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КОМПЬЮТЕРНЫЕ ТЕХНОЛОГИИ В ФИЗИКЕ

MULTIMODE SQUEEZING IN MICROSTRUCTURED NONLINEAR MEDIA

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We analyze the spectral properties of both squeezed light and twin-photon states produced in pulsed parametric down conversion in microstructured nonlinear media. We also analyze the multimode structure of biphoton spectrum and formation of effective squeezing modes by varying the spectral phase-matching in supperlattices.

Мы исследуем спектральные свойства сжатого света и двухфотонных состояний в импульсном режиме параметрической вниз-конверсии в микроструктурной нелинейной среде. Мы также исследуем многомодовую структуру бифотонного спектра и формирование характерных сжатых мод путем изменения спектрального фазового синхронизма в сверхрешетке.

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INTRODUCTION

Quantum communication protocols are usually realized in terms of photonic states as well as in terms of amplitudes of the electromagnetic fields by using the well-established experimental techniques of laser physics and quantum optics. In this way, quantum information resources such as entangled and squeezed states of light beams emerge often from the nonlinear optical interactions of a laser with various nonlinear crystals or/and atomic systems.

In this paper we consider the synthesis of various photonic states in down-conversion processes realized in quasi-phase matched nonlinear materials pumped by pulsed laser field. Microstructured materials such as photonic crystals or periodically poled crystals leading to quasi-phase-matching multiwave interactions are very perspective in the fields of laser technologies and telecommunication because they allow efficient nonlinear optical couplings in a broad range of wavelength. Recently, it has been shown [1, 2] that these materials, particularly, periodically poled nonlinear crystals (PPNC), are also extremely promising for generation of nonclassical states of light and open interesting perspectives for applied quantum information. A particularly effective structure in PPNC is the one in which the sign of the nonlinear susceptibility $\chi^{(2)}$ is periodically reversed through the medium.

We investigate spectral properties and multimode structure of joint states of photon pair produced by pulsed parametric down conversion in PPNC with linear dispersive segments.

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The results obtained are applied for investigation of multimode squeezed states. The key to this approach is the idea of manipulating overall group delay mismatches among the various fields in structured materials for synthesis of twin-photon states [3]. Recently, this approach has been developed in the series of papers (see [4] and references therein) devoted to controllable generation of entangled states as well as single-photon wave packets in superlattice structures of nonlinear and linear materials.

1. TWO-PHOTON SPECTRA IN STRUCTURED NONLINEAR CRYSTALS

The two-photon state at the output of the one-dimensional nonlinear medium with second-order susceptibility $\chi^{(2)}$ under laser field $E_0\,(z,t)$ is given by

$$|\psi\rangle = \frac{1}{2}\int d\omega_1 \int d\omega_2\, \Phi\,(\omega_1,\omega_2)\, a^+\,(\omega_1)\, a^+\,(\omega_2)|0\rangle, \tag{1}$$

where $a^+\,(\omega_1)$ and $a^+\,(\omega_2)$ are the creation photon operators for modes with frequencies ω_1 and ω_2, $|0\rangle$ is a vacuum state and $\Phi\,(\omega_1,\omega_2)$ is the spectral amplitude of two-photon radiation. We assume type-I collinear parametric interaction when both photons have the same linear polarizations and $\omega_0 = \omega_1 + \omega_2$, leading to a pulsed spontaneous parametric down conversion (SPDC). The amplitude of two-photon state is given by the product of the pump envelope function $E_0\,(\omega_0)$ and the phase matching function

$$\Phi\,(\omega_1,\omega_2) = E_0\,(\omega_1+\omega_2)\int dz \chi^{(2)}\,(z)\,\mathrm{e}^{\Delta k z}. \tag{2}$$

Here $\chi^{(2)}$ is the distribution of second-order nonlinearity along the longitudinal axis and Δk is the wave vector mismatch function: $\Delta k = k_0 - k_1\,(\omega_1) - k_2\,(\omega_2)$, where $k_i = |\mathbf{k}_i| = \dfrac{n\,(\omega_i)\,\omega_i}{c}$, $n\,(\omega_i)$ is the index of refraction ($i = 1,2$). The probability of twin-photon spontaneous parametric radiation is defined as $|\Phi\,(\omega_1,\omega_2)\,|^2$. The pump envelope function is modeled by a Gaussian pulse of duration τ_p in terms of the frequency detuning from the central SPDC frequency $\omega_1 + \omega_2 - \omega_0$.

