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ONE- AND TWO-RANK SEPARABLE KERNELS OF THE TWO-NUCLEON SYSTEM IN THE BETHE–SALPETER APPROACH

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We propose a completely covariant separable kernel for the nucleon–nucleon (NN) interaction in $J = 0$ ($^1S_0$, $^3P_0$) and $J = 1$ ($^3S_1$, $^1P_1$, $^3P_1$) states. We calculate the nucleon–nucleon (NN) T matrix in the framework of the covariant Bethe–Salpeter approach for a system of two spin-one-half particles with one-rank, extended one-rank, and two-rank Yamaguchi-type separable kernel of interaction. The explicit connection between parameters of the separable kernel and low-energy scattering and between parameters of deuteron binding energy and phase shifts is established. The obtained kernel is used for calculation of phase shifts. This approach can be applied to higher partail waves for NN scattering and other reactions.

В работе предлагаются ковариантные ядра нуклон-нуклонного взаимодействия для состояний $J = 0$ ($^1S_0$, $^3P_0$) и $J = 1$ ($^3S_1$, $^1P_1$, $^3P_1$). Рассчитывается T-матрица нуклон-нуклонного рассеяния в рамках ковариантного подхода Бете–Солпитера с двумя частицами со спином 1/2 с сепарабельным ядром с функциями Ямагучи первого и второго ранга. Получена прямая связь между параметрами сепарабельного ядра и низкоэнергетического рассеяния и между параметрами энергии связанного состояния и фазами. Найденные ядра используются для расчета фаз рассеяния. Формализм может быть применен для расчетов с более высокими парциальными состояниями нуклон-нуклонного рассеяния и других реакций.

INTRODUCTION

Relativistic investigations of the reactions with light nuclei (both with electromagnetic and hadron probes) in the Bethe–Salpeter approach [1] require constructing the amplitudes for bound states and for states of scattering. To construct them, one should solve Bethe–Salpeter equation for partial-wave decomposed T matrix. The separable kernels of interaction are very useful [2] in this case. The separable form of interaction is good not only for the technical reason but, in our opinion, also for the reflection of the nonlocal type of elementary nucleon–nucleon interaction (it was stressed first in paper [2]). This allows us not only to avoid the divergence peculiar to local theories but also to take into account the internal structure of the nucleon.

In the paper, we consider the BS equation for T matrix and solve it using the separable kernel of interaction for a spin-one-half particle system. This allows us to solve the BS equation without referring to the ladder approximation.

The paper is organized as follows: after describing the used formalism in Sec. 1, the result of constructing one- and two-rank separable kernels for partial states of the np system is given in Sec. 2. The summary is in Sec. 3.

1. FORMALISM

We start with the partial-wave decomposed Bethe–Salpeter equation for the nucleon–nucleon T matrix (in the rest frame of two-nucleon system):

$$T_{l'l}(p'_0, p', p_0, p; s) = V_{l'l}(p'_0, p', p_0, p; s) + \\ + \frac{i}{4\pi^3} \sum_{l''} \int dk_0 \int k^2\, dk\, \frac{V_{l'l''}(p'_0, p', k_0, k; s) T_{l''l}(k_0, k, p_0, p; s)}{(\sqrt{s}/2 - e_k + i\epsilon)^2 - k_0^2}. \quad (1)$$

Here $T_{l''l}$ is the partial-wave decomposed T matrix and $V_{l'l}$ is the kernel of the NN interaction, $e_k = \sqrt{k^2 + m^2}$. There is only one term in the sum for the singlet case ($L = J$), and there are two terms for the coupled triplet case ($L = J \mp 1$). We introduce square of the total momentum $s = P^2 = (p_1 + p_2)^2$ and the relative momentum $p = (p_1 - p_2)/2$ $[p' = (p'_1 - p'_2)/2]$ (for details, see Ref. [1]).

