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ON THE POSSIBILITY TO VERIFY MULTIDIMENSIONAL GRAVITY BY MEANS OF NEUTRONS

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О возможности проверки многомерной гравитации
с помощью нейтронов

Рассмотрены возможности экспериментальной проверки с помощью рассеяния нейтронов ньютоновского закона тяготения на малых расстояниях. Отклонение от закона можно ожидать в случае справедливости идеи о существовании многомерной гравитации. В рамках теоретической модели, связанной с длиной электрослабого взаимодействия, в борновском приближении получены выражения для амплитуд гравитационного рассеяния нейтронов. Вычислено влияние гравитационного рассеяния на асимметрию (вперед–назад) рассеяния нейтронов, взаимодействующих с ядрами при энергиях нейтронов от 10^{-10} эВ (ультрахолодные нейтроны) до нескольких ГэВ. Анализ результатов показывает, что эксперименты подобного рода при существующей точности вряд ли могут служить проверкой идеи существования многомерной гравитации.

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On the Possibility to Verify Multidimensional Gravity
by Means of Neutrons

The possibility to verify the Newtonian gravitational law for short distances with neutrons is discussed. Deviation from the law may be expected in case of validity of the idea on the existence of multidimensional gravity. Within the framework of the theoretical model related to the length of electroweak interaction in the Born approximation expressions for the amplitudes of gravitational neutron scattering have been obtained. Influence of the gravitational scattering on the asymmetry (forward–backward) of the neutron scattering interacting with nuclei at the neutron energies from ultracold ones (10^{-10} eV) up to several GeV has been calculated. Analysis of the results shows that such experiments may scarcely serve as verification of the idea of existence of multidimensional gravitation at the existing accuracy.

The investigation has been performed at the Frank Laboratory of Neutron Physics, JINR.

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In recent years growing interest has been shown by physicists in the question of possible existence of multidimensional gravity in nature. The significance of the given question is evidenced by the Russian School-Seminar (Kazan, September 2007) devoted to the modern problems of gravitation. The scientific program of this School includes topics related to multidimensional gravity. A number of papers [1–3] on the possibilities to test experimentally the multidimensional gravity hypothesis using neutron scattering have been published as well. As is known, the physicists studied only the free neutron fall due to the Earth's gravity, the dependence of the force of gravity on the neutron spin orientation, and verification of the equivalence principle in the quantum limit (in the domain, in which quantum theory plays an essential role) with the neutron interferometer.

The validity of the Newtonian three-dimensional gravitational law

$$F = -G\frac{m_1 m_2}{r^2}, \tag{1}$$

where F is the gravitational force, G is the Newtonian gravitation constant, m_1 and m_2 are the interacting masses, r is the distance between them, sing minus means that gravity force is the attractive force (see, e.g., [4]), has been thoroughly tested only for the distances of the order of a millimeter. It has not been verified for shorter distances. Meanwhile, for distances less than 1 mm any deviations are possible.

Let us suppose that masses as well as all other forces (except for gravitational ones) obey the laws established for the usual three-dimensional space and only the gravitational force requires that the multidimensional space ($n > 0$) should be invoked. Then, according to paper [1], using the supposition on the equality of the gravitational force between two particles, which are at the distance of electroweak length $R_e = \sqrt{G_F/\hbar c}$ with respect to each other and the usual Newtonian gravitational force at the distance between the particles equaling to the Planck length $R_P = \sqrt{\hbar G/c^3}$, the gravitational force may be written as

$$F_n = -G\frac{m_1 m_2}{r^{n+2}} R_c^n, \tag{2}$$

where R_c is some characteristic distance, $n = 0, 1, 2, \ldots$ characterizes the multidimensional gravity. At $n = 0$ equation (2) changes to the usual Newtonian equation (1). Equation (2) should be valid at $r \leqslant R_c$ and at r $> R_c$ the Newtonian three-dimensional gravitational law should be effective.

According to [1], we can obtain

$$R_c^n = \frac{c^{2-n/2} G_F^{1+n/2}}{G\hbar^{2+n/2}}, \tag{3}$$

where $G_F = (1.166 \cdot 10^{-5} \text{ GeV}^{-2})\ (\hbar c)^3 = G_f(\hbar c)^3$ is the Fermi constant.