At first, we consider an assembly of N $\chi^{(2)}$ crystals and linear media. Let each component m have thickness l_m, wave vectors k_0^m, k_1^m, k_2^m and nonlinear susceptibility χ_m. Some of the components can be linear with $\chi_m = 0$. The resulting two-photon amplitude is calculated as the sum of the partial amplitude [3]:

$$\Phi\,(\omega_1,\omega_2) = \chi E\,(\omega_1+\omega_2))\sum_m l_m \chi_m \exp\left[-i\left(\varphi_m + \frac{\Delta k_m l_m}{2}\right)\right]\operatorname{sinc}\left(\frac{\Delta k_m l_m}{2}\right),$$
$$\delta_m \equiv \Delta k_m l_m, \quad \varphi_m = \sum_n^{m-1}\delta_n, \quad \varphi_1 = 0, \tag{3}$$

Δk_m is the mismatch function in the mth component: $\Delta k_m = k_0^{(m)} - k_1^{(m)}\,(\omega_1) - k_2^{(m)}\,(\omega_2)$.

We consider below two important applications of these general expressions (see Fig. 1, *a, b*).

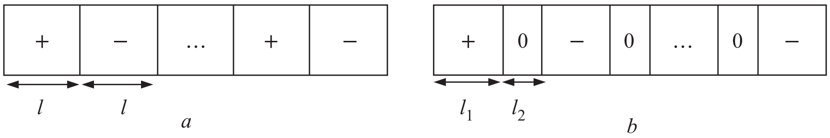

Fig. 1. PPNC which involves nonlinear domains of lengths l with $\chi_+ \equiv \chi^{(2)} > 0$ and $\chi_- \equiv \chi^{(2)} < 0$ (*a*), PPNC in which nonlinear domains with χ_+ and χ_- are interchanged by the linear domains with $\chi^{(1)} \neq 0$ and length l_2 (*b*)

2. SPDC IN PERIODICALLY POLED CONFIGURATIONS

We turn to the experimental arrangement shown in Fig. 1, *a* which consists of the periodic assembly of N $\chi^{(2)}$ crystals of length l with positive and negative susceptibilities: $\chi_{2n-1} = \chi_+ = \chi$, $\chi_{2n} = \chi_- = -\chi$, $n = 1, 2, \ldots, N/2$. We assume that the mismatch functions and lengths are the same in each domain, $\delta_n = \delta = l\Delta k$ and hence we have $\varphi_m = (m-1)\,\delta$. On the whole the amplitude is calculated from Eqs. (3) as

$$\Phi(\omega_1, \omega_2) = E_0(\omega_1 + \omega_2)\, l\chi \exp -i\frac{L\Delta k}{2} \operatorname{sinc}\left(\frac{l}{2}\Delta k\right) \frac{\sin \frac{L}{2}(\Delta k - q)}{\sin \frac{l}{2}(\Delta k - q)}. \tag{4}$$

Here $L = Nl$, $q = 2\pi/d$ is the spatial momentum, and $d = 2l$ is the period of the superlattice. Thus, in this case we obtain phase-matching condition $\Delta k \simeq q$ which is usually takes place in a PPNC.

The other composite system is shown in Fig. 1, *b*. It consists of $N/2$ crystals of length l_1, with positive and negative susceptibilities and $N/2$ linear optical $\chi^{(1)}$ spacers of length l_2. In this configuration we assume two mismatch functions Δk_1 and Δk_2 corresponding to nonlinear $n = 1, 3, 5, \ldots$ and linear $n = 2, 4, \ldots$ segments. The amplitude can be calculated as

$$\Phi(\omega_1, \omega_2) = E_0(\omega_1 + \omega_2)\, l_1 \chi \mathrm{e}^{-i\phi(\omega_1, \omega_2)} \operatorname{sinc}\left(\frac{l_1}{2}\Delta k_1\right) \frac{\sin\left(\frac{L\Delta K}{2}\right)}{\sin\left(\frac{(l_1 + l_2)\,\Delta K}{2}\right)}. \tag{5}$$

