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POLARIZED BEAMS IN STORAGE RINGS

Yu. M. Shatunov

Budker Institute of Nuclear Physics, Novosibirsk, Russia

This paper reviews modern technics to accelerate polarized particles to high energy and to preserve their polarization in the storage rings. Possibilities of the beam polarization control are discussed for proton and electron machines.

В докладе дается обзор современного состояния теории и экспериментальной техники для ускорения поляризованных частиц до высоких энергий и сохранения их поляризации в накопителях. Обсуждаются возможности управления направлением и степенью поляризации для протонных и электронных машин.

INTRODUCTION

Spin is a quantum number of an internal angular momentum connected with a particle magnetic moment. This statement in its time could solve many discrepancies in the atom theory. Later it has appeared that the spin considerably contributes to particle interactions at high energies and puts many puzzles to experimentalists and theorists. Due to this fact, there is increasing interest in the availability of spin-polarized beams at high energy and nuclear physics experiments.

It is well known that particle motion in modern accelerators is described with extremely high accuracy by the semiclassical approach. But for the spin, there is no quasi-classical limit when orbital quantum numbers are large. Even at highest energies, an electron or a proton remains in eigenstates «up» or «down» as particles with spin $1/2$ (in units of $\hbar$). The quantum operator for this spin is $\hat{\mathbf{S}} = 1/2\hat{\boldsymbol{\sigma}}$, where

$$\hat{\boldsymbol{\sigma}} = \boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z) = \left[\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right]$$

are Pauli matrices.

Following the Ehrenfest theorem, we determine in the particle rest frame a classical vector of spin in any state $|\psi\rangle$ as a quantum average of spin operator $\mathbf{S} = \langle \psi^\dagger | \hat{\boldsymbol{\sigma}} | \psi \rangle$. This vector $\mathbf{S}$ precesses in the rest frame around magnetic field $\mathbf{B}_c$ together with particle magnetic moment $\boldsymbol{\mu} = q\mathbf{S}$:

$$\frac{d\mathbf{S}}{d\tau} = \boldsymbol{\Omega}_c \times \mathbf{S}.$$

Spin precession frequency $\boldsymbol{\Omega}_c = -(q_0 + q')\mathbf{B}_c$, where $q_0 = e/m$ and q' are normal and anomalous parts of gyromagnetic ratio q. (We shall take later $c = \hbar = 1$.)

A relativistic generalization of this spin motion equation to a laboratory frame has been done in different ways by many authors (see, for example, [1]). The most easy-to-use for accelerator applications view can be presented in the following form:

$$\mathbf{S}' = \frac{d\mathbf{S}}{d\theta} = \mathbf{W}(\theta) \times \mathbf{S} \tag{1}$$

with

$$\mathbf{W}(\theta) = -\frac{q_0}{\gamma}\Big[\Big(1+\gamma\, a\Big)\,\mathbf{B}_\perp + (1+a)\,\mathbf{B}_\| + \Big(\frac{\gamma}{\gamma+1} + \gamma\, a\Big)\,\mathbf{E}\times\mathbf{V}\Big].$$

Here we decomposed the magnetic fields in two projections $\mathbf{B}_\|$ and $\mathbf{B}_\perp$ (along and perpendicular to particle velocity $\mathbf{V}$) and introduced so-called *magnetic anomaly* of a particle $a = q'/q_0$ and the generalized accelerator azimuth θ instead of time t.

The magnetic moment anomaly is a fundamental property of a particle on a level of its mass. By now, thanks to many measurements, magnetic anomalies of various particles are known with high accuracy. For example, for electron $a_e = 1.1596521859 \cdot 10^{-3} \pm 3.8 \cdot 10^{-12}$ and for proton $a_p = 1.792847351 \pm 2.8 \cdot 10^{-8}$.

Before proceeding further we analyze some of the essential features of the above expressions:

• It is important to note that $\mathbf{s}$ is expressed in the *rest frame*, whereas $\mathbf{E}$ and $\mathbf{B}$ are the fields in the *laboratory frame*.

