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КОМПЬЮТЕРНЫЕ ТЕХНОЛОГИИ В ФИЗИКЕ

SYMMETRY ENTANGLEMENT IN POLARIZATION BIPHOTON OPTICS

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We study kinematic and dynamic ways of forming entangled states of quantum light fields due to their local and global polarization $SU(2)$ symmetries. The kinematic entanglement is shown to be associated with particular polarization bases in the spaces of quantum states of multimode radiation, which are generated by the global $SU(2)$ symmetry. Dynamic entanglement is due to $SU(2)$ symmetries of the Hamiltonians of the matter–radiation interaction. We also define some entanglement measures, which are related to characteristics of light depolarization. Applications of results obtained in biphoton optics are briefly discussed.

Изучаются кинематический и динамический способы формирования «перепутанных» состояний квантовых световых полей, обусловленные их локальными и глобальными поляризационными $SU(2)$-симметриями. Показано, что «кинематическая перепутанность» ассоциируется с особыми поляризационными базисами в пространствах квантовых состояний многомодового излучения, порожденными глобальной $SU(2)$-симметрией. Динамическая перепутанность обусловлена $SU(2)$-симметриями гамильтонианов взаимодействия излучения с веществом. Определены некоторые меры «перепутанности», связанные с характеристиками деполяризации света. Кратко обсуждаются применения полученных результатов в бифотонной оптике.

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INTRODUCTION

The notion of entanglement introduced in quantum mechanics as early as 1935 [1, 2] plays an important role in current investigations of both quantum theory foundations and quantum information processing (see, e.g., [3–9] and references therein). Herewith, in the most of these investigations the main attention was paid to quantifying entanglement that is due to the quantum computing needs [6]. However, from the fundamental point of view, it is also of importance to analyze qualitative aspects, such as «sources» and mechanisms of entanglement for different quantum systems [1, 2, 8]. Amongst them one can distinguish symmetry considerations as «sources» (cf. [7]) and «pairing» (or «clusterization» in more general cases) of composite system components as mechanisms of entanglement. It is of interest to examine both quantitative and qualitative aspects of entanglement in polarization quantum optics which yielded elementary examples for setting the entanglement problem [2] and many modern studies [6]. However, all examples to be examined up to recently involved

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entangled states with finite and *little* photon numbers, while those for the macroscopical multi-mode radiation were not investigated. This lacuna was in part removed in our paper [10] where such states were constructed to describe unusual states of unpolarized light. Below we develop ideas [10] in more general context incorporating cases of states with both little and large photon numbers.

1. POLARIZATION OF QUANTUM LIGHT. THE P-QUASISPIN CONCEPT

At first, we briefly describe basic notions of quantum polarization optics. The quantum theory of light fields is based on the expansion of the operators of the vector potential $\hat{\mathbf{A}}(\mathbf{r},t)$, as well as the electric ($\hat{\mathbf{E}}(\mathbf{r},t)$) and the magnetic ($\mathbf{H}(\mathbf{r},t)$), in plane monochromatic waves, which specify the photon structure of radiation [11]. In the case of m spatiotemporal modes $\mathbf{k}_j$ this expansion has the form

$$\hat{\mathbf{A}}(\mathbf{r},t) = c\sum_{j=1}^{m}\sqrt{\frac{2\pi\hbar}{V\omega_j}}\left\{\hat{\mathbf{A}}^{(+)}(j)\,\mathrm{e}^{i(\mathbf{k}_j\mathbf{r}-\omega_j t)} + \hat{\mathbf{A}}^{(-)}(j)\,\mathrm{e}^{-i(\mathbf{k}_j\mathbf{r}-\omega_j t)}\right\},$$
$$\hat{\mathbf{E}}(\mathbf{r},t) = -\frac{1}{c}\frac{\partial\hat{\mathbf{A}}}{\partial t},\quad \hat{\mathbf{H}}(\mathbf{r},t) = \bigtriangledown\times\hat{\mathbf{A}}, \tag{1}$$
$$\hat{\mathbf{A}}^{(-)}(j) = \sum_{\alpha=\pm}\mathbf{e}_\alpha(j)\hat{a}^{\dagger}_{\alpha j} = (\hat{\mathbf{A}}^{(+)}(j))^{\dagger},\quad [\hat{a}_{\alpha i},\hat{a}^{\dagger}_{\beta j}] = \delta_{ij}\delta_{\alpha\beta},$$