Here $\Delta K = \bar{l_1}\Delta k_1 + \bar{l_2}\Delta k_2 - q$, $\phi(\omega_1, \omega_2) = \dfrac{l_1 \Delta k_1}{2} + \dfrac{N(l_1 + l_2)\,\Delta K}{4} + \dfrac{(l_1 + l_2)\,\Delta K}{2}$, $L = \dfrac{N}{2}(l_1 + l_2)$, $\bar{l_i} = \dfrac{l_i}{l_1 + l_2}$ $(i = 1, 2, \ldots)$, $\bar{l_1} + \bar{l_2} = 1$ and $q = \dfrac{2\pi}{d}$, $d = 2(l_1 + l_2)$.

As we see, the amplitude is the product of the phase-matching function of a single nonlinear segment and the phase-matching function which involves the combined effect of the $\chi^{(2)}$ segment, the spacer dispersion and the spatial momentum. Analogous results have been obtained in papers [4] however for the other configuration of the superlattice. For larger numbers of the segments: $N \gg 1$, $L \gg l$, the amplitude reads as

$$\Phi(\omega_1, \omega_2) = E_0(\omega_1 + \omega_2)\, l_1 \frac{N}{2} \chi \ \mathrm{e}\left(-i\frac{L\Delta K}{2}\right) \operatorname{sinc}\left(\frac{L\Delta K}{2}\right). \tag{6}$$

Then, we analyze the spectral structure of SPDC using Schmidt decomposition of the two-photon spectral amplitude

$$\Phi\left(\omega_1,\omega_2\right)=\sum_{n=0}^{\infty}\xi_n\psi_n^*\left(\omega_1\right)\psi_n^*\left(\omega_2\right). \quad (7)$$

Here ξ_n are nonnegative Schmidt coefficients, while $\psi_n\left(\omega_i\right)$ are mutually orthogonal Schmidt functions, normalizing to one. We expand the phase-matching functions Δk_1 and Δk_2 in Eqs. (5) up to the second order in derivation from the respective central frequencies. Then the approximate expression for the phase-matching functions in nonlinear segments $(i=1)$ and for linear segments $(i=2)$ takes the form

$$\Delta k_i=\left(\beta_{0,i}^{(1)}-\beta_i^{(1)}\right)\left(\omega_1+\omega_2-\omega_0\right)-\frac{1}{2}\beta_i^{(2)}\left[\left(\omega_1-\frac{\omega_0}{2}\right)^2+\left(\omega_2-\frac{\omega_0}{2}\right)^2\right]+ \\ +\frac{1}{2}\beta_{0,i}^{(2)}\left(\omega_1+\omega_2-\omega_0\right)^2, \quad (8)$$

where $\beta_i^{(j)}=\left.\frac{d^jk_i\left(\omega\right)}{d\omega^j}\right|_{\omega=\omega_0/2}$ and $\beta_{0,i}^{(j)}=\left.\frac{d^jk_{0,i}(\omega)}{d\omega^j}\right|_{\omega=\omega_0}$ are the dispersion coefficients.

In this approximation the amplitude (Eq. (6)) can be rewritten in the Gaussian form:

$$\Phi\left(\omega_1,\omega_2\right)=\sqrt{\frac{2M}{\pi\Omega\sigma}}\exp\left\{-\frac{\left(\omega_1+\omega_2-\omega_0\right)^2}{2\sigma^2}-\frac{\left(\omega_1-\omega_2\right)^2}{2\Omega^2}\right\}, \quad (9)$$

where spectral widths are

$$\begin{aligned}\frac{1}{\sigma^2}&=\frac{1}{10}\left[l_1\left(\beta_1^{(1)}-\beta_{0,1}^{(1)}\right)+l_2\left(\beta_2^{(1)}-\beta_{0,2}^{(1)}\right)\right]^2N^2+\tau_p^2,\\ \frac{1}{\Omega^2}&=\frac{1}{12}N\left[l_1\beta_1^{(2)}+l_2\beta_2^{(2)}\right],\end{aligned} \quad (10)$$

and the mean number of photon pairs is equal to

$$M=\int d\omega_1\,d\omega_2\left|\Phi\left(\omega_1,\omega_2\right)\right|^2=\frac{\chi^2l_1N^2E_0\left(\omega_1+\omega_2\right)\pi\sigma\Omega}{2}. \quad (11)$$

