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A NEW APPROACH FOR CALCULATION OF VOLUME CONFINED BY ECR SURFACE AND ITS AREA IN ECR ION SOURCE

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Новый подход к определению объема области, ограниченной ЭЦР-поверхностью, и ее площади в ионном источнике ЭЦР-типа

В модели уравнений баланса для расчета зарядовых распределений ионов (ЗРИ) в ионном источнике, основанном на электронно-циклотронном резонансе (ЭЦР), такие величины, как объем, ограниченный резонансной поверхностью, и ее площадь, являются важными параметрами. В данной работе предложен новый подход по определению данных величин, позволяющий уменьшить число параметров модели.

Работа выполнена в Лаборатории физики частиц ОИЯИ.

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Philippov A. V. E9-2007-161

A New Approach for Calculation of Volume Confined by ECR Surface and Its Area in ECR Ion Source

The volume confined by the resonance surface and its area are important parameters of the balance equations model for calculation of ion charge-state distribution (CSD) in the electron–cyclotron resonance (ECR) ion source. A new approach for calculation of these parameters is given. This approach allows one to reduce the number of parameters in the balance equations model.

The investigation has been performed at the Laboratory of Particle Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 2007

INTRODUCTION

In some transport models [1–3] for calculation of ion CSD in ECR ion sources [4] the values of volume V_p confined by resonance surface and its area S_p are important model parameters. For example, in model [3] these values are used in balance equation for neutral component of ECR plasma:

$$\frac{dn_{s,0}}{dt} = \frac{\bar{u}_{s,0}\, S_p}{V_p}\,(n_s - n_{s,0}) - \\ - \sum_{m=1}^{M} \left(\sum_{k=1}^{K} {}^{m}\nu^{\text{ion}}_{s,0\to m,k}\, n_{e,k} + \sum_{s'=1}^{S} \sum_{z=m+1}^{Z_{s'}} {}^{m}\nu^{\text{cx}}_{s',z\to z-m}\, n_{s',z} \right) n_{s,0}. \quad (1)$$

Here agreed notations are: s, s' are ion species indexes; z is an ion charge-state index; m is a process multiplicity; k is an electron component index; $\bar{u}_{s,0}$ is a neutral velocity; ${}^{m}\nu^{\text{ion}}_{s,0\to m,k}$, ${}^{m}\nu^{\text{cx}}_{s',z\to z-m}$ are ionization and charge exchange rates; $n_{s,z}$, $n_{e,k}$ are ions and electrons densities; $n_{s,0}$, n_s are neutral densities inside and outside the source chamber.

In this work the proper calculation of these important parameters is presented.

1. ECR ION-SOURCE MAGNETIC MAP APPROXIMATION

The approximation of the ECR ion-source magnetic map uses the following well-known fact: minimum–B field configuration is created by external magnetic system of ion source segmented in two different parts. One of these parts is solenoid magnet and the other one is multipole magnet, for example, sextupole magnet.

1.1. External Solenoid Field. In cylindrical coordinate system we describe the solenoidal magnetic field by $A_\theta = A_\theta\,(\rho,\ z)$ — azimuthal component of vector potential [5]:

$$A_\theta\,(\rho,\ z) = J_1 \left(\rho \frac{d}{dz} \right) \Phi\,(z)\,, \\ \Phi(z) = B_1 + z^2 B_2. \quad (2)$$

Here, $J_1\left(\rho\dfrac{d}{dz}\right)$ is a Bessel function of the first order; $\Phi(z)$ is a magnetic field at the axis; B_1 and B_2 are numerical coefficients in Gs and $\mathrm{Gs}\cdot\mathrm{cm}^{-2}$ units correspondingly. In decomposition (2) only the first order term is used.

1.2. External Field of Multipole Lens. We describe the external multipole magnet of sextupole lens by $A_z = A_z(\rho,\ \theta)$ — longitudinal component of vector potential [5]:

$$A_z(\rho,\ \theta) = \frac{\rho^3 B_0\ \sin 3\,\theta}{3\,R_0^2}. \tag{3}$$

Here, B_0 is a pole tip magnet field and R_0 is a lens radius. The dimension of a quantity B_0/R_0^2 is $\mathrm{Gs}\cdot\mathrm{cm}^{-2}$.