• q_0 and q' contribute differently to spin rotation by electric and magnetic fields (depending on parameter $\nu_0 = \gamma\, a$), whereas the particle revolution frequency is determined only by q_0:

$$\boldsymbol{\omega} = -\frac{q_0}{\gamma}\left[\mathbf{B}_\perp + \frac{\gamma^2}{\gamma^2-1}\,\mathbf{E}\times\mathbf{V}\right]. \tag{2}$$

• At low energies combinations of electric and magnetic fields are used to control spin orientation in polarized particle sources.

• In cases $\nu_0 = 1 \div 10$ combinations of longitudinal and transverse magnetic fields are applied to deliver the required beam polarization to an experiment area. For much higher energies ($\nu_0 > 10$) spin rotations by transverse magnetic fields are more effective.

1. SPIN CLOSED ORBIT

Let us remind the approach which is used for the orbital motion. Particle coordinates are given by the radius vector $\mathbf{R}(\theta) = \mathbf{R}_0(\theta) + \mathbf{r}$, where $\mathbf{R}_0(\theta)$ presents a periodical closed orbit. The vector $\mathbf{r} = x\mathbf{e}_x + z\mathbf{e}_z$ describes small deviation from CO — horizontal and vertical betatron oscillations with corresponding tunes ν_x and ν_z.

Following this approach, we share the spin precession frequency in two parts [2]: $\mathbf{W}(\theta) = \mathbf{W}_0(\theta) + \mathbf{w}$. The periodical part $\mathbf{W}_0(\theta + 2\pi) = \mathbf{W}_0(\theta)$ gives spin rotations by fields on the CO, whereas $\mathbf{w}$ is a small distortion ($|\mathbf{w}| \ll |\mathbf{W}_0|$) connected with momentum off particle oscillations. It is evident the solution of the spin motion equation (1) at any azimuth θ with $\mathbf{W}(\theta) = \mathbf{W}_0(\theta)$ is a periodical unit vector $\mathbf{n}_0(\theta + 2\pi) = \mathbf{n}_0(\theta)$, which is the spin precession axis. A spin rotation around $\mathbf{n}_0(\theta)$ by an angle ϕ substitutes all spin rotations by arbitrary local fields along the CO. Similarly to the betatron tunes, we define the spin tune $\nu = \phi/(2\pi)$. Two other (perpendicular to $\mathbf{n}_0$) eigen solutions of the spin equation are complex vectors $\boldsymbol{\eta}$ and $\boldsymbol{\eta}^*$ rotating clockwise and contraclockwise around $\mathbf{n}_0$ with the spin tune: $\boldsymbol{\eta}(\theta + 2\pi) = \boldsymbol{\eta}(\theta)\,\mathrm{e}^{-i\nu\theta}$.

A precession axis for spin of momentum off particles slightly differs from $\mathbf{n}_0$ and can be found in the form $\mathbf{n} = \sqrt{1 + |C|^2}\,\mathbf{n}_0 + \mathrm{Re}\,(iCbfg\eta^*)$; $|C| \ll 1$. Putting this $\mathbf{n}$ into (1), we come in the linear approximation to the short-cut equation

$$C' = w_\perp = (\mathbf{w} \cdot \boldsymbol{\eta}^*)\,. \tag{3}$$

2. IDEAL FLAT MACHINE

We proceed in our consideration to an ideal flat machine with a uniform vertical magnetic field $K_z = B_z/\langle B_z\rangle$. In this case it is clear that $\mathbf{n}_0$ coincides everywhere with the unit vector along the guiding field:

$$\mathbf{n}_0 = \mathbf{e}_z \quad \text{and} \quad \boldsymbol{\eta} = (\mathbf{e}_x - i\mathbf{e}_y)\,\mathrm{e}^{-i\nu_0\tilde{\theta}},$$

where

$$\tilde{\theta} = \int_0^\theta K_z d\theta.$$

From (1) and (2) we see that spin tune $\nu = 1 + \nu_0$. But in the accelerating frame $(\mathbf{e}_x;\, \mathbf{e}_y;\, \mathbf{e}_z)$ after one particle turn spin rotates only by angle $\phi = 2\pi\nu_0 = \gamma\, a$.