where $\mathbf{e}_\pm$ are the polarization orts; $\hat{a}^{\dagger}_{\alpha j}/\hat{a}_{\alpha j}$ are operators of creation/annihilation of photons with the wave vector $\mathbf{k}_j$, frequency $\omega_j = c|\mathbf{k}_j|$ and polarization $\alpha = \pm$, which are *basic quantities* for describing quantum radiation [11, 12].

In particular, the $2m$-mode Hilbert space $L_F(2m) = \mathrm{Span}\{|\{n_{+i}, n_{-i}\}\rangle\}$ of its quantum states is given as the tensor product $L_F(2m) = \prod_i^{\otimes m} L^i_F(2)$ of the two-mode Fock spaces

$$L^j_F(2) = \mathrm{Span}\left\{|\{n_{\pm j}\}\rangle = \prod_{\alpha=\pm}[n_{\alpha j}!]^{-1/2}(\hat{a}^{\dagger}_{\alpha j})^{n_{\alpha j}}|0\rangle\right\}, \tag{2}$$

where $|\{n_{\pm i}\}\rangle$ are eigenstates of the field operators of Hamiltonian $\hat{H}_f = \hbar\sum_{i=1}^{m}\omega_i\sum_{\alpha=\pm}\hat{n}_{\alpha i}$ and momentum $\hat{\mathbf{P}}_f = \hbar\sum_{i=1}^{m}\mathbf{k}_i\sum_{\alpha=\pm}\hat{n}_{\alpha i}$ ($\hat{n}_{\alpha i} \equiv \hat{a}^{\dagger}_{\alpha i}\hat{a}_{\alpha i}$) as well as of the relativistically invariant partial helicity operators $\hat{S}_{3\mathbf{k}_j} = (\hat{\mathbf{S}}\cdot\mathbf{k}_j)/|\mathbf{k}_j| = \hat{n}_{+j} - \hat{n}_{-j}$, which specify the physical sense of the polarization label α in (1). However, operators $\hat{S}_{3\mathbf{k}_j}$, being the only measurable components of the radiation spin [11], do not provide a complete polarization characterization of arbitrary states in $L_F(2m)$ because of the strong degeneracy of the $\hat{S}_{3\mathbf{k}_j}$ eigenvectors [12].

This drawback is canceled within the framework of the P-quasispin concept, which was proposed by the author in [10] and developed in [12, 13]. It is based on using the polarization gauge $SU(2)$ symmetry of light fields in the momentum representation described by means of the group

$$SU(2)_p^{ga} = \left\{ \exp\left(-i \sum_{j=1}^{m} \mathbf{w}(\mathbf{k}_j) \cdot \hat{\mathbf{P}}_{\mathbf{k}_j} \right) \right\} = \prod_{j=1}^{m} SU(2)_p^j,$$
$$[SU(2)_p^{ga}, \hat{H}_f] = 0 = [SU(2)_p^{ga}, \hat{\mathbf{P}}_f], \tag{3}$$

where the generators $\hat{P}_{i\,\mathbf{k}_j}$ of the partial polarization $SU(2)_p^j$ group are components of the partial P-quasispins $\hat{\mathbf{P}}_{\mathbf{k}_j} \equiv (\hat{P}_{1\,\mathbf{k}_j}, \hat{P}_{2\,\mathbf{k}_j}, \hat{P}_{3\,\mathbf{k}_j})$ defined as follows [10]:

$$2\hat{P}_1 \equiv \hat{n}_x - \hat{n}_y = \hat{a}_+^\dagger \hat{a}_- + \hat{a}_-^\dagger \hat{a}_+, \quad 2\hat{P}_2 \equiv \hat{n}_{x'} - \hat{n}_{y'} = i(\hat{a}_-^\dagger \hat{a}_+ - \hat{a}_+^\dagger \hat{a}_-),$$
$$2\hat{P}_3 \equiv \hat{n}_+ - \hat{n}_- = \hat{S}_{3\mathbf{k}_j} \tag{4}$$