Assuming the separable form (rank N) for the partial-wave decomposed kernels of NN interactions,

$$V_{l'l}(p'_0, p', p_0, p; s) = \sum_{i,j=1}^{N} \lambda_{ij} g_i^{[l']}(p'_0, p') g_j^{[l]}(p_0, p), \quad (2)$$

we can solve Eq. (1) and write for the T matrix:

$$T_{l'l}(p'_0, p', p_0, p; s) = \sum_{i,j=1}^{N} \tau_{ij}(s) g_i^{[l']}(p'_0, p') g_j^{[l]}(p_0, p). \quad (3)$$

Here the function $\tau(s)$ is

$$\tau^{-1}(s)_{ij} = \lambda_{ij}^{-1} + H_{ij}(s). \quad (4)$$

Function $H_{ij}(s)$ has the following form:

$$H_{ij}(s) = \sum_l H_{ij}^l(s) = -\frac{i}{4\pi^3} \int dk_0 \int k^2\, dk \sum_l \frac{[g_i^{[l]}(k_0, k)][g_j^{[l]}(k_0, k)]}{(\sqrt{s}/2 - e_k + i\epsilon)^2 - k_0^2}. \quad (5)$$

We use the following normalization condition for the on-mass-shell T matrix for the singlet case:

$$T_{ll}(s) \equiv T_{ll}(0, \bar{p}, 0, \bar{p}, s) = -\frac{16\pi}{\sqrt{s}\sqrt{s - 4m^2}}\, \mathrm{e}^{i\delta} \sin\delta; \quad (6)$$

and for the coupled triplet case:

$$T_{l'l}(s) = \frac{8\pi}{\sqrt{s}\sqrt{s - 4m^2}} \begin{pmatrix} \cos 2\epsilon\, \mathrm{e}^{2i\delta_<} - 1 & i \sin 2\epsilon\, \mathrm{e}^{i(\delta_< + \delta_>)} \\ i \sin 2\epsilon\, \mathrm{e}^{i(\delta_< + \delta_>)} & \cos 2\epsilon\, \mathrm{e}^{2i\delta_>} - 1 \end{pmatrix} \quad (7)$$

with $\bar{p} = \sqrt{s/4 - m^2} = \sqrt{m T_{\rm lab}/2}$. We introduce phase shifts $\delta \equiv \delta_{L=J}$, $\delta_{\lessgtr} \equiv \delta_{L=J\mp1}$, and mixing parameter ϵ.

A bound state, if exists, is described by a simple pole in the T matrix. Using Eq. (4), we can write ($M_b = 2m - E_b$, E_b is the energy of the bound state)

$$\det|\tau^{-1}(s = M_b^2)| = 0. \tag{8}$$

We also introduce the low-energy parameters — scattering length a_l and effective range r_l — by the following equation:

$$\bar{p}^{2l+1}\cot\delta_l(s) = -1/a_l + \frac{r_l}{2}\bar{p}^2 + \mathcal{O}(\bar{p}^3). \tag{9}$$

At this point by using Eqs. (6) and (7) and calculating T matrix on the mass shell ($p_0 = p_0' = 0, p = p' = \bar{p}$), we can connect internal parameters of the NN kernel and observables — phase shifts, bound state energy and low-energy parameters.

2. CALCULATIONS AND RESULTS

2.1. One-Rank Yamaguchi-Type Kernel. We use covariant generalization of the Yamaguchi [2] functions for $g^{[l]}(k_0, k)$:

$$g^{[S]}(k_0,k) = \frac{1}{k_0^2 - k^2 - \beta_0^2 + i\epsilon}, \tag{10}$$

$$g^{[P]}(k_0,k) = \frac{\sqrt{|-k_0^2 + k^2|}}{(k_0^2 - k^2 - \beta_1^2 + i\epsilon)^2}, \tag{11}$$

$$g^{[D]}(k_0,k) = \frac{C(k_0^2 - k^2)}{(k_0^2 - k^2 - \beta_2^2 + i\epsilon)^2}. \tag{12}$$

Now we can calculate internal parameters of the NN kernel by using the above equations to reproduce experimental values for the phase shifts (data from SAID program http://gwdac.phys.gwu.edu/), deuteron energy and quadroupole moment, and low-energy parameters (data from Ref. [5]).

1. To find parameters λ and β in ${}^1S_0^+$ channel, we solve a system of nonlinear equations (exp stands for experimental, s — for singlet):

$$a_s^{\rm exp} = a_s(\lambda,\beta), \qquad r_s^{\rm exp} = r_s(\lambda,\beta). \tag{13}$$

2. To find parameters λ, β_0, β_2, and C in ${}^3S_1^+ - {}^3D_1^+$ coupled channel, we solve a system of the nonlinear equations (t stands for triplet):

$$\begin{aligned} a_t^{\rm exp} &= a_t(\lambda,\beta_0,\beta_2,C), \qquad & E_d^{\rm exp} &= r_0(\lambda,\beta_0,\beta_2,C),\\ p_d &= p_d(\lambda,\beta_0,\beta_2,C), \qquad & q_d^{\rm exp} &= q_d(\lambda,\beta_0,\beta_2,C). \end{aligned} \tag{14}$$

Here we introduce D-wave pseudoprobability p_d.

