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MATHEMATICAL MODELING OF A STEADY GLASS FIBER DRAWING PROCESS

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Радев С. П., Бояджиев Т. Л., Онофри Ф. E11-2008-161
Математическое моделирование
процесса вытягивания стекловолокна

Предложен метод для численного изучения неизотермического процесса вытягивания стекловолокна из расплава. Математическая модель процесса основывается на одномерном уравнении движения и уравнении теплопередачи. При этом учитываются эффекты вязкости, гравитации, поверхностного натяжения, аэродинамического сопротивления, а также конвекции и радиационной теплопередачи. Исследованы изменение радиуса, аксиальной скорости и температуры вдоль волокна при варьировании параметров модели.

Работа выполнена в Лаборатории информационных технологий ОИЯИ.

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A spline differential numerical method is developed for studying the non-isothermal glass fiber drawing process. The method is based on the one-dimensional version of the equations of motion coupled by the heat transfer equation. The effects of the temperature-dependent viscosity, gravity, surface tension and air drag, as well as those of axial heat conduction, heat convection and radiative heat transfer are taken into account. Numerical results for the fiber radius, axial velocity and temperature are schown, illustrating the cooling effects of Stanton and radiation numbers.

The investigation has been performed at the Laboratory of Information Technologies, JINR.

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1. INTRODUCTION

The drawing of glass fibers is known (Kase & Matsuo [1], Glicksman [2], Shah & Pearson [3]) as non-isothermal process. In real conditions, due to different external and internal sources of disturbances (e.g., draw velocity and temperature fluctuations, take-up velocity and air-drag variations, etc.), the drawing process appears as unsteady. The undesirable effect of the disturbances manifests itself in variations of the fiber diameter. What is more, at very high extension ratios the diameter variations become (Gupta et al. [4]) self-sustained and are referred to as draw resonance.

In a series of papers Onofri et al. [5, 6] developed optical techniques for on-line measuring of the diameter of small glass fibers. The interferometric method was extended by Onofri et al. [7] for on-line measuring of the drawing tension. Using these methods, Onofri et al. [8] studied experimentally the weekly unsteady fiber drawing regime.

It will be interesting to develop a numerical method for predicting the diameter and tension variations in conditions similar to the drawing experiments in the above-mentioned papers. Simultaneously, we want to develop a tool for identifying the origin of the fiber fluctuations. To do this, we consider the drawing process as a superposition of a steady state drawing disturbed by small amplitude perturbations, originated from different external and internal sources.

As a first step, in the present paper we restrict ourselves to the case of steady drawing conditions. A spline differential method is proposed for calculating the radius, velocity and temperature profiles along the fiber. The method is based on one-dimensional equations of motion of the fiber (Gupta [9]) in which the effects of the temperature-dependent viscosity, gravity, surface tension and air drag are taken into account. These equations are coupled by heat transfer equation for the temperature profile in the fiber, accounting for the effects of the axial heat conduction, heat convection and radiative heat transfer. Selected numerical results are shown to demonstrate the role of cooling effects based on Stanton and radiation numbers.

2. STATEMENT OF THE PROBLEM

Geometrical configuration of the fiber forming process and the corresponding cylindrical coordinate system Orz are shown in Fig. 1. The fiber fluid is

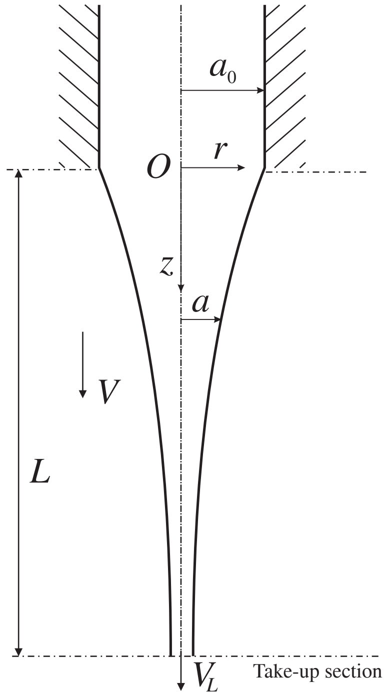

Fig. 1. Geometrical configuration of the fiber (sketch)

considered as Newtonian, incompressible and non-isothermal with variable non-dimensional temperature T and viscosity $\eta(T)$. For describing the fiber dynamics including the motion of the glass melt jet, one-dimensional equations in the form adopted by Gupta [9] are used. The latter are written as a system of coupled one-dimensional continuity, momentum and heat transfer equations, namely:

