mathbf{R})/5]_{\rm int}$.

For the opened boundaries (where $\mathbf{R}$ is from the outer region of $L \times L$ square) it is assumed $m(t, \mathbf{R}) = 0$. For $m(t, \mathbf{R}) \gg 1$ the mass equidistribution of the emissions becomes satisfied with $1/\sqrt{m}$ order (as a consequence of central limit theorem). Under these conditions, $I_{\rm nn}(t, \mathbf{R})$ acts as 2D Laplace of m, and thus CA rules describe the diffusive transport. No natural purification is assumed, the m-cell content is exclusively depleted by the outflow through opened bounds. The local concentration of emissions is permanently monitored by the $\mathbf{S}$-type *sensing* agents. The random walk rule ensures that the sensorial system is able to reach any lattice site independently of the system size. Mathematically, the feedback regulation is an integration of local pollution levels that affects the emission rates. The lattice position of random walker $\mathbf{S}(t, j) \equiv [S_x(t, j), S_y(t, j)]$ is given by $\mathbf{S}(t, j) = \mathbf{S}(t-1, j) + \mathbf{s}(t, j)$, where the shift $s(t, j)$ is chosen randomly from the set of four unit vectors $\{(0, 1), (1, 0), (-1, 0), (0, -1)\}$. The periodic boundary conditions are assumed for $\mathbf{S}(t, j)$. The sources of emissions (factory agents) (see Eq. (1)) $\mathbf{F}(t, k)$, $k = 1, 2, \ldots, N_F$ are localized at the random positions $\mathbf{F}(t, k) \equiv [F_x(t, k), F_y(t, k)]$. Their production needs are unlimited. The specific condition of the feedback system is that over the predetermined threshold $m(t, \mathbf{S}(t, j)) > m_c$, the agent $\mathbf{S}(t, j)$ communicates with the nearest (in the Euclidean sense) $k_{\rm near}^{(j)}$ factory. The request of $\mathbf{S}(t, j)$ instantly decreases the local emission rate $n(t, k_{\rm near}^{(j)})$ by the constant additive factor $\Delta > 0$. Without request (to satisfy economic needs) the production rate is increased automatically. Both alternatives of the rate change are described by the equation

$$n(t+1, k_{\rm near}^{(j)}) = \begin{cases} n(t, k_{\rm near}^{(j)}) + \Delta & \text{when no signal from } \mathbf{S}\text{(t,j) is received,} \\ n(t, k_{\rm near}^{(j)}) - \Delta & \text{signal received.} \end{cases} \tag{3}$$

The eventuality $n(t, k_{\rm near}^{(j)}) < \Delta$ yields the bankruptcy of $\mathbf{F}(t, k_{\rm near}^{(j)})$ and the appearance of a new factory (again with index $k_{\rm near}^{(j)}$) at random position within the lattice space with initial production $n(t, k_{\rm near}^{(j)}) = 0$. What remains unspecified in Eq. (1) is the emission production of $\mathbf{F}$ agents $I_F(t, \mathbf{R}) = n(t, k)\, \delta_{\mathbf{R}, \mathbf{F}(t,k)}$. It is worth noting that all the agent systems in common with CA subsystem are updated synchronously. The sharing of the same positions by several $\mathbf{S}$-type agents is allowed, but disallowed for the $\mathbf{F}$-type agents.

SIMULATIONS

The simulations have been performed for lattices $L = 20, 30, 40, 50$. To avoid the influence of transients, the initial 10^5 updates per site were discarded; about $\tau = 5 \cdot 10^7$ updates have been used to collect information about the steady-state for the fixed parameters $m_c = 5 \gg \Delta = 1$, $N_F = L^2/100$. In the world with large number of sensors N_S the emissions fluctuate around m_c. This oversaturated population of sensors prevents from unbearable contamination, but yields a non-efficient economy. Much different is the situation for small N_S, where the steady-state regime becomes unstable since the rate of emissions diverges in time. The dynamics corresponds to catastrophic regime. The outcome of our simulation experience is the decision to accumulate the statistics of those factories that have received at least one feedback request during the step t

$$\phi(t) = \frac{1}{N_F} \sum_{k=1}^{N_F} \sigma(t, k), \quad \langle \phi \rangle = \frac{1}{\tau} \sum_{t=0}^{\tau} \phi(t), \tag{4}$$

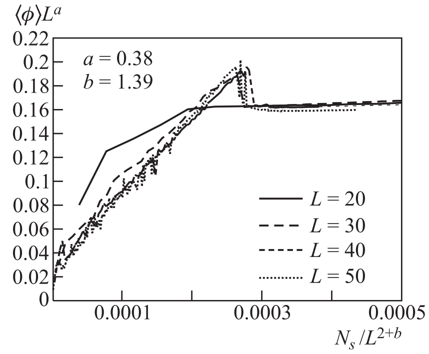

Fig. 1. Scaling of the averaged order parameter $\langle\phi\rangle$ for different L with fixed density of factories. The exceptional point, where $\langle\phi\rangle$ attains jump, is interpreted as a discontinuous transition from the safe to destructive economy

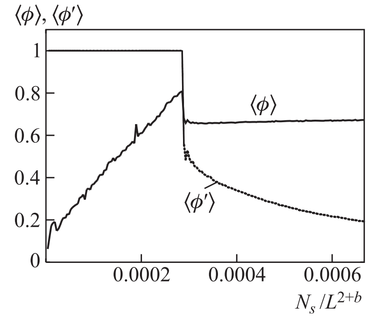

Fig. 2. The comparison of the two order parameters for $L = 40$. The difference $\langle\phi'\rangle - \langle\phi\rangle$ reflects the impact of screening effect

where $\langle\phi\rangle$ is the temporal average calculated over τ periods. If source $\mathbf{F}(t,k)$ received a feedback signal, $\sigma(t,k)$ is unity and zero otherwise. Figure 1 depicts $\langle\phi\rangle$ for different combinations of L and N_S. It has been checked numerically that data collapse onto the single finite-size scaling form $\langle\phi\rangle = L^{-a} f(\theta)$, $\theta = N_S/L^{2+b}$ with exponents $a = 0.38$, $b = 1.39$.

