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CHARM DISSOCIATION IN A RELATIVISTIC QUARK MODEL

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We calculate the amplitudes and the cross sections of the charm dissociation processes $J/\psi+\pi\to D\bar{D}, D^*\bar{D}\,(\bar{D}^*D), D^*\bar{D}^*$ within a relativistic constituent quark model. We consistently account for the contributions from both the box and triangle diagrams to the dissociation processes. The cross section is dominated by the $D^*\bar{D}$ and $D^*\bar{D}^*$ channels. By summing up the four channels, we find a maximum total cross section of about 2.3 mb at $\sqrt{s}\approx 4.1$ GeV. We compare our results to the results of other model calculations.

Производится расчет амплитуд и сечений процессов распада чарма $J/\psi+\pi\to D\bar{D}$, $D^*\bar{D}\,(\bar{D}^*D), D^*\bar{D}^*$ в рамках релятивистской модели конституентного кварка. Последовательно учитываются вклады диаграмм «ящиков» и треугольных диаграмм в процессы распада. В сечении доминируют $D^*\bar{D}$- и $D^*\bar{D}^*$-каналы. После суммирования четырех каналов оказывается, что сечение становится максимальным, около 2,3 мб, при $\sqrt{s}\approx 4{,}1$ ГэВ. Приводится сравнение наших результатов с результатами других модельных вычислений.

The study of the J/ψ dissociation cross sections is important for the understanding of J/ψ suppression observed in Pb–Pb collisions by the NA50 collaboration at CERN-SPS [1]. There are a number of theoretical calculations on the $c\bar{c}+$ light hadron cross sections (see, e.g., the review [2]). However, they give widely divergent results, which implies that we are still far away from a real understanding of the scattering mechanism. The nonrelativistic quark model has been applied in [3] and [4–6] for the calculation of the dissociation process $c\bar{c}+q\bar{q}\to c\bar{q}+q\bar{c}$ cross sections. The calculated cross sections for the reactions $J/\psi+\pi\to D\bar{D}$, $D^*\bar{D}+\bar{D}^*$, $D^*\bar{D}^*$ have the following common features: they rise very fast from zero at threshold to a maximum value and finally fall off due to the Gaussian form of the potential. The magnitude of the maximum total cross section was found to be ≈ 7 mb at $\sqrt{s}\approx 4.1$ GeV in [3] and a somewhat smaller value of ≈ 1.4 mb at $\sqrt{s}\approx 3.9$ GeV in [4–6].

Another direction to explore the charm dissociation process has been taken in [7–12], where an effective chiral $SU(4)$ Lagrangian was employed. Such an approach looks quite dubious for several reasons: $SU(4)$ is badly broken, mesons are treated as pointlike particles, some of the couplings in the Lagrangian are unknown. Nevertheless, this is a relativistic approach which allows one to study the above processes in a systematic fashion. In this

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approach the mesons were assumed to be pointlike, resulting in very large values of the cross sections ($\approx$ 30 mb at $\sqrt{s} \approx 4.5$ GeV) which rise with energy. By adopting rather arbitrary damping form factors, these cross sections were significantly reduced.

It appears that the microscopic nature of hadrons is important in the charm dissociation processes. The first step is to calculate the relevant form factors corresponding to the triple and quartic meson vertices in the kinematical region of the dissociation reaction. QCD sum rules have been used in Refs. [14–17] to evaluate those form factors and to determine the charm cross section. It was found [17] that the cross section is about 1 mb at $\sqrt{s} \approx 4.1$ with a monotonic growth when the energy is increased.

We also mention the work of Deandrea et al. where the strong couplings $J/\psi D^* D^*$ and $J/\psi D^* D^* \pi$ were evaluated in the constituent quark model [18, 19]. Finally, an extension of the finite-temperature Dyson–Schwinger equation (DSE) approach to heavy mesons and its application to the reaction $J/\psi + \pi \to D + \bar{D}$ was considered in [20].

We employ a relativistic quark model [21] to calculate the charm dissociation amplitudes and cross sections. This model is based on an effective Lagrangian which describes the coupling of hadrons H to their constituent quarks. The coupling strength is determined by the compositeness condition $Z_H = 0$ [23], where Z_H is the wave function renormalization constant of the hadron H. One starts with an effective Lagrangian written down in terms of quark and hadron fields. Then, by using Feynman rules, the S-matrix elements describing the hadronic interactions are given in terms of a set of quark diagrams. In particular, the compositeness condition enables one to avoid a double counting of the hadronic degrees of freedom. The approach is self-consistent and universally applicable. All calculations of physical observables are straightforward. The model has only a small set of adjustable parameters given by the values of the constituent quark masses and the scale parameters that define the size of the distribution of the constituent quarks inside a given hadron. The values of the fit parameters are within the window of expectations.