This fact has a very important practical consequence. Since the values of magnet anomalies a are known to great accuracy, one can deduce the γ-factor with high precision from spin tune measurement. A knowledge of the particle mass immediately gives an absolute beam energy calibration. An experimental technique for that is radiofrequency magnetic field applied in horizontal plane to kick the spin. On resonant frequency the kicks add up to a beam depolarization. This can generally be done with great accuracy. This energy calibration method was called *resonant depolarization* and by now it has been applied with success at many accelerators [3].

2.1. Spin Resonances. Focusing elements are unavoidable in any storage ring, as also are the betatron oscillations. It's appeared immediately as a distortion for the spin motion. A vertically deviated particle meets the radial component of the focusing magnetic fields. By using the particle motion equation, this distortion can be described by $w_\perp = \nu_0 z'' = \nu_0 g_z z$. Putting that in (3), one finds that $\mathbf{n}$ oscillates around $\mathbf{n}_0$ with the betatron tunes ν_z. In the resonance case $\nu = \nu_k = k \pm \nu_z$ (so-called «intrinsic resonances») spin will rotates around the horizontal axis with a precession frequency w_k, which is the resonance strength

$$|w_k| = |A_z|\frac{\nu_0}{2\pi}\oint g_z|f_z|\,\mathrm{e}^{i(\nu_k \mp \nu_z)\tilde{\theta}}d\theta,$$

where we used the Floke form for the solution of z-motion equation: $z = A_z f_z + \mathrm{c.c.}$ One can see that the strength of the intrinsic resonances enhances with the particle energy and the vertical oscillation amplitude. Calculations showed, when $\nu_0 \simeq 100$ at any resonances $k = mP$ (P is a machine periodicity) $|w_k| \sim (0.1 \div 0.3)\omega_0$ by $A_z \simeq 1$ mm (see Fig. 1).

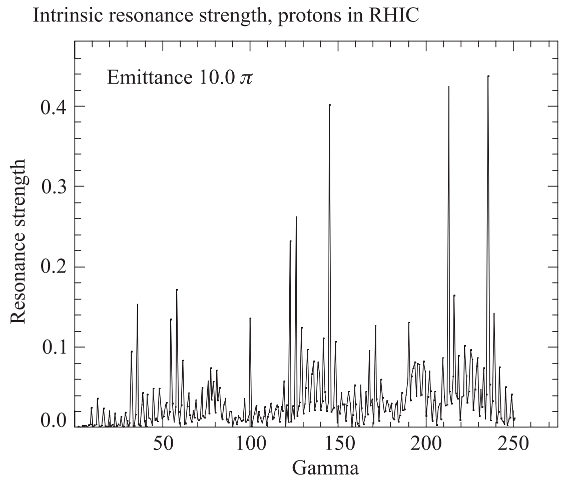

Fig. 1. Intrinsic resonance strengths in a RHIC lattice (normalized vertical emittance of 10 $\pi \cdot$ mm $\cdot$ mrad)

Other spin distortions connect with vertical deviations of the CO caused by radial imperfection fields $K_x = B_x/\langle B_z \rangle$. In this case, putting into (3) the forced periodical part of the vertical motion Z_s, we get strengths of imperfection resonances $\nu = k$:

$$|w_k| = \frac{\nu_0}{2\pi} \oint Z_s'' \, \mathrm{e}^{-ik\tilde{\theta}} d\theta = \frac{\nu_0}{2\pi} \oint K_x F_3(\theta) \, \mathrm{e}^{-ik\tilde{\theta}} d\theta,$$

where $F_3(\theta) = dn/dz'$ is a spin response function, which reflects a sensitivity of $\mathbf{n}$ vector to vertical kicks [4]. Most strong imperfection resonances have also $k = mP$ (m — *integer)*. They increase with the energy faster than the intrinsics ($F_3 \sim \gamma$). But unlike the intrinsic resonances, one can adjust the vertical closed orbit to minimize w_k up to level depending on his experimental technics.

3. SPIN RESONANCE CROSSING

Since the spin tune is proportional to the energy spin resonance crossings are unavoidable at an acceleration. A gap between two imperfection resonances is equal to 440.652 MeV for electrons and 523.342 MeV for protons. The intrinsic resonances are located symmetrically around each imperfection resonance. So, the acceleration of polarized particles looks like a very complicated issue. In the simplest case of a separate resonance $\nu = \nu_k$ with a strength w_k a final polarization ζ_F after one crossing with tune rate $\dot{\delta} = (d(\nu - \nu_r))/dt$ will be different from the initial one: $\zeta_F = \zeta_0 \left(2\mathrm{e}^{-\Psi} - 1\right)$, where $\Psi = \pi w_k^2/2\dot{\delta}$ is a spin phase advance in the resonance zone (tune $\delta \sim w_k$) [5].