3. To find parameters λ and β in uncoupled ${}^3P_0^+$, ${}^1P_1^+$, and ${}^3P_1^+$ channels, we use procedure to minimize function:

$$\chi^2 = \sum_{i=1}^{n}(\delta^{\rm exp}(s_i) - \delta(s_i))^2/(\Delta\delta^{\rm exp}(s_i))^2, \tag{15}$$

where n is the number of the experimental points. The results of calculations are given in Tables 1 and 2 and Fig. 1.

Table 1. **Parameters for $^1S_0^+$ and $^3S_1^+ - ^3D_1^+$ channels. One-rank Yamaguchi-type functions**

Parameters	$^1S_0^+$	$^3S_1^+ - ^3D_1^+$ ($p_d = 4\%$)	$^3S_1^+ - ^3D_1^+$ ($p_d = 5\%$)	$^3S_1^+ - ^3D_1^+$ ($p_d = 6\%$)
λ, GeV^2	–0.285549	–0.502690	–0.429637	–0.364905
β_0, GeV	0.221858	0.251241	0.246706	0.242285
β_2, GeV		0.293994	0.324341	0.350007
C		1.6471	2.4071	3.2735

Table 2. **Parameters for $^3P_0^+$, $^1P_1^+$ and $^3P_1^+$ channels. One-rank Yamaguchi-type functions**

Parameters	$^3P_0^+$, $n = 3$	$^3P_0^+$, $n = 5$	$^1P_1^+$, $n = 4$	$^1P_1^+$, $n = 5$	$^3P_1^+$, $n = 7$	$^3P_1^+$, $n = 9$
λ, GeV^2	–0.029420	–0.016123	0.091535	0.19513	0.31263	0.65701
β_1, GeV	0.23833	0.21861	0.27673	0.30891	0.33890	0.38191

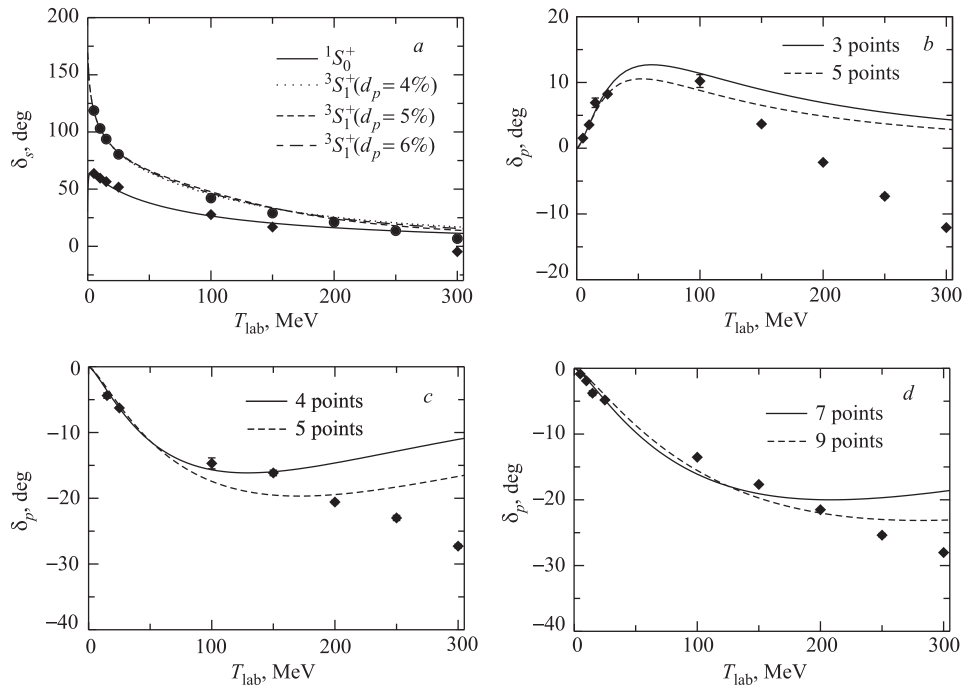

Fig. 1. $^1S_0^+$ and $^3S_1^+$ (*a*), $^3P_0^+$ (*b*), $^1P_1^+$ (*c*), and $^3P_1^+$ (*d*) channel phase shifts. One-rank Yamaguchi-type functions