The shape of the vertex functions and the quark propagators can in principle be found from an analysis of the Bethe–Salpeter and Dyson–Schwinger equations as was done, e.g., in [26]. In this paper, however, we choose a phenomenological approach where the vertex functions are modelled by a Gaussian form, the size parameter of which is determined by a fit to the leptonic and radiative decays of the lowest lying charm and bottom mesons. For the quark propagators we use the local representation.

We calculate the amplitudes and the cross sections of the charm dissociation processes

$$\begin{aligned} J/\psi + \pi &\to D + \bar{D}, \\ J/\psi + \pi &\to D^* + \bar{D} \;\; (\bar{D}^* + D), \\ J/\psi + \pi &\to D^* + \bar{D}^*. \end{aligned}$$

These processes are described by both box and resonance diagrams which are calculated straightforwardly in our approach. We compare our results with the results of other studies.

The coupling of a meson $H(q_1\bar{q}_2)$ to its constituent quarks q_1 and $\bar{q}_2$ is determined by the Lagrangian

$$\mathcal{L}_{\rm int}^{\rm Str}(x) = g_H H(x) \int\! dx_1 \!\int\! dx_2 F_H(x, x_1, x_2) \bar{q}_2(x_2) \Gamma_H \lambda_H q_1(x_1) \; + \text{h.c.} \tag{1}$$

Here, λ_H and Γ_H are Gell-Mann and Dirac matrices which describe the flavor and spin quantum numbers of the meson field $H(x)$. The function F_H is related to the scalar part of the Bethe–Salpeter amplitude and characterizes the finite size of the meson. To satisfy translational invariance, the function F_H has to fulfil the identity $F_H(x+a, x_1+a, x_2+a) = F_H(x, x_1, x_2)$ for any 4-vector a. In the following we use a particular form for the vertex function

$$F_H(x, x_1, x_2) = \delta(x - c_{12}^1 x_1 - c_{12}^2 x_2)\Phi_H((x_1 - x_2)^2), \tag{2}$$

where Φ_H is the correlation function of two constituent quarks with masses m_1, m_2 and $c_{ij}^i = m_i/(m_i + m_j)$.

The coupling constant g_H in Eq. (1) is determined by the so-called *compositeness condition* originally proposed in [23], and extensively used in [22]. The compositeness condition requires that the renormalization constant of the elementary meson field $H(x)$ is set to zero:

$$Z_H \, = \, 1 - \frac{3g_H^2}{4\pi^2}\,\tilde{\Pi}'_H(M_H^2) \, = \, 0, \tag{3}$$

where $\tilde{\Pi}'_H$ is the derivative of the meson mass operator. In order to clarify the physical meaning of this condition, we note that $Z_H^{1/2}$ is also interpreted as the matrix element between a physical particle state and the corresponding bare state. For $Z_H = 0$ it then follows that the physical state does not contain the bare one and is described as a bound state. The interaction Lagrangian in Eq. (1) and the corresponding free Lagrangian describe both the constituents (quarks) and the physical particles (hadrons) which are bound states of the quarks. As a result of the interaction, the physical particle is dressed; i.e., its mass and wave function have to be renormalized. The condition $Z_H = 0$ also effectively excludes the constituent degrees of freedom from the physical space and thereby guarantees that a double counting of physical observables is avoided. The constituent quarks exist in virtual states only. One of the corollaries of the compositeness condition is the absence of a direct interaction of the dressed charge particle with the electromagnetic field. Taking into account both the tree-level diagram and the diagrams with the self-energy insertions into the external legs yields a common factor Z_H which is equal to zero. We refer the interested reader to our previous papers [21, 22, 24] where these points are discussed in more detail.