(for the sake of simplicity subindices $\mathbf{k}_j$ in $\hat{P}_{i\,\mathbf{k}_j}$ are omitted). Herewith, operators $\hat{a}_{\alpha j}^\dagger$ are transformed as components of polarization spinors under the $SU(2)_p^{ga}$ group actions,

$$\hat{a}_{\alpha j}^\dagger \to \hat{a}_{\alpha j}^\dagger(\mathbf{w}) = \hat{U}(\mathbf{w}) \hat{a}_{\alpha j}^\dagger \hat{U}(\mathbf{w})^\dagger = \sum_{\beta=\pm} U_{\alpha\beta}^{1/2}(\mathbf{w})\, \hat{a}_{\beta j}^\dagger, \quad \hat{U}(\mathbf{w}) \in SU(2)_p^{ga}, \tag{5}$$

where $U_{\alpha\beta}^{1/2}(\mathbf{w})$ are matrix elements of the spinor representation of the $SU(2)$ group [12, 14]. Hence, the group $SU(2)_p^{ga}$ equivalent to the $SU(2)$ group of local transformations, which is used in examining entanglement of qubit systems [6].

The introduction of the $SU(2)_p^{ga}$ group and the use of the appropriate P-quasispin formalism [14] allow one to describe polarization properties of arbitrary light fields completely because all polarization observables are expressed in terms of P-quasispins $\hat{\mathbf{P}}_{\mathbf{k}_j}$ and the above degeneracy of the $\hat{S}_{3\mathbf{k}_j}$ eigenvectors is removed. Moreover, the P-quasispin formalism allows one to determine new types of entanglement in polarization optics. Herewith, besides the $SU(2)_p^{ga}$ group and partial P-quasispins $\hat{\mathbf{P}}_{\mathbf{k}_j}$, an important role belongs to the *global* polarization group $SU(2)_p^{gl} = \{\exp(-i\mathbf{w} \cdot \hat{\mathbf{P}})\}$ and to the **total** field P-quasispin $\hat{\mathbf{P}} \equiv (\hat{P}_1, \hat{P}_2, \hat{P}_3) = \sum_{j=1}^{m} \hat{\mathbf{P}}_{\mathbf{k}_j}$ [10, 12].

2. THE P-QUASISPIN FORMALISM AND ENTANGLEMENT IN POLARIZATION OPTICS

The starting point of our analysis of the entanglement problem in polarization optics is the introduction of two types of the *polarization* bases in the Hilbert spaces $L_F(2m)$ [12].

The first is defined as the tensor product $\prod_{j=1}^{\otimes m} |P_j; \mu_j\rangle \equiv |\{P_j; \mu_j\}\rangle$ of the «partial» polarization bases in the spaces $L_F^j(2)$ where $|P_j; \mu_j\rangle$ are eigenvectors of the commuting operators $\hat{\mathbf{P}}_{\mathbf{k}_j}^2 \equiv \hat{P}_{1\mathbf{k}_j}^2 + \hat{P}_{2\mathbf{k}_j}^2 + \hat{P}_{3\mathbf{k}_j}^2 \equiv \hat{P}_j(\hat{P}_j + 1)(\hat{P}_j = \frac{1}{2}\hat{n}_j, \hat{n}_j \equiv \hat{n}_{+j} + \hat{n}_{-j})$, $\hat{P}_{3\mathbf{k}_j}$:

$$\hat{\mathbf{P}}_{\mathbf{k}_j}^2 |P_j; \mu_j\rangle = P_j(P_j + 1)|P_j; \mu_j\rangle, \quad \hat{P}_{3\mathbf{k}_j}|P_j; \mu_j; \rangle = \mu_j |P_j; \mu_j; \rangle,$$
$$2P_j = 0, 1, \ldots, \infty, \quad |\mu_j| \leqslant P_j, \tag{6}$$

which are explicitly given via re-numbering the Fock states in $L_F^j(2)$: $|P_j;\mu_j\rangle = |n_{+j} = P_j+\mu_j, n_{-j} = P_j-\mu_j\rangle$. Evidently, the label $2P_j = n_{+j}+n_{-j} \equiv \hat{n}_j$ removes the degeneracy of the $\hat{S}_{3\mathbf{k}_j}$ eigenvalues $2\mu_j$, and the basis $\{|\{P_j;\mu_j\}\rangle\}$ provides a complete polarization analysis in $L_F(2m)$.