1.3. Fitting of Total Magnetic Field. The total vector potential $\mathbf{A} = \mathbf{A}(\rho,\ \theta,\ z)$ of minimum–B configuration is in the form

$$\mathbf{A}(\rho,\ \theta,\ z) = \begin{pmatrix} 0 \\ A_\theta \\ A_z \end{pmatrix}. \tag{4}$$

Here, A_θ, A_z are defined in (2) and (3) correspondingly. The total magnetic field $\mathbf{B} = \mathbf{B}(\rho,\ \theta,\ z)$ of ECR ion source can be expressed as follow:

$$\mathbf{B} = \nabla \times \mathbf{A}, \tag{5}$$

where ∇ is a gradient operator.

Fitting of the numerical coefficients in formulas (2) and (3) was performed separately for solenoid field and for sextupole magnet. These coefficients were found for magnetic field maps of three different ECR ion sources: for INFN, LNS, SERSE ion sources of two different working frequencies 14, 18 GHz and for ECR ion source of the Frankfurt University (UNI), IKF with working frequency 14.4 GHz. The results of this calculation are presented below in Table 1.

Table 1. Numerical coefficients B_0/R_0^2, B_1, B_2

ECR ion source	B_0/R_0^2	B_1	B_2
INFN, LNS, SERSE 14 GHz	270	4813	27
INFN, LNS, SERSE 18 GHz	334	5374	42
Frankfurt UNI, IKF 14.4 GHz	251	4797	21

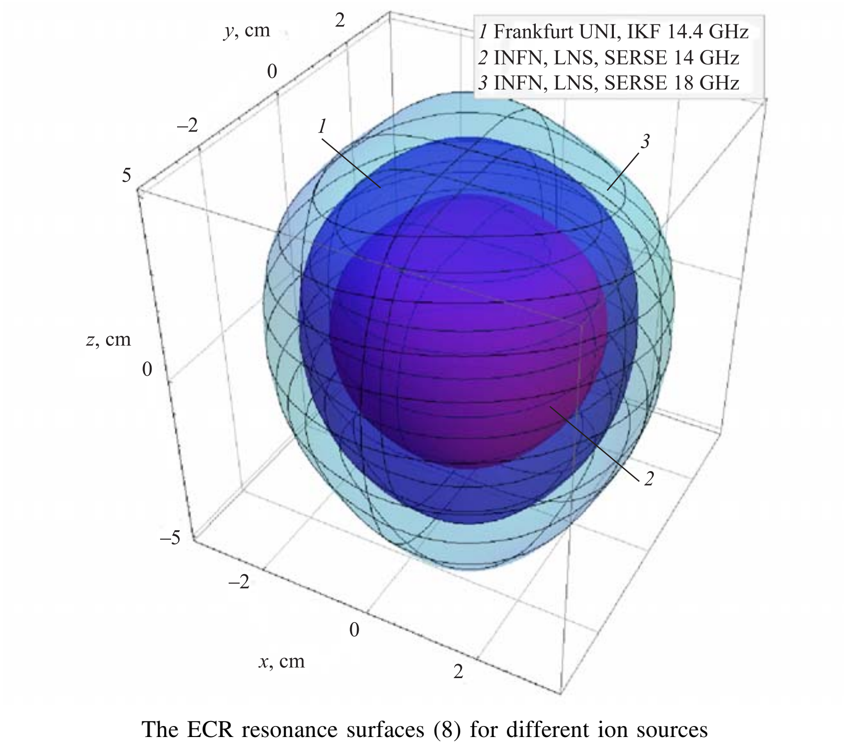
2. ECR RESONANCE SURFACE

The subsequent discussion we advance in Cartesian coordinate system. The ECR resonance surface $F = F\,(x,\ y,\ z)$ is determined by condition that the absolute value $B = B\,(x,\ y,\ z)$ of total magnetic field (5) is equal to the resonance value $B_{\rm res}$, i. e.,

$$B = B_{\rm res}. \tag{6}$$

For absolute value of total magnetic field $B\,(x,\ y,\ z)$ we have

$$\begin{gathered} B\,(x,\ y,\ z) = \sqrt{B_x^2\,(x,\ y,\ z) + B_y^2\,(x,\ y,\ z) + B_z^2\,(z)}, \\ B_x\,(x,\ y,\ z) = \left(x^2 - y^2\right)\ B_0 - x\,z\,B_2, \\ B_y\,(x,\ y,\ z) = -y\ \left(2\,x\,B_0 + z\,B_2\right), \\ B_z\,(z) = \Phi\,(z)\,. \end{gathered} \tag{7}$$