When $\Psi \ll 1$ (fast crossing) polarization loss is small: $\delta\zeta \simeq \zeta_0 \Psi$. More interesting is the opposite case: $\Psi \gg 1$. It leads to a spin flip ($\zeta_F \simeq -\zeta_0$) with an exponentially low depolarization: $|\delta\zeta| = 2\zeta_0 \, \mathrm{e}^{-\Psi}$. Both situations are widely used in accelerator practice. A suppression of the resonance strength (orbit corrections) or increasing tune rate (tune jump) leads to the fast crossing and vice versa an artificial resonance enhancement helps to safely reverse the polarization.

Let us insert in the ideal flat machine at a part of the orbit $0 \div \theta_1$ a solenoid with the longitudinal magnet field $K_y = B_y/\langle B_z \rangle$, which rotates spin by an angle

$$\phi = \frac{q}{2\pi} \int_0^{\theta_1} K_y d\theta.$$

Exploring this simple model, we shall give an easy-to-use way to find the spin tune and the $\mathbf{n}_0$ axis. A general form of a unitary $SU2$ matrix of a vector rotation by an angle ϕ around a unit vector $\mathbf{n}_j$ can be presented as $T_j = \cos(\phi/2) - i(\mathbf{n}_j \cdot \boldsymbol{\sigma}) \sin(\phi/2)$. The one turn spin map will be product of the local matrices T_j: $T = T_N T_{N-1} \cdots T_2 T_1$, which has the same unitary form with $\mathbf{n}_j = \mathbf{n}_0$. Hence, one finds easily $\cos(\pi\nu) = 1/2\,\mathrm{Tr}\,(T)$ and $\mathbf{n}_0 = -i/\sin(\pi\nu)\,\mathrm{Tr}\,(\boldsymbol{\sigma} \cdot T)$.

Practising to a machine with one solenoid, we see that the solenoid shifts the spin tune $\cos(\pi\nu) = \cos(\pi\nu_0)\cos(\phi/2)$ and a direction of the vector $\mathbf{n}_0$ is no longer vertical and depends on the solenoid strength and spin tune.

If the solenoid is short, $\theta_1 \ll 1$, we can take it as δ function like distortion and expand it in the series of resonances $\nu = k$ with equal strengths $|w_k| = \phi/2\pi$.

So, it is possible to provide the adiabatic resonance crossing by increasing the solenoidal field. This method was called a partial Siberian snake and is successfully used at many machines after its first test at VEPP-2M storage ring in 1976 [6].

The same adiabatic approach to an intrinsic resonance crossing can be realized by RF dipole working nearby the vertical tune. Under such a condition the spin response function blows up $F_3 \gg 1$ and enhances the resonance harmonics many times. This method is successfully used on strongest intrinsic resonances at AGS [7].

4. SIBERIAN SNAKES

An interesting situation occurs when the rotation angle by the solenoid $\phi = \pi$. As is easy to see, the spin tune in this case is always equal to $1/2$ independently of ν_0. It means a total exclusion of the spin resonances at acceleration. Moreover, the vector $\mathbf{n}_0$ is longitudinal along the drift opposite to the snake insertion [8].

Spin rotators with similar properties have been named «Siberian snakes». They can be designed from combinations of longitudinal and transverse fields suitable for each specific case with one main requirement to be matched with machine lattice. First test of the Siberian snake idea (solenoid at the IUCF proton storage ring) has shown full suppression imperfection resonances as well as intrinsic resonances [9]. To minimize machine optics and orbit distortions, a snake transfer matrix has been done equal to the matrix of a drift L occupied by the insertion:

$$M_x = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}; \quad M_z = -M_x.$$

A similar approach is proposed for future HESR storage ring ($3.5 \div 15$ GeV). Two pairs of solenoids and rotating quads together with the electron cooling solenoid will realize the optics of the 56 m straight. An angle of each quad rotation depending on fields strengths and beam energy can be designed as a field superposition of two superconducting coils — regular quadrupole one and second rotated by 45°.