However, another (collective) polarization basis in $L_F(2m)$ is of more importance for analyzing the polarization entanglement. By analogy with Eq. (6), it is defined as a set $\{|P;\mu;\lambda\rangle\}$ of eigenvectors of two commuting operators $\hat{\mathbf{P}}^2, \hat{P}_3$ related to the global group $SU(2)_p^{gl}$. At the same time, there are essential distinctions with the previous case, namely: 1) $\hat{P} \neq \hat{n}/2$, $\hat{n} = \sum_{j=1}^{m} \hat{n}_j, 0 \leqslant P \leqslant n/2$, 2) total eigenvalues P,μ are strongly degenerate and 3) the composite label λ, removing this degeneracy, includes the partial P-quasispins $P_j, j=1,\ldots,m$, and a set σ of $m-2$ additional quantum numbers connected with the group $SO^*(2m)$, which acts complementarily to the $SU(2)_p^{gl}$ group on the space $L_F(2m)$ [10]. The vectors $|P;\mu;\lambda\rangle$ can be expressed via linear combinations of the partial basis vectors $|P_j;\mu_j\rangle$: $|P;\mu;\lambda\rangle \equiv \sum_{\mu_j} C^{P;\mu;\lambda}_{\{P_j;\mu_j\}} |\{P_j;\mu_j\}\rangle$ where $C^{P;\mu;\lambda}_{\{P_j;\mu_j\}}$ are products of the $SU(2)$ Clebsch–Gordan coefficients and the set σ consists of intermediate («cluster») P-quasispins [13].

However, an alternative form was proposed for the vectors $|P;\mu;\lambda\rangle$ in [10]:

$$\begin{gathered}|P;\mu;\lambda\rangle = \sum C^{P;\mu;\lambda}(\{\alpha_{\pm j};\beta_{ij};\gamma_{ij}\}) \prod_j (a^\dagger_{+j})^{\alpha_{+j}} (a^\dagger_{-j})^{\alpha_{-j}} \prod_{i\leqslant j} (\hat{Y}^\dagger_{ij})^{\beta_{ij}} (\hat{X}^\dagger_{ij})^{\gamma_{ij}} |0\rangle, \\ \sum \alpha_{\pm j} = |\mu| \pm \mu, \quad \sum \beta_{ij} = P - |\mu|, \quad \sum \gamma_{ij} = n/2 - P, \ n = \sum_{j=1}^{m} n_j,\end{gathered} \tag{7}$$

where composed operators $\hat{Y}^\dagger_{ij} = \frac{1}{2}[\hat{a}^\dagger_{+i}\hat{a}^\dagger_{-j} + \hat{a}^\dagger_{-i}\hat{a}^\dagger_{+j}]$, $\hat{X}^\dagger_{ij} = \frac{1}{2}[\hat{a}^\dagger_{+i}\hat{a}^\dagger_{-j} - \hat{a}^\dagger_{-i}\hat{a}^\dagger_{+j}]$ satisfy the equations $[\hat{Y}^\dagger_{ij}, \hat{P}_3] = 0$, $[\hat{X}^\dagger_{ij}, \hat{P}_{a=1,2,3}] = 0$ and, hence, may be interpreted as «creation operators» of «helicityless» and P-scalar biphotons, accordingly. The form (7) shows that the basis $\{|P;\mu;\lambda\rangle\}$ contains the subset of entangled states, which are specified by coupling partial spatiotemporal modes via the explicit occurrence of operators $\hat{Y}^\dagger_{ij}, \hat{X}^\dagger_{ij}$ in $|P;\mu;\lambda\rangle$. By way of example, we write down in the case of $m=2$ the states $|\Phi_0\rangle = \left|P=1;\mu=0;P_1=\frac{1}{2}=P_2\right\rangle = \sqrt{2}\,\hat{Y}^\dagger_{12}|0\rangle$, $|\Psi_0\rangle = \left|P=0;\mu=0;P_1=\frac{1}{2}=P_2\right\rangle = \sqrt{2}\,\hat{X}^\dagger_{12}|0\rangle$ which coincide with two «diagonal» Bell states [6]. (Two other Bell states are $|\Phi_\pm\rangle = \frac{1}{\sqrt{2}}\left[\left|P=1;\mu=1;P_1=\frac{1}{2}=P_2\right\rangle \pm \left|P=1;\mu=-1;P_1=\frac{1}{2}=P_2\right\rangle\right]$.) Evidently, the entanglement, manifesting in Eq. (7), may be named kinematic because it is due to the $SU(2)_p^{gl}$ symmetry and does not depend on any dynamics. It can be implemented dynamically in processes of biphoton optics [15] with interaction Hamiltonians