2.2. Extended One-Rank Yamaguchi-Type Functions. Let us now extend the form of functions $g^{[l]}(k_0, k)$ and add one more term

$$g^{[S]}(k_0, k) = \frac{1}{(k_0^2 - k^2 - \beta_{01}^2 + i\epsilon)} + \frac{C_{02}(k_0^2 - k^2)}{(k_0^2 - k^2 - \beta_{02}^2 + i\epsilon)^2}, \tag{16}$$

Table 3. **Parameters for $^3P_0^+$, $^1P_1^+$ channels. Extended one-rank Yamaguchi-type functions**

Parameters	$^3P_0^+$, $n=4$	$^3P_0^+$, $n=5$	$^1P_1^+$, $n=6$	$^1P_1^+$, $n=8$	$^1P_1^+$, $n=10$
λ, GeV^2	–2.09	–0.0108	$0.416{\cdot}10^{-2}$	$0.8323{\cdot}10^{-2}$	$0.706{\cdot}10^{-2}$
C_{12}, GeV	0.91	9.74	–25.7	–31.549	–28.92
β_{11}, GeV	0.401	0.209	0.172	0.18716	0.183
β_{12}, GeV	0.31	0.495	0.388	0.44545	0.428

$$g^{[P]}(k_0,k) = \frac{\sqrt{|-k_0^2+k^2|}}{(k_0^2-k^2-\beta_{11}^2+i\epsilon)^2} + \frac{C_{12}\sqrt{|(-k_0^2+k^2)^3|}}{(k_0^2-k^2-\beta_{12}^2+i\epsilon)^3}, \tag{17}$$

$$g^{[D]}(k_0,k) = \frac{C_{21}(k_0^2-k^2)}{(k_0^2-k^2-\beta_{21}^2+i\epsilon)^2} + \frac{C_{22}(k_0^2-k^2)^2}{(k_0^2-k^2-\beta_{22}^2+i\epsilon)^3}. \tag{18}$$

Now we can calculate internal parameters of the NN kernel by using above equations to reproduce experimental values for the phase shifts (data from SAID program http://gwdac.phys.gwu.edu/), deuteron energy and quadroupole moment, and low-energy parameters (data from Ref. [5]).

