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МЕТОДИКА ФИЗИЧЕСКОГО ЭКСПЕРИМЕНТА

INFORMATIONAL CONTENT OF THE HIGH ORDER DIFFRACTION PATTERN

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The information content of the diffraction pattern in the region of very high orders is considered. It is shown that high order diffraction pattern represents a superresolution width indicator of the particle track in nuclear emulsion. A principally new experimental setup designed for width measurements of the wires and particle tracks is described. The first experiments performed for tungsten wire as an object are presented. It is shown that the relative error of the measurement made by this new technique is as small as 0.03% for tungsten wire of the diameter $\cong 26$ μm.

Рассмотрено информационное содержание дифракционной картины в области очень высоких порядков. Показано, что дифракционная картина высоких порядков представляет собой сверхразрешающий индикатор ширины следа частицы в ядерной фотоэмульсии. Описана принципиально новая экспериментальная установка, предназначенная для измерения ширины проволоки и следов частиц. Изложены первые эксперименты, выполненные для вольфрамовой проволоки в качестве объекта. Показано, что относительная погрешность измерений, проведенных этим новым методом, равна всего 0,03 % для вольфрамовой проволоки диаметром $\cong 26$ мкм.

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INTRODUCTION

The observation of the diffraction pattern of the straight line objects is a typical one in the Fourier optics [1]. This method gives very large depth of focus both for registration [2] and for processing of the track information [3]. The convergent light beam, which accomplishes the spatial Fourier transformation, is used widely in the meso-optical Fourier transform microscopes (MFTM) [4]. In all these papers the object of the observation was a diffraction pattern in the region of the first diffraction order. The zero diffraction order is suppressed with ease when we use the convergent illuminated light beam. The high orders of the diffraction pattern were used implicitly in [5] for observation of the meso-optical images of the open circles, rectangular slit, shutter and straight line set of many open circles. As was shown in [5], the meso-optical images of these objects have the form of the one-dimensional derivative of the input object. The results were explained in terms of the one-dimensional Foucault–Hillbert transform by the meso-optical element with ring response. It has been also proved that the existence of the central dead part in the meso-optical element with ring response is indeed a minor handicap for using this element in the MFTM for nuclear emulsion.

In this paper we treat the informational content of the diffraction pattern in the region of very high orders. It is shown that high order diffraction pattern represents a superresolution width indicator of the particle track in nuclear emulsion. A new experimental setup designed for width measurements of the particle track is described. The main feature of this new technique is the observation of the diffraction pattern of the particle track in the region, which covers very high diffraction orders. To accomplish this observation the support of the photosensitive layer must be chosen in the form of the cylindrical surface with center which coincides with convergent point of the illuminated light beam. Some quantitative tolerances to the mutual arrangement of the main parts of the experimental setup are explained. The first experiments performed with tungsten wire as an object are described. The results of the direct measurements show that the relative error of this new technique is as small as $\pm 0.03\%$ for tungsten wire of the diameter $\cong 26\ \mu$m.

1. THEORY

To treat the informational content of the diffraction pattern in the region of very high diffraction orders, we consider the one-dimensional model of the light diffraction on the spectral slit, the amplitude transmission function of which $f(x)$ is equal to

$$f(x) = \begin{cases} 1, & |x| \leqslant d/2, \\ 0, & |x| < d/2, \end{cases} \tag{1}$$

where d is the width of the spectral slit.

The far field diffraction pattern, expressed in terms of spatial frequency ω_x, can be written as one-dimensional Fourier transformation

$$f(x) \to F(\omega_x) \int_{-\infty}^{\infty} f(x) \exp(-ix\omega_x)\, dx. \tag{2}$$

For spectral slit we have

$$F_s(\omega_x) = \int_{-d/2}^{d/2} \exp(-ix\omega_x)\, dx = \sin\left(\frac{\omega_x d}{2}\right) \Big/ \left(\frac{\omega_x d}{2}\right). \tag{3}$$

The intensity distribution of the diffraction light has the form

$$I_s(\omega_x) = \sin^2\left(\frac{\omega_x d}{2}\right) \Big/ \left(\frac{\omega_x d}{2}\right)^2 \tag{4}$$

and represents a periodic function $\sin^2(\bullet)$ with envelope $(\omega_x)^{-2}$. The distance between two successive minima of the function $I_s(\omega_x)$ is equal to

$$\Omega_x = \frac{2\pi}{d}. \tag{5}$$

To detect this period in the region of small spatial frequencies ω_x, a plane oriented perpendicular to the optical axis of the convergent light is used commonly.