The algebraic combination $\theta_{\rm crit} = N_S/L^{2+b}$, where the scaling function $f(\ldots)$ attains abrupt jump $f(\theta_{\rm crit})$ and $\langle\phi\rangle$ tends to the L^{-a} asymptotics, is interpreted as critical point. For given ρ_F we have estimated $\theta_{\rm crit} \simeq 2.8 \cdot 10^{-4}$. The inability of sensorial system to limit the emission rates of factories manifests itself when $\theta < \theta_{\rm crit}$. In this regime the sensor multiply affects the same nearest factory and lowers its emission production, while other hidden sources, maybe more influential polluters, could left without feedback requests. Hereafter the process is referred as *spatial screening*. The statistical significance of discussed effect is clarified in Fig. 2, where the auxiliary order parameter $\langle\phi'\rangle$ is plotted against θ. The quantity averaged is

$$\phi'(t) = \frac{1}{N_S}\sum_{j=1}^{N_S}\sigma'(t,j), \tag{5}$$

where $\sigma'(t,j)$ is unity, if a feedback request comes from the jth sensor and zero otherwise. The situation is the most pronounced at $\theta \leqslant \theta_{\rm crit}$. In this context the ratio N_S/N_F reflects the *screening* efficiency. At the critical point we have $N_S/N_F = \theta_{\rm crit}L^b/\rho_F = \theta_{\rm crit}\rho_F^{-(b+2)/2}N_F^{b/2}$. The proportionality $N_S/N_F \sim N_F^{b/2}$ offers the physical meaning of index b. The vicinity of criticality is corroborated by the broad-scale statistics of the production rates (see Fig. 3, a). We see that the most pronounced power-law dependence belongs to $\theta \simeq \theta_{\rm crit}$. In the economic models Ref. [6], the emission rates are usually related to the overall production of goods with a proportionality factor called emission-output ratio. Standard and widely accepted assumption is that this ratio is fixed and irrespective to the investments in clean technology. With such an idealization, the critical fluctuations of $n(t,k)$ can be assigned

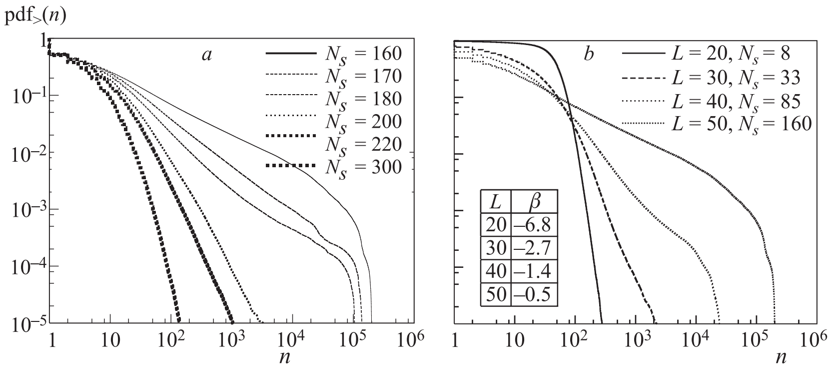

Fig. 3. *a*) The log–log plot of the cumulative distribution of emissions $n(t,k)$ for $L = 50$. Thereupon for $N_{\rm S} < 160$ the dynamics has no steady state, the distributions for these parameters have no meaning; *b*) The cumulative pdf's of the production rates $n(t,k)$ obtained for different $N_{\rm S}$ and L

to the empirical Zipf's law. As we see from Fig. 3, *b*, the cumulative probability distribution function $\mathrm{pdf}_>(n)$ corresponding to (N_S, L)-parametric domain has been characterized by the effective exponents within excessively broad range $\beta \simeq 0.5{-}6.8$. What is lacking the present version of model is the narrower specification of β. It is worth noting the goal empirical values (see, e.g., $\beta = 0.84,\ 0.995$ in Ref. [10]). Since the place for new factory is chosen randomly, the snapshots showing the spatial distributions of $\mathbf{F}$ agents evoke the images of the short-range ordered molecular patterns. From this perspective the system of factories has features of 2D molecular liquid or «vapor». Within such a viewpoint the «vapor» of $\mathbf{F}(t,k)$ agents prepared for $\theta > \theta_{\rm crit}$ belongs to the safe economy, whereas higher correlated «liquid condensate» ($\theta < \theta_{\rm crit}$) corresponds to the catastrophic regime.

CONCLUSIONS

The statistical properties of the multiagent system including the environmental-economic interactions have been studied. The simple and nondynamic rules of sensorial agents ensure that the extent of sensorial system is an order parameter of environment. The main implication from the present simulation is the identification of the power-law (nearly catastrophic) pdf's of fluctuations of emission rates. We believe that focus to power-law distributions will possibly open new perspectives of the environmental diagnostics.

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