Письма в ЭЧАЯ. 2005. Т. 2, № 5(128). С. 69–74

УДК 539.126

THERMODYNAMICS OF RESONANCES WITH FINITE WIDTH

D. B. Blaschke [a,b], *K. A. Bugaev* [a,c]

[a] Fachbereich Physik, Universität Rostock, Rostock, Germany

[b] Joint Institute for Nuclear Research, Dubna

[c] Bogolyubov Institute for Theoretical Physics, Kiev

The Hagedorn resonance gas model is generalized in order to include a medium-dependent resonance width. It is shown that a model with a vanishing width below the Hagedorn temperature T_H and a Hagedorn spectrum-like width above T_H not only eliminates the divergence of the thermodynamic functions above T_H, but also gives a satisfactory description of lattice quantum chromodynamics (QCD) data above the deconfinement transition. In addition, this model explains the absence of heavy-resonance contributions in the fit of experimental particle ratios at SPS and RHIC energies.

Производится обобщение модели резонансного газа Хагедорна так, чтобы учесть зависящую от среды ширину резонанса. Показано, что модель с шириной, исчезающей ниже температуры Хагедорна T_H и имеющей хагедорн-подобный спектр выше T_H, не только устраняет расходимость термодинамических функций выше T_H, но и дает удовлетворительное описание данных решеточной квантовой хромодинамики (КХД) выше перехода деконфайнмента. Кроме того, эта модель объясняет отсутствие вкладов тяжелых резонансов в экспериментальных соотношениях для частиц при энергиях SPS и RHIC.

1. The lattice QCD simulations not only provide the strongest theoretical support of the quark–gluon plasma (QGP) existence, but they also give detailed information on the properties of strongly interacting matter over a wide range of temperatures. A recent analysis [1] of the lattice energy density showed that a resonance gas model can perfectly explain the steep rise in the number of degrees of freedom at $T \approx T_c$. On the other hand, lattice QCD has also revealed that hadronic correlations persist for $T > T_c$ [2]. The question arises whether it is more appropriate to describe hot QCD matter in terms of hadronic correlations rather than in terms of quarks and gluons. Therefore, in the present contribution, we would like to discuss a generalization of the Hagedorn resonance gas (statistical bootstrap) model which allows for the extension of a hadronic description above T_c.

The statistical bootstrap model (SBM) [3] is based on the hypothesis that hadrons are made of hadrons, with constituent and compound hadrons being treated on the same footing. This implies an exponentially growing form of the hadronic mass spectrum $\rho_H(m) \approx C_H m^{-a} \exp[m/T_H]$ for $m \to \infty$. The parameter T_H, the Hagedorn temperature, was interpreted as a limiting temperature reached at infinite energy density. The extensive investigation of the SBM has led to a formulation of both the important physical ideas and the mathematical methods for modern statistical mechanics of strongly interacting matter [4].

However, up to now the formulation of the SBM had some severe problems. The first one is the absence of a width for the heavy resonances. From the Particle Data Group [5] we know that heavy resonances with masses $m \geqslant 3.5$ GeV may have widths comparable to their masses. Taking the widths into account will effectively reduce the statistical weight of the resonance.

The second problem arises while discussing the results of the hadron gas (HG) model [6,7]. The HG model accounts for all strong decays of resonances according their partial width given in [5], and, hence, it describes remarkably well the light hadron multiplicities measured in nucleus–nucleus collisions at CERN SPS [6] and BNL RHIC [7] energies. This model is nothing else than the SBM of light hadrons which accounts for the proper volume of hadrons with masses below 2.5 GeV, but neglects the contribution of the exponentially growing mass spectrum.

Thus, one immediately faces a severe problem: «Why do the heavy resonances with masses above 2.5 GeV predicted by the SBM not contribute to the particle spectra measured in heavy-ion collisions at SPS and RHIC energies?» Note that the absence of heavy-resonance contributions in the particle ratios cannot be due to the statistical suppression of the Hagedorn mass spectrum because the latter should not be strong in the quark–hadron phase transition region, where those ratios are believed to be formed [6, 7].

2. According to QCD, hadrons are not elementary, pointlike objects but rather color singlet bound states of quarks and gluons with a finite spatial extension of their wave function. While at low densities a hadron gas description can be sufficient, at high densities and temperatures, when hadronic wave functions overlap, nonvanishing quark exchange matrix elements between hadrons occur in order to fulfil the Pauli principle. This leads to a Mott–Anderson type delocalization transition with frequent rearrangement processes of color strings (string-flip [8]) so that hadronic resonances become off-shell with a finite, medium-dependent width. Such a Mott transition has been thoroughly discussed for light hadron systems in [9] and has been named *soft deconfinement*. The Mott transition for heavy mesons may serve as the physical mechanism behind the anomalous J/ψ suppression phenomenon [10].