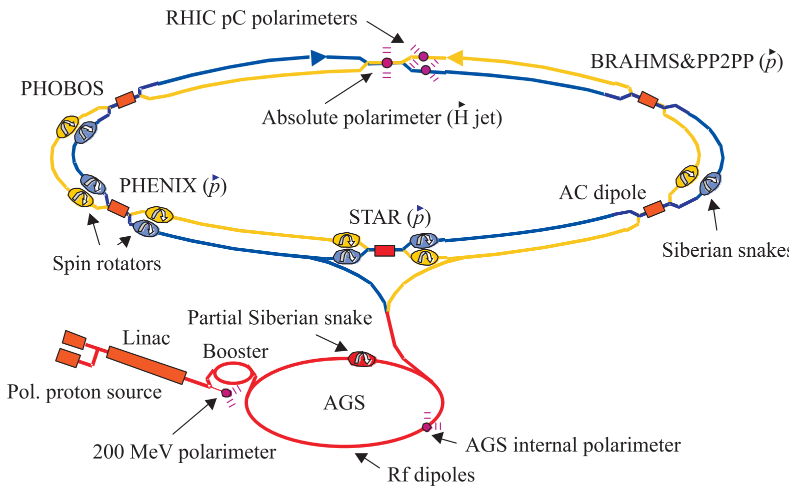

Fig. 2. Schematic layout of BNL complex for polarized proton operations

For much higher energy, as we noticed before, transverse fields are more effective for spin rotations. It turned out that helical magnetic field configuration is most convenient for these purposes, because it gives less orbit deviations in comparison with regular dipoles.

A scheme from four full twist helical magnets with mirror symmetry of the fields polarity and helicity is able to rotate spin by 180° around arbitrary axis in the horizontal plane. Moreover, the same magnet combination can work as a spin rotator from vertical direction to a position in the medium plane by any angle to the velocity [10]. Such a scheme of Siberian snakes and spin rotators was realized at RHIC (see Fig. 2). Two snakes in each ring with axis angles $\pm 45^\circ$ also provide the spin tune $\nu = 1/2$ and opposite vertical polarization in the arcs. Four pairs of the spin rotators deliver the longitudinal polarization in collision points for detectors STAR and PHENIX. Recently at RHIC, polarized protons in both rings were accelerated up to the top energy 200 GeV.

5. RADIATIVE POLARIZATION OF ELECTRONS

Electron radiates energy while accelerated. An intensity of this synchrotron radiation enhances $\sim \gamma^4$ and at high energy it determines all beam parameters. Due to the quantum nature of this radiation the orbital motion gets stochastic kicks. For their turn, the orbit jumps lead to fluctuations of the precession axis $\mathbf{n}(\theta)$ and to a spin diffusion — random changes of spin projections S_n. This effect spreads considerably the spin resonances, thus the acceleration of polarized electrons is possible only up to a few GeV.

Fortunately aggravation of electron spin by radiative effects is more than compensated for by electron self-polarization due to small difference in probabilities of spin-flip quantum emission in states «spin up» and «spin down». A calculation of this effect in the homogeneous magnetic field [11] has shown that an equilibrium degree of beam polarization $\zeta = 8/5\sqrt{3} \simeq 0.924$ builds up with a characteristic time τ_p, which can be written (in practical units) as $\tau_p(\mathrm{h}) \approx \dfrac{(R/\rho)}{[B(\mathrm{T})]^3[E(\mathrm{GeV})]^2}$. It is easy to estimate that the radiative polarization is available in most of the electron–positron colliders and light source storage rings. It is important to remark that radiative polarization is unique method for obtaining high-energy polarized positrons.