The drop of the intensity distribution $I_s(\omega_x)$ for large spatial frequencies ω_x is the main factor which restricts the power of the method. The second factor is the emergence of the reflected light at very high ω_x.

From Eq. (4) we see that one single measurement of the first intensity minimum coordinate is enough to estimate parameter d, that is the width of the spectral slit. There is an essential obstacle for measuring of the high order diffraction minima. In all cases commonly encountered in the practice, the surface of the photosensitive layer support has the form of a plane. In such a condition we are faced with defocusing effect which reduces drastically the contrast of the diffraction pattern in the region of high diffraction orders.

In this paper a new technique for observation of any high diffraction orders is proposed. The main point of this approach is the nontraditional form of the observation surface.

2. EXPERIMENTAL SETUP

The experimental setup is shown in Fig. 1. The object is oriented perpendicular to the optical axis of the convergent light beam and is directed along the axis of the cylindrical surface of the photosensitive layer. This configuration is very adequate for detection without any

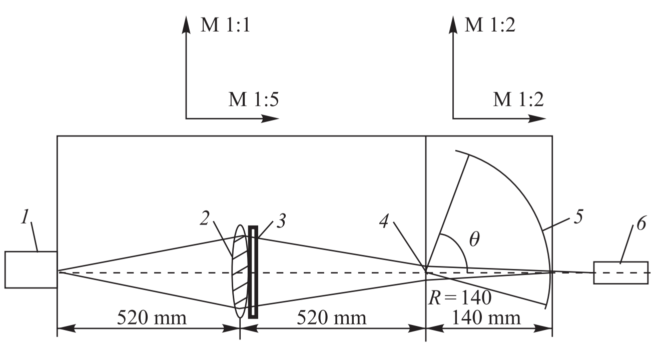

Fig. 1. The experimental setup for detection of very high diffraction order: *1* — point light source; *2* — Fourier transform lens; *3* — aperture diaphragm; *4* — object (wire); *5* — photosensitive layer; *6* — absorption stop of the direct light beam

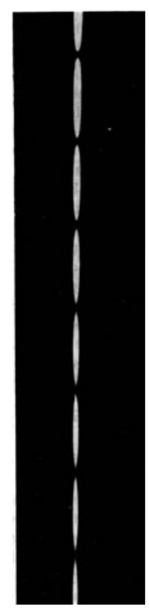

Fig. 2. Fragment of the diffraction pattern produced by the tungsten wire of the diameter ≅ 26 μm. The distance between two successive minima is equal to about 4 mm on the real photo-negative

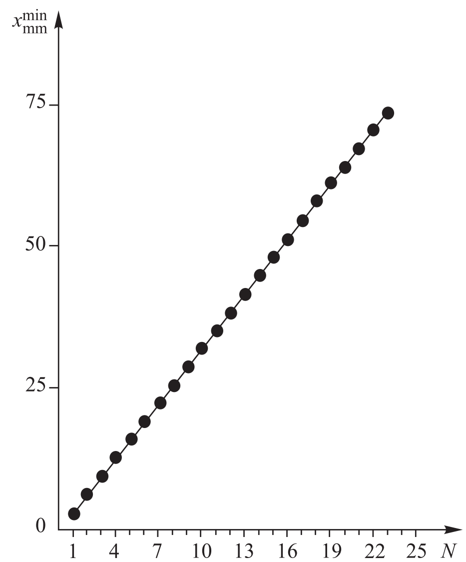

Fig. 3. The coordinates of the intensity minima $x_{\mathrm{mm}}^{\mathrm{min}}$ measured for various index number N from $N = 2$ to $N = 24$