$$\hat{H}_{bf} = \hbar \sum_{i,j=1}^{m} \sum_{\alpha,\beta=\pm} \left[g^{\alpha\beta}_{ij}\hat{a}^\dagger_{\alpha i}a^\dagger_{\beta j} + g^{\alpha\beta *}_{ij}\hat{a}_{\alpha i}\hat{a}_{\beta j}\right], \tag{8}$$

when the coupling constants $g^{\alpha\beta}_{ij}$ have symmetry properties: $g^{\alpha\alpha}_{ij} = 0, g^{+-}_{ij} = \pm g^{-+}_{ij} = \tilde{g}_{ij}$ [13].

The introduction of the basis $\{|P;\mu;\lambda\rangle\}$ entails the decomposition

$$L_F(2m) = \sum_{2P=0}^{\infty} \sum_{\lambda} L(P;\lambda), \quad L(P;\lambda) \equiv \left\{ |\Psi_{P,\lambda}\rangle = \sum\nolimits_{\mu} C_{\mu}^{P,\lambda} |P;\mu;\lambda\rangle \right\} \tag{9}$$

of $L_F(2m)$ in $SU(2)_p^{gl}$-invariant subspaces $L(P;\lambda)$ as it follows from the transformation law

$$|P;\mu;\lambda\rangle \overset{SU(2)_p^{gl}}{\longrightarrow} |P;\mu;\lambda\rangle(\mathbf{w}) \equiv \hat{U}(\mathbf{w})\,|P;\mu;\lambda\rangle = \sum_{\mu'=-P}^{P} U_{\mu,\mu'}^{P}(\mathbf{w})|P;\mu';\lambda\rangle \tag{10}$$

of the vectors $|P;\mu;\lambda\rangle$ with respect to the $SU(2)_p^{gl}$ group [12]. Equation (10) provides possibilities to find new entangled states (dynamically) by means of $SU(2)_p^{gl}$ transformations of certain initial ones. For example, transforming in such a way the Bell state $|\Phi_0\rangle$, one can get two other Bell states $|\Phi_{\pm}\rangle$: $|\Phi_+\rangle = \hat{U}(\mathbf{w}_+ = (0,\pi/2,0))\,|\Phi_0\rangle$, $|\Phi_-\rangle = -i\hat{U}(\mathbf{w}_- = (\pi/2,0,0))\,|\Phi_0\rangle$. Note that the transformations (10) maintain the total P-quasispin value and the number $n/2 - P$ of «biphoton clusters» $\hat{X}_{ij}^{\dagger}$, whereas it is not the case for the local $SU(2)_p^{ga}$ group transformations:

$$|P;\mu;\lambda\rangle \overset{SU(2)_p^{ga}}{\longrightarrow} |P;\mu;\lambda\rangle(\{\mathbf{w}_j\}) = \sum_{\{P',\mu';\mu_j,\mu_j'\}} C_{\{P_j';\mu_j'\}}^{P';\mu';\lambda} C_{\{P_j;\mu_j\}}^{P;\mu;\lambda} \prod_j U_{\mu_j,\mu_j'}^{P_j}(\mathbf{w}_j)|P';\mu';\lambda\rangle. \tag{11}$$