For the pseudoscalar and vector mesons treated in this paper, the derivatives of the mass operators are written as

$$\begin{aligned}
\tilde{\Pi}'_P(p^2) &= \frac{1}{2p^2}\, p^\alpha \frac{d}{p^\alpha} \int \frac{d^4k}{4\pi^2 i} \tilde{\Phi}_P^2(-k^2)\, \mathrm{tr}\left[\gamma^5 S_1(\not k + c_{12}^1 \not p)\gamma^5 S_2(\not k - c_{12}^2 \not p)\right], \\
\tilde{\Pi}'_V(p^2) &= \frac{1}{3}\left[g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right] \frac{1}{2p^2}\, p^\alpha \frac{d}{p^\alpha} \times \\
&\times \int \frac{d^4k}{4\pi^2 i} \tilde{\Phi}_V^2(-k^2)\, \mathrm{tr}\left[\gamma^\nu S_1(\not k + c_{21}^1 \not p)\gamma^\mu S_2(\not k - c_{12}^2 \not p)\right].
\end{aligned} \tag{4}$$

The leptonic decay constant f_P is calculated from

$$\frac{3g_P}{4\pi^2} \int \frac{d^4k}{4\pi^2 i} \tilde{\Phi}_P(-k^2)\, \mathrm{tr}\left[O^\mu S_1(\not k + c_{12}^1 \not p)\gamma^5 S_2(\not k - c_{12}^2 \not p)\right] = f_P\, p^\mu.$$

$$\frac{3g_V}{4\pi^2} \int \frac{d^4k}{4\pi^2 i} \tilde{\Phi}_V(-k^2)\, \mathrm{tr}\left[O^\mu S_1(\not k + c_{12}^1 \not p)\gamma\epsilon_V S_2(\not k - c_{12}^2 \not p)\right] = m_V\, f_V\, \epsilon_V^\mu\,,$$

We use free fermion propagators for the valence quarks

$$S_i(\not k) = \frac{1}{m_i - \not k} \tag{5}$$

with an effective constituent quark mass m_i. As discussed in [21, 22] we assume for the meson mass M_H that

$$M_H < m_1 + m_2 \tag{6}$$

in order to avoid the appearance of imaginary parts in the physical amplitudes. This holds true for the light pseudoscalar mesons but is no longer true for the light vector mesons. We shall therefore employ identical masses for the pseudoscalar mesons and the vector mesons in our matrix element calculations, but use physical masses in the phase space calculation. This is quite a reliable approximation for the heavy mesons, e.g., D^* and B^* whose masses are almost the same as the D and B, respectively.

The shape of the vertex functions and the quark propagators can in principle be found from an analysis of the Bethe–Salpeter and Dyson–Schwinger equations as was done, e.g., in [25, 26].

We choose a phenomenological approach where the vertex functions are modelled by a Gaussian form, the size parameter of which is determined by a fit to the leptonic and radiative decays of the lowest lying charm and bottom mesons. As discussed above, we use the local representation for the quark propagators. Our previous studies [21, 22] have shown that such an approximation is successful and reliable with its application to phenomena involving the low-lying hadrons. We employ a Gaussian for the vertex function $\tilde{\Phi}_H(k_E^2) \doteq \exp(-k_E^2/\Lambda_H^2)$, where k_E is an Euclidean momentum. The size parameters Λ_H^2 are determined by a fit to experimental data, when available, or to lattice results for the leptonic decay constants f_P where $P = \pi, D, B$. Here we improve the fit by using the MINUIT code in a least-square fit. The values of the fitted parameters are displayed in Table 2, whereas the quality of the fit may be seen from Table 1.

Table 1. **The physical quantities used in the least-square fitting our parameters. The values are taken from the PDG [28] or from lattice simulations [27]. The value of f_{B_c} is our average of QCD sum rules calculations [13]. All numbers are given in MeV except for $g_{\pi^0\gamma\gamma}$**

Quantity	This model	Experiment/Lattice	Quantity	This model	Experiment/Lattice
f_π	130.7	$130.7 \pm 0.1 \pm 0.36$	$g_{\pi^0\gamma\gamma}$	0.272 GeV^{-1}	0.273 GeV^{-1}
f_K	159.8	$159.8 \pm 1.4 \pm 0.44$	$f_{J/\psi}$	405	405 ± 17
f_D	211	203 ± 14 226 ± 15	f_B	182	173 ± 23 198 ± 30
f_{D_s}	244	230 ± 14 250 ± 30	f_{B_s}	209	200 ± 20 230 ± 30
f_{B_c}	360	360	f_Υ	710	710 ± 37