The calculation results are given in the following Table:

n	1	2	3	4
R_c, cm	$1.2 \cdot 10^{17}$	2.8	$8 \cdot 10^{-6}$	$1.4 \cdot 10^{-8}$

It should be mentioned that the above reasoning is not a rigorous theory. It is of evaluation character. However, even from the purely experimental point of view the verification of Newtonian law of gravitation at short distances is of significance.

The value $n = 1$ presented in the Table should be rejected, since the magnitude of R_c exceeds the sizes of the Solar System and planetary orbits would be unstable. The sizes of $R_c < 1$ cm can be hardly tested by macroscopic experiments, though experiments with neutrons are possible in principle. In any case it might be well to search for deviations from the law $1/r^2$ using neutrons, even if the above theoretical considerations proves to be wrong.

For distances $r \leqslant R_c$, the gravitational potential can be written as

$$V(r) = -\frac{m_1 m_2 G (R_c)^n}{(n+1) r^{n+1}}. \tag{4}$$

The amplitude of gravitational scattering can be found in the Born approximation. For $n = 2$, it takes the form:

$$f(\varphi) = \frac{2}{3} m_n^2 m_t G_f^2 c^7 \hbar \int_0^{R_c} \frac{\sin qr}{qr^2} dr, \tag{5}$$

where $q = 2k \sin\varphi/2$, k is the wave number (unrelativistic $k = 2.197 \cdot 10^9 \sqrt{E}$ cm^{-1}, if E is in eV), φ is the scattering angle, m_n and m_t are the masses of neutron and target, respectively.

The calculation result is as follows:

$$\int_0^{R_c} \frac{\sin qr}{qr^2} dr = \frac{\sin qR}{(qR)^3} - \frac{\cos qR}{(qR)^2} + \frac{\sin qR}{qR} - ci(qR) - \frac{\sin qR_c}{qR_c} + ci(qR_c), \tag{6}$$

where R is the nuclear radius.

The value of (6) increases with decreasing neutron energy and at small qR:

$$\int_0^{R_c} \frac{\sin qr}{qr^2} dr = 0.756 - \ln(qR). \quad (7)$$

Thus, the most favourable conditions for measurements (comparatively large amplitude $f(\varphi)$) are created at small energies of scattered neutrons.

Since the amplitude of gravitational scattering is a real quantity (see (5)), it will interfere with a real part of the nuclear scattering amplitude and the differential cross section of neutron scattering will take the form:

$$\sigma(\varphi) = \sigma_n(\varphi) + 2\mathrm{Re} f_n f(\varphi), \quad (8)$$

where the first term is the nuclear scattering cross section, and the second one includes the gravitational scattering. For the case of neutron scattering with the energy of 10^{-10} eV by xenon nuclei ($R = 6.6 \cdot 10^{-13}$ cm) at the angles of 15 and 165° we can obtain:

$$q_1 R = 2kR \sin\varphi/2 = 3.79 \cdot 10^{-9} \text{ and } q_2 R = 2.88 \cdot 10^{-8}.$$

For such small qR the values of gravitational scattering amplitudes (see (7) and (5)) turn out to be equal to

$$f_1(15^\circ) = 4.03 \cdot 10^{-21} \text{ cm and } f_2(165^\circ) = 3.62 \cdot 10^{-21} \text{ cm}^*.$$

The expected value of scattering asymmetry $\alpha = \sigma(\varphi_1)/\sigma(\varphi_2)$ will be

$$\alpha = 1 + \frac{2\mathrm{Re} f_n [f_1(15^\circ) - f_2(165^\circ)]}{\sigma(\varphi)} = 1 + 1.75 \cdot 10^{-9}, \quad (9)$$

i.e., measurements with an uncertainty in the tenth significant digit are required, which is hardly possible at present.