And now we briefly discuss the physical nature of kinematic polarization entanglement and its «measures». First, we note that the decomposition (9) contains the subspace $L_X(0) = \sum\limits_{\lambda} L(P=0;\lambda)$ generated by the biphotons $\hat{X}_{ij}^{\dagger}$ only, and its states $|\Psi_X\rangle$ satisfy equations

$$\langle\Psi_X|\hat{P}_{i\,\mathbf{k}_j}|\Psi_X\rangle = 0, \quad i=1,2,3; \quad \langle\Psi_X|\hat{P}_1^{a_1}\hat{P}_2^{a_2}\hat{P}_3^{a_3}|\Psi_X\rangle = 0, \quad a_1+a_2+a_3 \geqslant 1, \tag{12}$$

demonstrating all features of unpolarized light (the first equality) and the full absence of the polarization noises of the total P-quasispin components (that is an indicator of the total polarization squeezing of light). However, the mechanism of light depolarization is here due to strong phase correlations between photons (phase matching), unlike the randomization of light waves for unpolarized light in classical optics [13]. An example of microscopic realization of states $|\Psi_X\rangle$ is the Bell singlet $|\Psi_0\rangle$, and their macroscopic implementation is given by the generalized coherent states $|\Psi_X(z_{ij})\rangle = \exp\left[\sum\limits_{ij}(z_{ij}\hat{X}_{ij}^{\dagger} - z_{ij}^{*}\hat{X}_{ij})\right]|0\rangle$ of the $SO^{*}(2m)$ group which determine the unique class of unpolarized quantum light (P-scalar light) [10]. The second example of such unusual (coherent!) mechanisms of light depolarization is yielded by the subspace $L_Y(0) = \left\{|\Psi_Y\rangle = \sum\limits_{P,\lambda} C_{\mu}^{P,\lambda}\,|P = \frac{n}{2};\mu=0;\lambda\rangle\right\}$ generated by the biphotons $\hat{Y}_{ij}^{\dagger}$ only and determining a class of «helicityless» unpolarized quantum light because its states $|\Psi_Y\rangle$ satisfy the first equality in Eqs. (12) and the equation $\langle\Psi_Y|\hat{P}_3^{a(\geqslant 1)}|\Psi_Y\rangle = 0$ [10]. By analogy with states $|\Psi_X\rangle$, a microscopic and some macroscopic realizations of states $|\Psi_Y\rangle$ are given the Bell state $|\Phi_0\rangle$ and states $|\Psi_Y(z_{ij})\rangle = \exp\left[\sum\limits_{ij}(z_{ij}\hat{Y}_{ij}^{\dagger} - z_{ij}^{*}\hat{Y}_{ij})\right]|0\rangle$, respectively.

So, we showed that new (coherent!) mechanisms of light depolarization describe also the origin of kinematic entanglement in polarization optics. Therefore, we conjecture that as «measures» of latter it is worthwhile to use $SU(2)_p$-invariant characteristics of light depolarization $1-\mathcal{P}$ and $1-\mathcal{P}_j$ where total ($\mathcal{P}$) and partial ($\mathcal{P}_j$) degrees of polarization are defined as follows [13]:

$$\mathcal{P} = 2\frac{\sqrt{\langle\hat{\mathbf{P}}\rangle^2}}{\langle\hat{n}\rangle}, \quad \mathcal{P}_j = 2\sqrt{\frac{\langle\hat{\mathbf{P}}_{\mathbf{k}_j}\rangle^2}{\langle\hat{n_j}\rangle}}, \quad j = 1, 2, \ldots, m. \tag{13}$$

A good premise for that is the relation $C^2 = 1 - \frac{1}{2}[\mathcal{P}_1^2 + \mathcal{P}_2^2]$ between Wooter's concurrence C [4] and partial degrees of polarization $\mathcal{P}_i$ of two spatiotemporal modes which was obtained in [16]. Besides, one can try to use for this aim other $SU(2)_p$-invariant quantities. Amongst them one can distinguish the «content» $\mathcal{C}_X = 1 - \frac{2\bar{P}}{\langle\hat{n}\rangle}$, $2\bar{P} = -1 + \sqrt{1 + 4\langle\hat{\mathbf{P}}\rangle^2}$ of $X_{ij}^\dagger$ biphotons in the state under study [10] as well as its partial and «cluster» analogs.