The ECR resonance surfaces (8) for different ion sources

The coefficient B_0/R_0^2 here was redefined as B_0, and therefore the expression for $F(x,\ y,\ z)$ is given by

$$F(x,\ y,\ z) = x^4 B_0^2 + y^4 B_0^2 + B_1^2 - 2\,x^3 z\,B_0\,B_2 + 6\,x\,y^2 z\,B_0\,B_2 + \\ + 2\,z^2 B_1\,B_2 + y^2 z^2 B_2^2 + z^4 B_2^2 + x^2\left(y^2 B_0^2 + z^2 B_2^2\right) - B_{\text{res}}^2. \quad (8)$$

The equality to zero of expression (8) defines the implicit equation of ECR surface.

The ECR resonance surfaces for different ion sources are shown in the figure. The fitted numerical coefficients are taken from Table 1 above.

3. DEFINITION OF VOLUME CONFINED BY RESONANCE SURFACE AND ITS AREA

Now when the equation is known we can develop the method for calculation of volume confined by resonance surface and its area. The volume can be defined as

$$V_p = \iiint\limits_{\Omega} dV,\ dV = dx\,dy\,dz, \\ \Omega = \{(x,\ y,\ z):\ F(x,\ y,\ z) < 0\}, \quad (9)$$

and for resonance surface area as

$$S_p = \oint\!\!\int\limits_{S} \mathbf{n}\cdot d\mathbf{S}, \\ d\mathbf{S} = \mathbf{n}\,dS,\ dS = dx dy, \quad (10) \\ S = \{(x,\ y,\ z):\ F(x,\ y,\ z) = 0\}.$$

Using the Ostrogradsky–Gauss theorem we reduce the last expression, i. e.,

$$\oint\!\!\int\limits_{S} \mathbf{n}\cdot d\mathbf{S} = \iiint\limits_{\Omega} \nabla\cdot\mathbf{n}\,dV, \\ S_p = \iiint\limits_{\Omega} \nabla\cdot\mathbf{n}\,dV, \quad (11) \\ \mathbf{n} = \mathbf{n}(x,\ y,\ z),\ \mathbf{n}(x,\ y,\ z) = \frac{\nabla B(x,\ y,\ z)}{|\nabla B(x,\ y,\ z)|}.$$

We use formulas (9), (11) for calculation of V_p and S_p. This calculation was produced using Monte-Carlo method and tested for surfaces with analytical expression for volume and area, i. e., sphere with given radius and ellipsoid with given semi-axis.

The numerical results of V_p and S_p values were found for set of numerical parameters of approximated magnetic field B_0, B_1, B_2 of three different ECR ion sources. The results of this calculation are presented in Table 2.

Table 2. Result of calculation of V_p and S_p

ECR ion source	Volume, cm^3	Area, cm^2
INFN, LNS, SERSE 14 GHz	64	79
INFN, LNS, SERSE 18 GHz	262	207
Frankfurt UNI, IKF 14.4 GHz	148	144

CONCLUSION

From the point of view of the author the calculation problem of the volume confined by resonance surface and its area is important.

There are some works where the assumption about the ellipsoidal shape of resonance surface is given and for this case these parameters were calculated. Also, in some works [1] the numerical estimation is given:

$$\begin{aligned} V_p &= 0.15\,L\,d^2, \\ S_p &= 2.79\,L\,d, \end{aligned} \tag{12}$$

where L is the mirror-to-mirror distance and d is a working chamber diameter, and two numerical factors in (12) are for a very particular geometry of magnetic system of ion source. But all of these examples have special cases. Therefore, the above-presented technique for calculation of volume confined by resonance surface and its area without any assumption of ECR surface shape in general allows defining these parameters using only an ion-source magnetic field map.

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