Table 2. **The fit values of the model parameters**

Quark masses, GeV		Λ_H, GeV		Λ_H, GeV	
$m_u = m_d$	0.223	Λ_π	1.074	Λ_{B_c}	1.959
m_s	0.356	Λ_K	1.514	$\Lambda_{J/\psi}$	2.622
m_c	1.707	$\Lambda_D = \Lambda_{D_s}$	1.844	Λ_Υ	3.965
m_b	5.121	$\Lambda_B = \Lambda_{B_s}$	1.887		

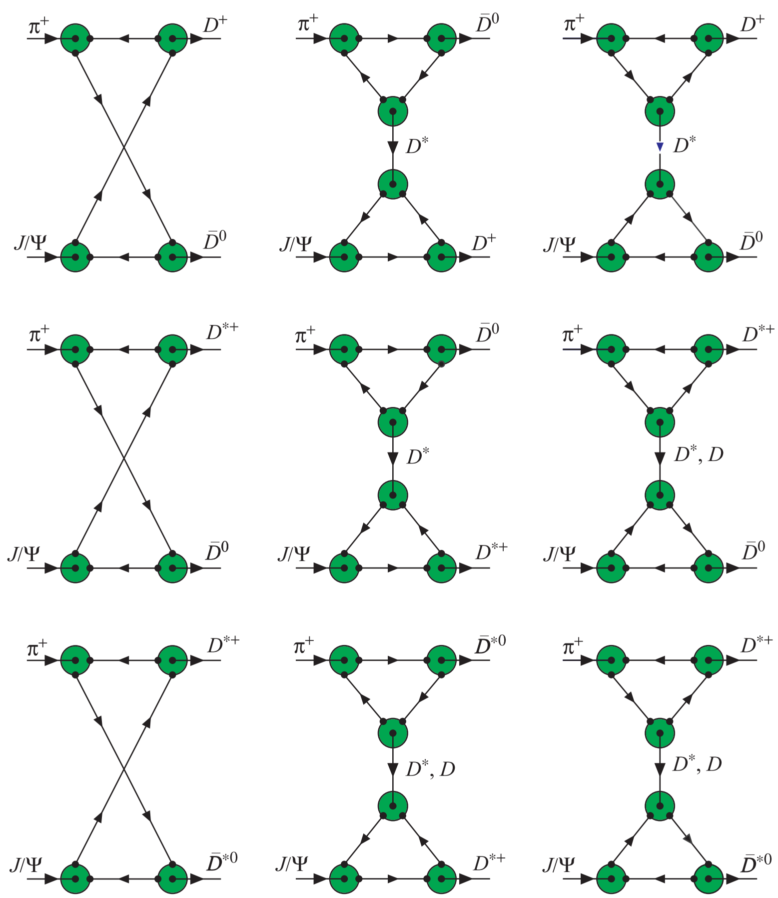

Fig. 1. The Feynman diagrams describing charm dissociation processes $J/\psi + \pi^+ \to D^+ + \bar{D}^0$, $D^{*+} + \bar{D}^0$, $D^{*+} + \bar{D}^{*0}$

In our approach the dissociation processes $J/\psi + \pi^+ \to D + \bar{D}^0$, $D^{*+} + \bar{D}^0$ and $D^{*+} + \bar{D}^{*0}$ are described by the diagrams in Fig. 1.

The kinematics of the processes is defined as

$$J/\psi(p_1) + \pi^+(p_2) \to D_3^+(q_1) + \bar{D}_4^0(q_2), \tag{7}$$

where $D_3^+ = D^+$ or D^{*+}, $\bar{D}_4^0 = \bar{D}^0$ or $\bar{D}^{*0}$, $p_1^2 = m_1^2 \equiv m_{J/\psi}^2$, $p_2^2 = m_2^2 \equiv m_\pi^2$, $q_1^2 = m_3^2 \equiv m_{D^+}^2\,(m_{D^{*+}}^2)$, $q_2^2 = m_4^2 \equiv m_{\bar{D}^0}^2\,(m_{\bar{D}^{*0}}^2)$.