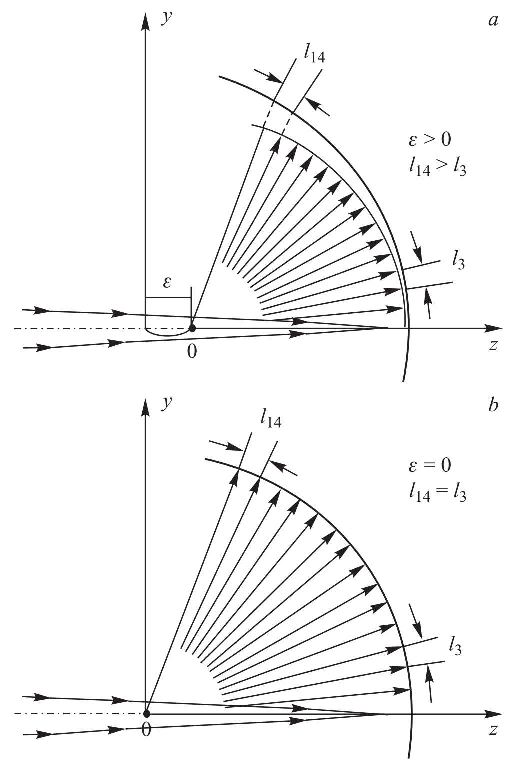

Fig. 4. The explanation of the nonlinear effect induced by the eccentricity ε between the object axis and the axis of the cylindrical support of the photosensitive layer. For $\varepsilon > 0$, as shown in this figure, we observe $l_{m+1} > l_m > l_{m-1}$ for every index number m

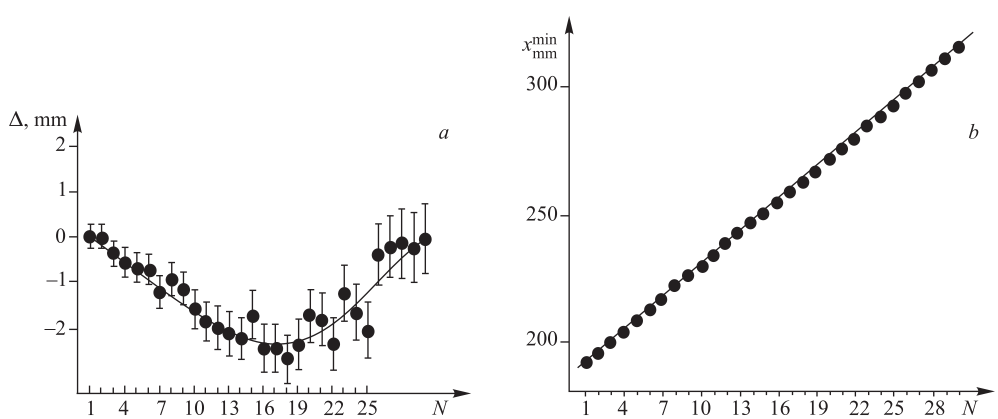

Fig. 5. The extra effect of the nonlinearity Δ (mm) observed in the experiments with $\varepsilon > 0$ (see Fig. 3)

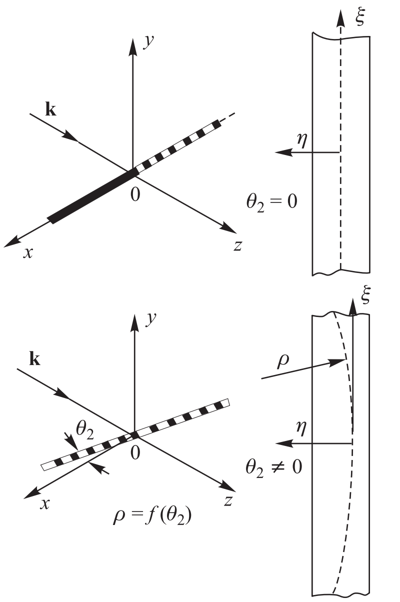

Fig. 6. The bending of the diffraction picture, produced by the wire in the case of wrong orientation of the wire with respect to the axis of the cylindrical surface

defocusing effect of diffraction pattern in the region of high diffraction orders. The fragment of the diffraction pattern produced by the tungsten wire of the diameter $\cong 26\ \mu$m is presented in Fig. 2.

This experimental setup is designed for estimation of the width d of the straight line object with error as small as possible from the data on positions of many intensity minima of the diffraction pattern in the region of very high orders. The coordinates of these intensity minima $x_{\mathrm{mm}}^{\mathrm{min}}$ versus the index number N of the diffraction minimum are shown in Fig. 3. This dependence is a pure linear one.

Another picture is observed in the case of the accidental eccentricity between the object axis and the axis of the cylindrical support of the photosensitive layer. A nonlinear effect is shown in Fig. 4.

The differences between the observed minima positions and the expected linear dependence are shown in the upper part of Fig. 5.

Another effect observed in this experimental setup is the bending of the diffraction pattern induced by the wrong orientation of the object axis with respect to the axis of the cylindrical support of the photosensitive layer (Fig. 6). The curvature radius ρ of this diffraction pattern is defined by the angle $\theta_{я}$ between the object axis and the axis of the cylindrical support. Just this phenomenon was used to measure the dip angle of the straight particle track in the nuclear emulsion by means of MFTM [6].