We introduce a resonance width Γ in the statistical model with the Hagedorn mass spectrum through the spectral function $A(s,m) = N_s \Gamma\ m/[(s-m^2)^2 + \Gamma^2\ m^2]$, a Breit–Wigner type attenuation of virtual mass with a maximum at $\sqrt{s} = m$ and a normalization factor

$$N_s = \int_{m_B^2}^{\infty} ds\ A(s,m) = \left[\frac{\pi}{2} + \arctan\left(\frac{m^2 - m_B^2}{\Gamma m}\right)\right]^{-1}.$$

Then the energy density of this model with zero resonance proper volume for a given temperature T and baryonic chemical potential μ can be cast in the form

$$\varepsilon(T,\mu) = \sum_{i=\pi,\rho,\ldots} g_i \int \frac{d^3k}{(2\pi)^3} \frac{\sqrt{k^2+m_i^2}}{\exp\left(\dfrac{\sqrt{k^2+m_i^2}-\mu_i}{T}\right)+\delta_i} + \\ + \sum_{A=M,B} \int_{m_A}^{\infty} dm \int_{m_B^2}^{\infty} ds\, \rho_H(m)\, A(s,m) \int \frac{d^3k}{(2\pi)^3} \frac{\sqrt{k^2+s}}{\exp\left(\dfrac{\sqrt{k^2+s}-\mu_A}{T}\right)+\delta_A}, \quad (1)$$

with the degeneracy g_A and the baryonic chemical potential μ_A of hadron A. For mesons, $\delta_M = -1$, $\mu_M = 0$ and for baryons $\delta_B = 1$ and $\mu_B = \mu$, respectively. According to Eq. (1)

the energy density of hadrons consists of the contribution of light hadrons for $m_i < m_A$ and the contribution of the Hagedorn mass spectrum $\rho_H(m)$ for $m \geqslant m_A$.

A new element of Eq. (1) in comparison to the SBM is the presence of the $\sqrt{s}$-dependent spectral function. The analysis shows that, depending on the behavior of the resonance width Γ in the limit $m \to \infty$, there are the following possibilities:

I. For vanishing resonance width, $\Gamma = 0$, Eq. (1) evidently recovers the usual SBM.

II. For final values of the resonance width, $\Gamma = \text{const}$, Eq. (1) diverges for all temperatures T because, in contrast to the SBM, the statistical factor in Eq. (1) behaves as $\{\exp[(m_B - \mu_A)/T] + \delta_A\}^{-1}$ so that it cannot suppress the exponential divergence of the Hagedorn mass spectrum $\rho_H(m)$.

III. For a resonance width growing with mass like the Hagedorn spectrum $\Gamma \sim \sim C_\Gamma \exp[m/T_H]$ or faster, Eq. (1) converges again.

Indeed, in the latter case the Breit–Wigner spectral function behaves as

$$N_s \frac{\Gamma m}{(s - m^2)^2 + \Gamma^2 m^2} \Bigg|_{m \to \infty} \to \frac{2}{\pi\,\Gamma} \sim \exp\left(-\frac{m}{T_H}\right) \tag{2}$$

and cancels the exponential divergence of the Hagedorn mass spectrum. Hence, the energy density remains finite. Note that both the analytical properties of model (1) and the right-hand side of Eq. (2) remain the same, if a Gaussian shape of the spectral function is chosen instead of the Breit–Wigner one.

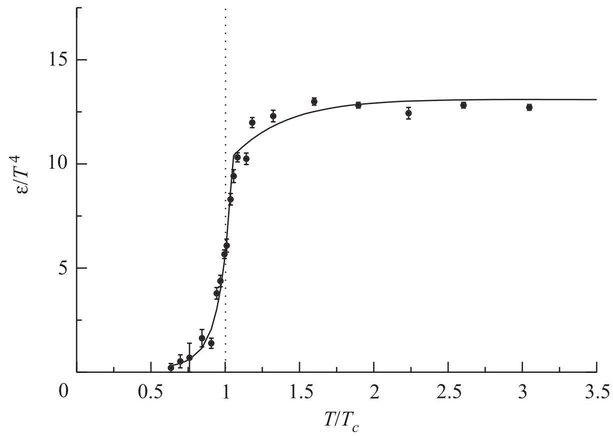
It can be shown that the behavior of the width at finite resonance masses is not essential for the convergence of the energy density (1). In other words, for a convergent energy density (1) above T_H it is sufficient to have a very small probability density (2) (or smaller) for a resonance of mass m to be found in the state with the virtual mass $\sqrt{s}$. Since there is no principal difference between the high and low mass resonances, we can use the same functional dependence of the width Γ for all masses. Thus, for the following model ansatz:

$$\Gamma(T) = \begin{cases} 0, & \text{for} \quad T \leqslant T_H, \\ C_\Gamma \left(\dfrac{m}{T_H}\right)^{N_m} \left(\dfrac{T}{T_H}\right)^{N_T} \exp\left(\dfrac{m}{T}\right), & \text{for} \quad T > T_H, \end{cases} \tag{3}$$

the energy density (1) is finite for all temperatures and the divergence of the SBM is removed. At $T = T_H$, depending on choice of parameters, either it may have a discontinuity or its partial T derivative may be discontinuous. As discussed above, for $T \leqslant T_H$ such a model corresponds to the usual SBM, but for high temperatures $T > T_H$ it remains finite for a wide choice of powers N_m.

Note that for heavy resonances having the widths (3) the resulting mass distribution will be a power law which is seen both in hadron–hadron reactions [11] and in nucleus–nucleus reactions [12] at high energies.

3. As one can see from figure, the Hagedorn gas model correctly reproduces the lattice QCD results below the critical temperature T_c and just in a vicinity above T_c, but not for large temperatures. Figure shows a comparison of the same lattice QCD data [1] with the Mott–Hagedorn gas (3) where the parameters of the spectral function are $N_T = 2.325$, $N_m = 2.5$ and $T_H = 165$ MeV and $m_A = m_B = 1$ GeV. The successful description of the lattice energy density [1] indicates that above T_c the strongly interacting matter may be well described in terms of strongly correlated hadronic degrees of freedom. This result is based on the concept of soft deconfinement and provides an alternative to the conventional explanation of the deconfinement transition as the emergence of quasi-free quarks and gluons.



Fit of the lattice QCD data [1] (dots) with the Mott–Hagedorn resonance gas model (3) (solid curve). For details see text

Another interesting feature of the model (3) is that it allows one to explain naturally the absence of heavy-resonance contributions to the particle yields measured at highest SPS and all RHIC energies, where QGP conditions are expected [6, 7]. In order to find out whether a given resonance has a chance to survive till the freeze-out, it is necessary to compare its lifetime with the typical timescale in the system. There are two typical timescales usually discussed in nucleus–nucleus collisions, the equilibration time $\tau_{\rm eq}$ and the formation time τ_f. The equilibration time indicates when the matter created in collision process reaches a thermal equilibrium which allows one to use the hydrodynamic and thermodynamic descriptions. For Au + Au collisions at RHIC energies it was estimated to be about $\tau_{\rm eq} \approx 0.5$ fm [13]. On the other hand, in transport calculations the formation time is used: the time for constituent quarks to form a hadron. The formation time depends on the momentum and energy of the created hadron, but is of the same order $\tau_f \approx 1{-}2$ fm [14] as the equilibration time.

Since within our model the QGP is equivalent to a resonance gas with medium-dependent widths, all hadronic resonances with lifetime $\Gamma^{-1}(m)$ shorter than $\max\{\tau_f, \tau_{\rm eq}\}$ will have no chance to be formed in the system. Therefore, the upper limit of the integrals over the resonance mass m and over the virtual mass $\sqrt{s}$ in Eq. (1) should be reduced to a resonance mass defined by

$$\Gamma(m)^{-1} = \max\{\tau_f, \tau_{\rm eq}\}. \tag{4}$$

This reduction may essentially weaken the energy density gap at the transition temperature or even make it vanish. Thus, the explicit time dependence should be introduced into the resonance width model (1) while applying it to nuclear collisions, and this finite time (size) effect, as we discussed, may change essentially the thermodynamics of the hadron resonances formed in the nucleus–nucleus collisions.

4. The statistical bootstrap model allows one to interpret the QGP as the hadron resonance gas dominated by the state of infinite mass (and infinite volume). As we show, it is necessary to include the resonance width in the SBM in order to avoid the contradiction with experimental data on hadron spectroscopy. We found that the simple model (1)–(3) may not only eliminate the divergence of the thermodynamic functions above T_H, but it is also able to successfully describe the lattice QCD data [1] for energy density. Such a model also explains the absence of heavy-resonance contributions in the fit of the experimentally measured particle ratios at SPS and RHIC energies.