CONCLUSION

So, we have discussed the notion of entanglement in polarization quantum optics as well as kinematic and dynamic ways of forming entangled states due to local and global polarization $SU(2)$ symmetries. Specifically, the kinematic entanglement is related to the synergetic role of the global $SU(2)$ symmetry, while the dynamic one is due to $SU(2)$ transformations properties of the Hamiltonians of the matter–radiation interaction. We also established relationships between polarization entanglement and unusual (*coherent!*) mechanisms of light depolarization and proposed appropriate polarization entanglement measures which can be used for a classification of entangled states.

In conclusion we mention some possible directions of developing the above results. The first one is in examining relations between the above polarization entanglement measures and those which were proposed in literature for qubit systems using other considerations [4–9]. The second, applied direction is in the use of results obtained to analyze entangled states produced in spontaneous parametric scattering in multiply domained crystals (cf. [16]) and to generate macroscopic entangled states like $|\Psi_X(z_{ij})\rangle$ and $|\Psi_Y(z_{ij})\rangle$.

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REFERENCES

1. *Schrödinger E.* Die Gegenwärtige Situation in der Quantenmechanik // Naturwiss. 1935. Bd. B23. S. 807–849.

2. *Einstein A., Podolsky B., Rosen N.* Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? // Phys. Rev. 1935. V. 47. P. 777–780.

3. *Werner R. F.* Quantum States with Einstein–Podolsky–Rosen Correlations Admitting a Hidden-Variable Model // Phys. Rev. A. 1989. V. 40. P. 4277–4281.

4. *Wooters W. K.* Entanglement of Formation of an Arbitrary State of Two Qubits // Phys. Rev. Lett. 1998. V. 80. P. 2245–2248.

5. *Vedral V., Plenio M. B.* Entanglement Measures and Purification Procedures // Phys. Rev. A. 1998. V. 57. P. 1619–1633.

6. The Physics of Quantum Information: Quantun Cryptography, Quantum Teleportation, Quantum Computation / Eds. D. Boumeester, A. K. Ekert, A. Zeilinger. Berlin: Springer Verlag, 2000.

7. *Vollbrecht K. G. H., Werner R. F.* Entanglement Measures under Symmetry // Phys. Rev. A. 2001. V. 64. P. 062307-1–062307-15.

8. *Vedral V., Kashefi E.* Uniqueness of the Entanglement Measure for Bipartite Pure States and Thermodynamics // Phys. Rev. Lett. 2002. V. 89. P. 037903-1–037903-4.

9. *Kazakov A. Ya.* The Geometric Measure of Entanglement of Three-Partite Pure States // Intern. J. Quant. Inform. 2006. V. 4. P. 907–915.

10. *Karassiov V. P.* Polarization Structure of Quantum Light Fields: A New Insight: 1. General Outlook // J. Phys. A. 1993. V. 26. P. 4345–4354.

11. *Jauch J. M., Rohrlich F.* Theory of Photons and Electrons. Berlin: Springer, 1976.

12. *Karassiov V. P.* P-Quasispin Formalism in Polarization Optics // JETP Lett. 2006. V. 84. P. 640–644.

13. *Karassiov V. P.* Polarization of Light in Classical and Quantum Optics: Concepts and Applications // Opt. Spektrosk. 2007. V. 103. P. 143–150.

14. *Varshalovich D. A., Moskalev A. N., Khersonsky V. K.* Quantum Theory of Angular Momentum. L.: Nauka, 1975; Singapore: World Sci., 1988.

15. *Klyshko D. N.* Photons and Nonlinear Optics. M.: Nauka, 1980.

16. *Kalashnikov D. A. et al.* Generation of Entangled States in Multiply Domained KH_2PO_4 Crystals // JETP Lett. 2008. V. 87. P. 66–71.