It is of interest to compare the value of expected asymmetry (9) with the available experimental data on slow (thermal) neutron scattering by xenon nuclei. The most accurate of them were published in 1966 in [5]. The uncertainty in the determination of asymmetry in them was $\pm 10^{-4}$. To achieve such an accuracy, the second term in (9) should be increased approximately by five orders of magnitude. Xenon is unsuitable for this task. However, $\mathrm{Re} f_n$ value can be decreased using suitable isotopic mixture of, for instance, titan ($R = 4.7 \cdot 10^{-13}$ cm), having both positive and negative real parts of amplitudes

*Newtonian gravitational scattering amplitude ($n = 0$) is no more than 10^{-38} cm.

of neutron nuclear scattering by nuclei of various isotopes. The mixture of 44.6 % of ^{48}Ti isotope and 55.4 % of ^{46}Ti isotope has $\mathrm{Re} f_n \approx 5 \cdot 10^{-18}$ cm. According to formula (9), the second term of the asymmetry value will increase and will be $6 \cdot 10^{-5}$, which is already close to the desired value. However, measurements of scattering asymmetry in this case will be extremely difficult since the value of $2\mathrm{Re} f_n\ f(\varphi)$ at neutron energy of the order of 10^{-10} eV will be only $1.4 \cdot 10^{-38}$ cm^2. It is well to bear in mind that in experiments with condensed matter, in particular with titan, in addition to the corrections for the Doppler effect, the measured value of asymmetry should be corrected for interaction of neutrons with a collective of atoms. These corrections at energies below thermal neutron energy usually present difficulties and can be hardly taken into account with a high degree of accuracy.

Let us consider the case of diffraction of the small energy neutrons on the monocrystal prepared from the mixture of several isotopes of tungsten and having very small length of nuclear scattering (in the most accurate paper [6] it was approximately $0.02 \cdot 10^{-12}$ cm).

Let us suppose that there is a tungsten monocrystal with the neutron scattering length of the order of $0.002 \cdot 10^{-12}$ cm. Then, using the above-mentioned formula for the second term of the value of α asymmetry of the forward–backward scattering one may obtain the value $6 \cdot 10^{-7}$ (at the neutron energy of 0.017 eV and scattering angles of 12.5 and $158.5°$). However, it should be noted that it is hardly possible to record such a small value of asymmetry against the essential background of other interactions (thus, for example, the value of scattering asymmetry caused by interaction of a neutron with an electron of tungsten atom will be at least by six orders of magnitude greater).

Thus, multidimensional gravity at $n = 2$ can be hardly tested by means of neutron scattering.

Let us consider the case when $n = 3$ ($R_c \approx 8 \cdot 10^{-6}$ cm). In the Born approximation the amplitude of gravitational scattering will take the form:

$$f(\varphi) = \frac{1}{2} m_n^2 m_t c^8 \hbar^2 G_f^{5/2} \int\limits_0^{R_c} \frac{\sin\, qr}{qr^3} dr. \qquad (10)$$

The calculation result in the expression (at small qR) is as follows:

$$\int\limits_0^{R_c} \frac{\sin qr}{qr^3} dr = \frac{1}{R}\{-\frac{\cos qR}{(qR)^2} + \frac{1}{(qR)^2} + \\ + \frac{\sin qR}{2qR} + \frac{\cos qR}{2} + \frac{1}{2} qRSi(qR)\} - \frac{q}{2} Si(qR_c), \qquad (11)$$

where $Si(qR) = \int\limits_0^{qR} \frac{\sin\, qR}{qR} d(qR)$ is the integral sine.

The asymmetry value for the case of xenon at neutron energy of the order of 10^{-10} eV will be

$$\alpha = 1 + \frac{2[f_1(15^\circ) - f_2(165^\circ)]}{\mathrm{Re} f_n} = 1 + 1.3 \cdot 10^{-21}. \qquad (12)$$

This small value is nonmeasurable in practice.

Measurements at high energies are possible. Calculated values of neutron scattering asymmetry by xenon at neutron energy of the order of 100 GeV may be given as an example.

At $n = 3$

$$q_1 R = 2kR \sin 7.5^\circ = 873.5,$$

$$q_2 R = 2kR \sin 82.5^\circ = 6635,$$

and the value of asymmetry will be

$$\alpha = 1 + 2.4 \cdot 10^{-18}.$$

Thus, multidimensional gravitation at $n = 3$ can hardly be verified using neutron scattering.

In conclusion, it should be noted once more that despite the negative results of estimations carried out within the framework of electroweak approximation (see paper [1]) the search for possible manifestations of multidimensional gravitation using neutrons should be continued owing to the importance of the assigned task.

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