This Gaussian form of the amplitude allows us to find analytically the Schmidt decomposition. Particulary, in this case the Schmidt coefficients read as

$$\sinh\xi_n=\frac{\sqrt{M}}{\cosh r}\tanh^n r,\quad r=\frac{\ln\left(\Omega/\sigma\right)}{2}. \quad (12)$$

3. SQUEEZING OF INDEPENDENT MODES

The final two-photon state can now be written as

$$|\psi^{(2)}\rangle=\frac{1}{2}\sum_n\xi_n\left(b_n^+\right)^2|0\rangle, \quad (13)$$

through the nonmonochromatic operators

$$b_n = \int d\omega \, \psi_n(\omega) \, \hat{a}(\omega), \tag{14}$$

which satisfy standard bosonic commutation relations.

Let us discuss a multimode quadrature squeezing. As the modes $\psi_n(\omega)$ are mutually orthogonal, the quadrature fluctuations are given by a sum of independent contributions from all the modes. Thus, if the local oscillator is prepared in one of the modes $\psi_n(\omega)$, the maximum $V_n^{(+)}$ and minimum $V_n^{(-)}$ variances of the quadrature amplitude $\frac{1}{\sqrt{2}}\left(b_n \, \mathrm{e}^{-i\Theta} + b_n^{(+)} \, \mathrm{e}^{-i\Theta}\right)$ read as

$$V_n^{(\pm)} = \frac{1}{4} \, \mathrm{e}^{\pm 2\xi_n}. \tag{15}$$

Note that the spectral properties of squeezed light produced by pulsed SPDC in ordinary nonlinear crystals have been analyzed on the basis of Schmidt decomposition (see, for example, [5]). Thus, we discuss here some multimode characteristics of squeezing for the superlattice. The number of squeezing eigenmodes depends on the spectral widths σ and Ω. In a typical arrangement $\sigma \ll \Omega$ for an ordinary $\chi^{(2)}$ crystal, which leads to excitation of a broad range of modes. In the case of PPNC with linear space (see Fig. 1, *b*) it is possible to realize almost factorizable $\Phi(\omega_1, \omega_2)$, i.e., SPDC with a few effective modes, by varying the spectral phase-matching structure in Eq. (9). The factorizable amplitude $\Phi(\omega_1, \omega_2)$ is realized, if $\sigma \simeq \Omega$. For increasing of σ (Eq. (10)) the crystal and spacer materials should be chosen with opposite-signed group velocity mismathes, $\beta_1^{(1)} - \beta_{0,1}^{(1)}$ and $\beta_2^{(1)} - \beta_{0,2}^{(1)}$. In this case, the width σ may reach inverse length of pulse $1/\tau_p$ by compensation of the dispersion effects. On the other hand, we can suppress the width Ω by varying the thickness and dispersion coefficients $\beta_2^{(2)}$ of the spacer material and hence reach the range $\sigma \simeq \Omega$. Some illustrations are shown in Fig. 2. For calculations we assume $M = 0.9$, i.e., an average 0.9 photon pairs per pulse [6].

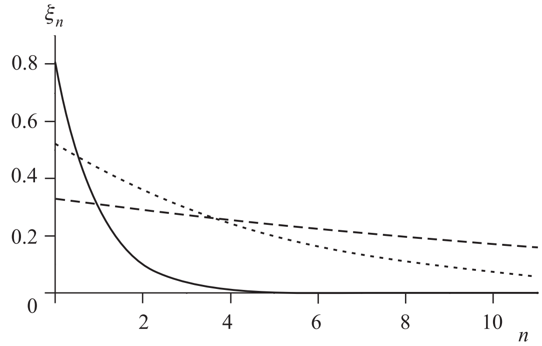

Fig. 2. The squeezing parameter as a function of the mode number: solid line — $\Omega/\sigma = 2$; dotted line — $\Omega/\sigma = 30$; dashed line — $\Omega/\sigma = 10$

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