3. RESULTS

The photo of the diffraction pattern produced by two tungsten wires on the cylindrical surface of the radius $R = 140$ mm over the region of diffraction orders, which includes the diffraction angles from $\theta = 0$ to $\theta_0 \approx 1$ rad, is given in Fig. 7. The central spot in the region of zero diffraction order is produced by the Fourier transform lens *2* (Fig. 1) with aperture diaphragm *3* having six iris stops. The rays going from the focus point in the photo (Fig. 7) are produced by those iris stops. We see that the diffraction picture, ranged from $\theta = 0$ to $\theta_0 \approx 1$ rad, goes into the reflection component at very high angles θ. A local interference between diffraction and reflection components can be seen as well.

This photo was proceeded as follows. We have estimated the coordinates of the intensity minima at different index number N from $N = 1$ to $N = 24$. Each measurement has been repeated many times to calculate the real r.m.s. spread or standard error for each N. To get the measurement error of the average period L of the diffraction picture we must divide the received r.m.s. spread by the corresponding N. Thus, we get the effective standard error $\sigma_{\mathrm{eff}}^{\mathrm{exp}}$ of the period L (but not the diameter of the wire!). From these data we can calculate the relative measurement error of the period L, $\sigma_{\mathrm{eff}}^{\mathrm{exp}}/L$.

Fig. 7. The photo of the diffraction pattern, in the scale 1:1, produced by two tungsten wires. The radius of the cylindrical surface $R = 140$ mm. The maximal elongation from the central order is equal to 170 mm ($\theta >$ 1 rad)

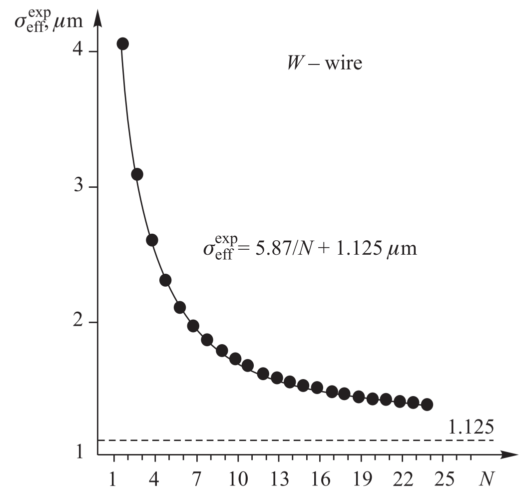

Fig. 8. The effective standard errors $\sigma_{\rm eff}^{\rm exp}$, measured for index numbers from $N_1 = 2$ to $N_2 = 24$, in the photo shown in Fig. 7

It is evident that

$$\frac{\Delta L}{L} = \frac{\Delta d}{d}, \tag{6}$$

where Δd is the measurement error of the diameter of the wire.

The effective standard errors $\sigma_{\rm eff}^{\rm exp}$ for different index number N are shown in Fig. 8. These data can be approximated for tungsten wire of the diameter $\cong 26$ μm by the function

$$\sigma_{\rm eff}^{\rm exp} = \left(\frac{5.87}{N} + 1.125\right) \mu\text{m}. \tag{7}$$

The existence of the constant term 1.125 μm is due to the fact that the contrast of the diffraction minima decreases at high Ns. For $N = 24$ we have $\sigma_{\rm eff}^{\rm exp} = 1.35$ μm $= \Delta L$.

The period of the diffraction pattern shown in Fig. 2 and in Fig. 7 equals $L \approx 4$ mm. Therefore,

$$\frac{\Delta L}{L} = \frac{1.35 \cdot 10^{-3}}{4} = 3.4 \cdot 10^{-4} = \frac{\Delta d}{d}. \tag{8}$$

From Eq. (6) we see that application of the new technique by observation of very high diffraction orders is equivalent to superresolution. This technique gives the possibility to estimate the diameter of the wire of the order 26 μm with error as small as 0.03%. Until now we discussed the diffraction error of the measurements. But if all other geometrical factors of our experimental setup can be fixed or measured with some error $\Delta\xi$, then the total relative error $(\Delta d)^t$ of the diameter d will be equal to

$$\frac{(\Delta d)^t}{d} = \sqrt{(3 \cdot 10^{-4})^2 + (\Delta\xi)^2}. \tag{9}$$

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