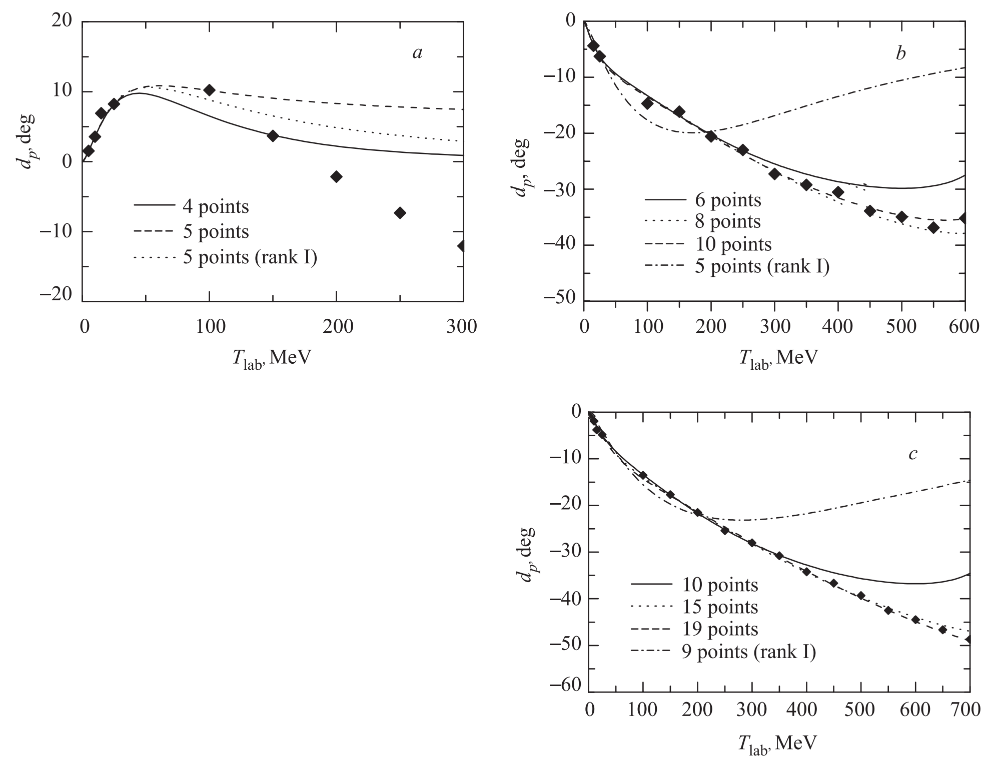

Fig. 2. $^3P_0^+$ (*a*), $^1P_1^+$ (*b*), and $^3P_1^+$ (*c*) channel phase shifts. Extended one-rank Yamaguchi-type functions

1. To find parameters λ, C_{12}, β_{11}, and β_{12} in uncoupled ${}^3P_0^+$, ${}^1P_1^+$, and ${}^3P_1^+$ channels, we use procedure to minimize the χ^2 value:

$$\chi^2 = \sum_{i=1}^{n} (\delta^{\exp}(s_i) - \delta(s_i))^2 / (\Delta\delta^{\exp}(s_i))^2, \tag{19}$$

The results of calculations are given in Table 3 and Fig. 2. We also show former results (with two parameters) for comparison, where n is the number of the experimental points.

2.3. Two-Rank Yamaguchi-Type Kernel. We use covariant generalization of the Yamaguchi [2] functions for $g^{[l]}(k_0, k)$:

$$\begin{aligned}
g_1^{[S]}(k_0, k) &= \frac{1}{(k_0^2 - k^2 - \beta_1^2 + i\epsilon)}, \\
g_2^{[S]}(k_0, k) &= \frac{(k_0^2 - k^2)}{(k_0^2 - k^2 - \beta_2^2 + i\epsilon)^2}, \\
g_1^{[P]}(k_0, k) &= \frac{\sqrt{|-k_0^2 + k^2|}}{(k_0^2 - k^2 - \beta_1^2 + i\epsilon)^2}, \\
g_2^{[P]}(k_0, k) &= \frac{\sqrt{|-k_0^2 + k^2|^3}}{(k_0^2 - k^2 - \beta_2^2 + i\epsilon)^3}.
\end{aligned} \tag{20}$$

1. To find parameters β_1, β_2, λ_{11}, λ_{12}, and λ_{22} in ${}^1S_0^+$ channel, we use procedure to minimize function:

$$\chi^2 = \sum_{i=1}^{n} (\delta^{\exp}(s_i) - \delta(s_i))^2 / (\Delta\delta^{\exp}(s_i))^2 + (a_{0s}^{\exp} - a_{0s}^{\mathrm{cal}})^2 / (\Delta a_{0s}^{\exp})^2. \tag{21}$$

2. To find parameters β_1, β_2, λ_{11}, λ_{12}, and λ_{22} in uncoupled ${}^3S_1^+$ channels, we use procedure to minimize function:

$$\chi^2 = \sum_{i=1}^{n} (\delta^{\exp}(s_i) - \delta(s_i))^2 / (\Delta\delta^{\exp}(s_i))^2 + (a_{0t}^{\exp} - a_{0t}^{\mathrm{cal}})^2 / (\Delta a_{0t}^{\exp})^2 + \\ + (E_d^{\exp} - E_d^{\mathrm{cal}})^2 / (\Delta E_d^{\exp})^2. \tag{22}$$

3. To find parameters β_1, β_2, λ_{11}, λ_{12}, and λ_{22} in uncoupled ${}^3P_0^+$, ${}^1P_1^+$, and ${}^3P_1^+$ channels, we use procedure to minimize function:

$$\chi^2 = \sum_{i=1}^{n} (\delta^{\exp}(s_i) - \delta(s_i))^2 / (\Delta\delta^{\exp}(s_i))^2. \tag{23}$$

The results of calculations are given in Tables 4, 5 and Fig. 3.