However, such a modification of the SBM requires an essential change in our view on QGP: it is conceivable that hadrons of very large masses which should be associated with a QGP cannot be formed in nucleus–nucleus collisions because of their very short lifetime.

It is also necessary to mention that the presented model should be applied to experimental data with care: it can be successfully applied to describe either the quantities associated with the chemical freeze-out, i.e. particle ratios, or spectra of Ω hyperons, ϕ, J/ψ and ψ' mesons that are freezing out at hadronization [15–18]. But as discussed in Refs. [19–21], the model presented here should not be used for the post freeze-out momentum spectra of other hadrons produced in the nucleus–nucleus collisions. Perhaps only such weakly interacting hadrons as Ω, ϕ, J/ψ and ψ' will allow us to test the model presented here.

Acknowledgements. We are grateful to M. I. Gorenstein for interesting discussions and important comments. K. A. B. acknowledges the financial support of DFG grant No. 436 UKR 17/13/03 and the Ministry for Culture and Education of Mecklenburg-Western Pomerania.

REFERENCES

1. *Karsch F., Redlich K., Tawfik A.* // Eur. Phys. J. C. 2003. V. 29. P. 549.
2. *Wetzorke I. et al.* // Nucl. Phys. Proc. Suppl. 2002. V. 106. P. 510;
   Asakawa M., Hatsuda T., Nakahara Y. // Nucl. Phys. A. 2003. V. 715. P. 863.
3. *Hagedorn R.* // Suppl. Nuovo Cim. 1965. V. 3. P. 147;
   Hagedorn R., Ranft J. // Suppl. Nuovo Cim. 1968. V. 6. P. 169.
4. More references can be found in *Letessier J., Rafelski J., Tounsi A.* // Phys. Lett. B. 1994. V. 328. P. 499.
5. *Particle Data Group* // Phys. Rev. D. 2002. V. 66.
6. *Braun-Munzinger P., Heppe I., Stachel J.* // Phys. Lett. B. 1999. V. 465. P. 15;
   Yen G. D., Gorenstein M. I. // Phys. Rev. C. 1999. V. 59. P. 2788;
   Becattini F. et al. // Phys. Rev. C. 2001. V. 64. P. 024901.
7. *Braun-Munzinger P. et al.* // Phys. Lett. B. 2001. V. 518. P. 41;
   Xu N., Kaneta M. // Nucl. Phys. A. 2002. V. 698. P. 306.
8. *Blaschke D. et al.* // Phys. Lett. B. 1985. V. 151. P. 439;
   Röpke G., Blaschke D., Schulz H. // Phys. Rev. D. 1986. V. 34. P. 3499.

9. *Hüfner J., Klevansky S. P., Rehberg P.* // Nucl. Phys. A. 1996. V. 606. P. 260.

10. *Burau G., Blaschke D., Kalinovsky Yu.* // Phys. Lett. B. 2001. V. 506. P. 297.

11. *Gazdzicki M., Gorenstein M. I.* // Phys. Lett. B. 2001. V. 517. P. 250.

12. *Schaffner-Bielich J. et al.* // Nucl. Phys. A. 2002. V. 705. P. 494.

13. *Wang X.-N., Gyulassy M., Plümer M.* // Phys. Rev. D. 1995. V. 51. P. 3436;
Baier R. et al. // Phys. Lett. B. 1995. V. 345. P. 277.

14. See, for instance, *Bass S. A. et al.* // Prog. Part. Nucl. Phys. 1998. V. 41. P. 225.

15. More references can be found in *Bugaev K. A., Gazdzicki M., Gorenstein M. I.* // Phys. Lett. B. 2001. V. 523. P. 255; 2002. V. 544. P. 127; Phys. Rev. C. 2003. V. 68. P. 017901.

16. *Gorenstein M. I., Bugaev K. A., Gazdzicki M.* // Phys. Rev. Lett. 2002. V. 88. P. 132301.

17. *Bass S., Dumitru A.* // Phys. Rev. C. 2000. V. 61. P. 064909.

18. *Teaney D., Lauret J., Shuryak E. V.* // Phys. Rev. Lett. 2001. V. 86. P. 4783; nucl-th/0110037. 2001.

19. *Bugaev K. A.* // Nucl. Phys. A. 1996. V. 606. P. 559; J. Phys. G. 2002. V. 28. P. 1981; Phys. Rev. Lett. 2003. V. 90. P. 252301.

20. *Bugaev K. A., Gorenstein M. I.* nucl-th/9903072.

21. *Bugaev K. A., Gorenstein M. I., Greiner W.* // J. Phys. G. 1999. V. 25. P. 2147; Heavy Ion Phys. 1999. V. 10. P. 333.