The cross section is calculated by using the formula

$$\sigma(s) = \frac{1}{192\pi s}\frac{1}{p_{1,\mathrm{cm}}^2}\int\limits_{t_-}^{t_+} dt\,|M(s,t)|^2 \tag{8}$$

where $M(s,t)$ is an invariant amplitude and

$$\begin{aligned} t_\pm &= (E_{1,\mathrm{cm}} - E_{3,\mathrm{cm}})^2 - (p_{1,\mathrm{cm}} \mp q_{1,\mathrm{cm}})^2, \\ E_{1,\mathrm{cm}} &= \frac{s + m_1^2 - m_2^2}{2\sqrt{s}}, \qquad E_{3,\mathrm{cm}} = \frac{s + m_3^2 - m_4^2}{2\sqrt{s}}, \\ p_{1,\mathrm{cm}} &= \frac{\lambda^{1/2}(s, m_1^2, m_2^2)}{2\sqrt{s}}, \qquad q_{1,\mathrm{cm}} = \frac{\lambda^{1/2}(s, m_3^2, m_4^2)}{2\sqrt{s}}. \end{aligned}$$

The reaction threshold is equal to $s_0 = (m_3 + m_4)^2$. Note that Eq. (8) contains the statistical factor 1/3 which comes from averaging over the J/ψ polarizations.

The dissociation processes are described by both the box and the resonance diagrams as shown in Fig. 1. The resonance diagrams depend only on t or u variables, whereas the box diagrams are the functions of s and t variables.

The behavior of the $F_{PVP}^{(1a)}(t)$ and $F_{VVP}^{(a)}(t)$ in the kinematical region is shown in Fig. 2. In order to be able to compare with other calculations, we quote the value $F_{PVP}^{(1a)}(t)$ at $t = m_{D^*}$ which is equal to 22. We cannot go on mass-shell in the corresponding form factor $F_{J/\psi D^*\pi}^{(a)}(t)$ due to the presence of an anomalous threshold.

The dependence of $F_{VPPP}(s,t)$ on $\sqrt{s}$ at $t = 0$ is shown in Fig. 3.

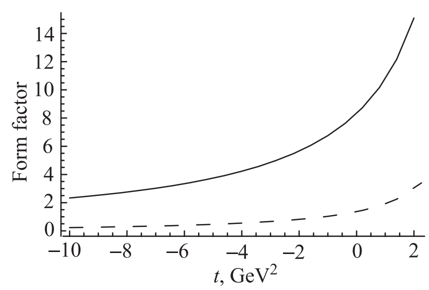

Fig. 2. Calculated forms of $F_{\pi D^* D}^{(1a)}(t)$ (solid line) and $F_{J/\psi D^* D}^{(a)}(t)$ (dashed line) in the physical region of the invariant variable t which is the D^*-momentum squared

The total cross section is a sum over all channels:

$$\sigma_{\mathrm{tot}}(s) = \sigma_{D^+\bar{D}^0}(s) + \sigma_{D^{*+}\bar{D}^0}(s) + \sigma_{D^+\bar{D}^{*0}} + \sigma_{D^{*+}\bar{D}^{*0}}(s). \tag{9}$$

Note that $\sigma_{D^+\bar{D}^{*0}} = \sigma_{D^{*+}\bar{D}^0}$. We plot $\sigma_{\mathrm{tot}}(s)$ as a function of $\sqrt{s}$ in Fig. 4. One can see that the maximum is about 2.3 mb at $\sqrt{s} \approx 4.1$ GeV. This is close to the result obtained in [4–6].

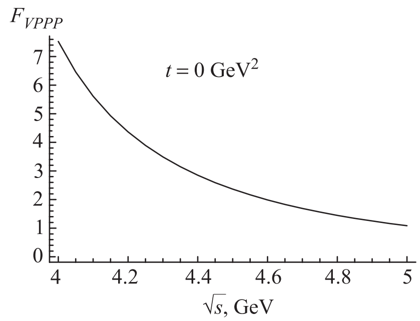

Fig. 3. Form factor $F_{VPPP}(s,t)$ at $t = 0$

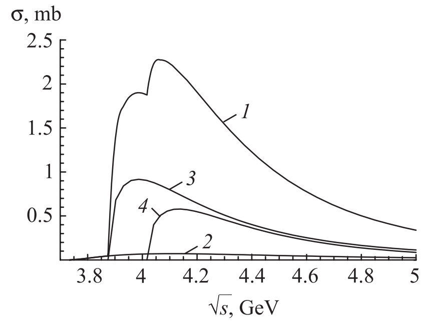

Fig. 4. The total cross section (*1*) with all contributions: $D\bar{D}$ (*2*), $D^*\bar{D}$ (*3*), $D^*\bar{D}^*$ (*4*)

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