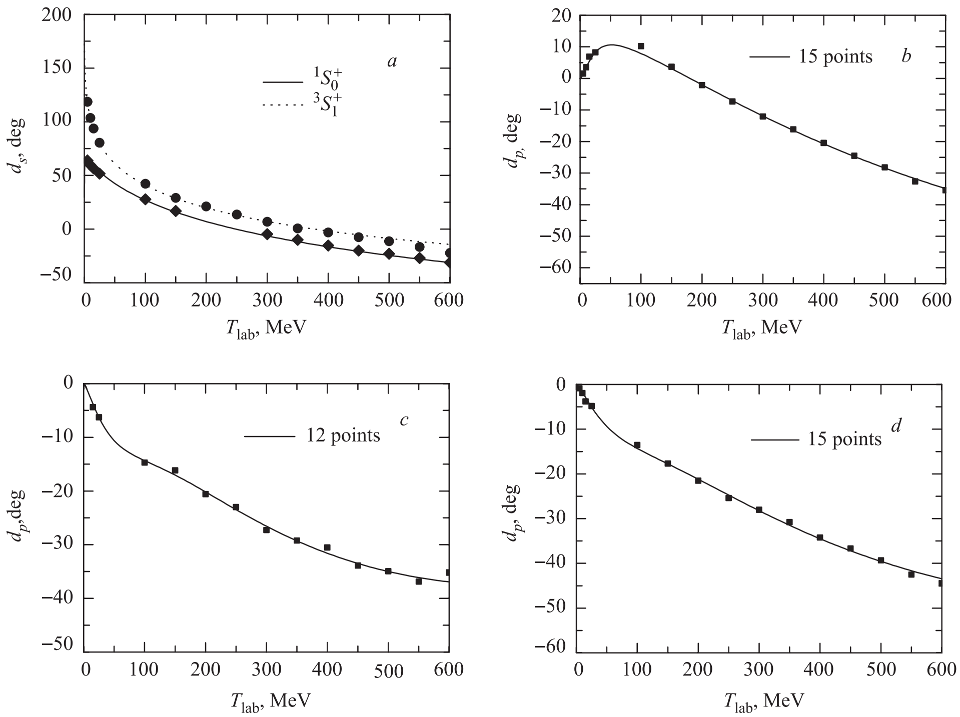

Fig. 3. $^1S_0^+$ and $^3S_1^+$ (*a*), $^3P_0^+$ (*b*), $^1P_1^+$ (*c*), and $^3P_1^+$ (*d*) channel phase shifts. Two-rank Yamaguchi-type functions

Table 4. **Parameters for $^3P_1^+$ channel. Extended one-rank Yamaguchi-type functions**

Parameters	$^3P_1^+$, $n = 10$	$^3P_1^+$, $n = 15$	$^3P_1^+$, $n = 19$
λ, GeV^2	$0.861 \cdot 10^{-2}$	0.0342	0.0632
C_{12}, GeV	–22.7	–24.3	–26.7
β_{11}, GeV	0.200	0.235	0.250
β_{12}, GeV	0.409	0.503	0.535

Table 5. **The binding energy and low-enegy parameters for singlet and triplet channels. Two-rank Yamaguchi-type functions**

$^1S_0$	a_{0s}, Fm	$^3S_1$	a_{0t}, Fm	E_d, MeV
Calculation	–23.745	Calculation	5.419	2.224606
Experiment	–23.748±0.010	Experiment	5.424±0.004	2.224644±0.000046

CONCLUSION

We constructed completely covariant separable kernels for the nucleon–nucleon interaction in singlet and triplet channels. We have found that the use of the one-rank and the extended

Table 6. **Parameters for $^1S_0$, $^3S_1$, $^3P_0^+$, $^1P_1^+$, and $^3P_1^+$ channels. Two-rank Yamaguchi-type functions**

Parameters	$^1S_0^+$, $n=10$	$^3S_1^+$,$n=13$	$^3P_0^+$, $n=15$	$^1P_1^+$, $n=12$	$^3P_1^+$, $n=15$
β_1, GeV	0.3849	0.3095	0.21425	0.37036	0.3741
β_2, GeV	0.6877	0.8376	0.51963	0.363609	0.40618
λ_{11}, GeV^2	0.866	–0.394	–0.0116239	0.9343	0.7125
λ_{12}, GeV^2	21.89	17.654	0.36589	1.8371	1.57199
λ_{22}, GeV^2	–7.5	21.982	43.887	6.9663	11.523

one-rank kernels can reproduce the $^1S_0$, $^3S_1$, and $^3P_0$ phase shifts till $T_{\rm lab} = 200{-}300$ MeV. The two-rank Yamaguchi-type kernels are able to reproduce the deuteron static properties and the phase shifts up to $T_{\rm lab} = 600$ MeV.

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