However, real fields in storage rings are far from homogeneous. More full study of the radiative polarization in real machine fields brings to the equilibrium polarization level $\zeta = \alpha_-/\alpha_+$ and polarization time $\tau = \alpha_+$ with [12]:

$$\alpha_- = q_0^5\gamma^2\langle|\mathbf{B}|^3\,\mathbf{b}\,(\mathbf{n}_0 - \mathbf{d})\rangle_\theta; \quad \alpha_+ = \frac{5\sqrt{3}}{8}q_0^5\gamma^2\left\langle|\mathbf{B}|^3\left[1 - \frac{2}{9}(\mathbf{n}_0\cdot\mathbf{V}) + \frac{11}{18}\mathbf{d}^2\right]\right\rangle_\theta, \quad (4)$$

where $\mathbf{b}$ is the unit vector along $\mathbf{B}$ and $\mathbf{d} = \gamma(\partial\mathbf{n}/\partial\gamma)$ is a spin-orbit coupling vector that shows a sensitivity of $\mathbf{n}$ axis to the orbital kicks by quantum emission. It is clear that a rate of the spin diffusion has to be $\sim \mathbf{d}^2$.

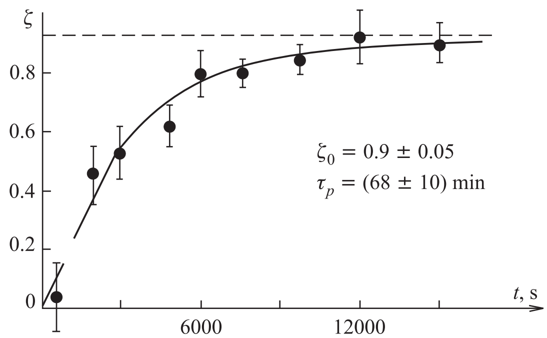

Fig. 3. The polarization buildup in VEPP-2M (1976)

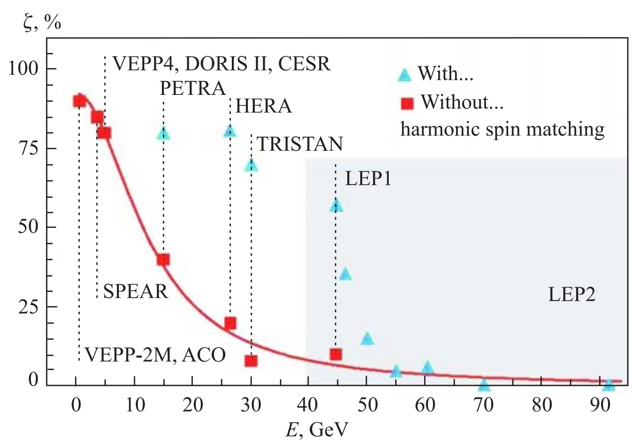

Fig. 4. The maximum attained asymptotic polarization levels in different high energy e^+e^- storage rings, with and without harmonic spin matching

An early observation of radiative polarization buildup in an e^+e^- storage ring VEPP-2M is presented in Fig. 3 [13], from where it is seen the asymptotic polarization ($\zeta \simeq 0.92$) is nearby the predicted value for the homogeneous field. However, with increasing particle energy a depolarizing influence of the spin diffusion grows up ($|\mathbf{d}| \sim \gamma^2$) and one has to take specific measures to maximize an achievable degree of polarization. It appeared that the most effective method is «harmonic spin matching». If the spin tune is far away from dangerous intrinsic resonances, it is possible to compensate two nearest imperfection resonances by adjusting corresponding harmonics of the closed orbit and using data from a polarimeter as a feedback.

By now the radiative polarization has been observed and used in many storage rings in the energy range from 500 MeV (ACO, VEPP-2M) up to 50 GeV (LEP). As can be seen from Fig. 4, the polarization level dropped precipitously at high energies at LEP. It indicates a full overlapping of spin resonances due to the spin diffusion.

CONCLUSION

In many laboratories around the world there is increasing interest in the availability of spin-polarized beams at high energy and nuclear physics experiments. This is due to a considerable progress that has been achieved in the field of polarized beams during the last 3–4 decades.

The classical theory for full description of spin behavior in accelerators and storage rings has been developed, as well as a number of simulating spin tracking codes. On this base a number of practical approaches have been invented and applied in the experiments. The invention of Siberian snakes opened ways to the acceleration of polarized protons up to a few hundreds of GeV.

The phenomena of radiative polarization of electrons and positrons have been discovered and deeply studied. Now they are used in many machines in the energy range from a few hundreds of MeV up to tens of GeV. The method of resonance depolarization was performed to carry out unique metrological experiments of particle masses measurements.

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