$$(AV)' = 0, \tag{1a}$$

$$-AVV' + \frac{3}{\text{Re}} (\eta A V')' + \frac{1}{\text{Fr}} A + \frac{1}{\text{We}} \left(\sqrt{A}\right)' - 2D_{R}^{*} V^2 \sqrt{A} = 0, \tag{1b}$$

$$-AVT' + \frac{1}{\text{Pe}} (AT')' - 2\sqrt{A} \left[\text{St}^*(T - T_a) + H \left(T^4 - T_a^4\right)\right] = 0, \tag{1c}$$

where superscript prime denotes the derivative in respect to the axial coordinate z, while $A(z), V(z)$ and $T(z)$ — the cross-section, axial velocity and temperature, respectively, assumed unknown functions of z. The above equations are written in non-dimensional form, based on the following characteristic scales: length L in axial and radius a_0 in radial direction, velocity V_0, temperature T_0 and viscosity

η_0 as defined in Table 1 at the end of the paper. Note that the momentum Eq. (1b) takes into account the effects of axial viscous stress (proportional to $1/\mathrm{Re}$), gravity ($1/\mathrm{Fr}$) and surface tension ($1/\mathrm{We}$), where non-dimensional parameters

$$\mathrm{Re} = \frac{\rho L V_0}{\eta_0}, \quad \mathrm{Fr} = \frac{V_0^2}{gL}, \quad \mathrm{We} = \frac{\rho a_0 V_0^2}{\sigma},$$

stand for Reynolds, Froude and Weber number, respectively. Also, the last term in Eq. (1b) accounts for the effect of air drag assumed proportional to the local drag coefficient C_f as follows:

$$D_R^* = C_f \frac{L\rho_a}{2a_0\rho}. \tag{2}$$

Similarly in heat transfer Eq. (1c) the cooling effects of heat convection, controlled by local Stanton number St^* and heat radiation, proportional to the radiation number H, are included together with the axial heat transfer (proportional to inverse Peclet number Pe):

$$\mathrm{St}^* = \frac{hL}{\rho c_p a_0 V_0}, \quad H = \frac{\varepsilon \sigma_{SB} L T_0^3}{\rho c_p a_0 V_0}, \quad \mathrm{Pr} = \frac{c_p \eta_0}{\kappa}, \quad \mathrm{Pe} = \mathrm{Pr}\,\mathrm{Re},$$

where h denotes the convective heat transfer coefficient, Pr is Prandtl number, while the remaining quantities are given in table. In Eq. (1c) the non-dimensional temperature of the surrounding air is denoted as $T_a = \hat{T}_a/T_0$.

Dimensional parameters of the problem

Input		Physico-chemical properties of the melt		Physico-chemical properties of the surrounding air	
a_0	Tip (nozzle) radius	c_p	Specific heat capacity	g	Gravity acceleration
L	Forming zone length	$\hat{k}$	Viscosity exponential coefficient	κ_a	Thermal conductivity
T_0	Mean temperature of the melt	ε	Hemispherical total emissivity	μ_a	Viscosity
$\hat{T}_a$	Temperature of the ambient air	$\hat{\eta}_0$	Melt viscosity	ν_a	Kinematics viscosity
V_0	Mean velocity of the melt	κ	Thermal conductivity	ρ_a	Density
V_L	Take-up velocity	ρ	Density		
		σ	Surface tension coefficient		
		σ_{SB}	Stefan–Boltzmann constant		

Following Kase & Matsuo [1], the local Stanton number is expressed as a function of the local air Reynolds number ($\mathrm{Re}_l = \mathrm{Re}_a A^{1/2} V$) and as a result is related to the local cross-section (radius) and axial velocity in the form

$$\mathrm{St}^* = \mathrm{St}\, A^{m-1/2} V^m,$$

where

$$\mathrm{St} = 0.2 \frac{\kappa_a L V_0}{\rho c_p \nu_a^2}\, \mathrm{Re}_a^{m-2}$$

is the fixed Stanton number, $\mathrm{Re}_a = a_0 V_0/\nu_a$ — air Reynolds number related to the local nozzle conditions, while typically $m = 1/3$.

In a similar way the local drag coefficient is chosen proportional to local air Reynolds number in the form proposed by Glicksman [10]

$$C_f = 0.4 \left(\mathrm{Re}_a \sqrt{A}\, V\right)^{m-1}. \tag{3}$$

Substituting Eq. (3) into (2), we obtain $D_R^* = D_R \left(\sqrt{A}\, V\right)^{m-1}$, where

$$D_R = 0.2\, (\mathrm{Re}_a)^{m-1}\, \frac{L\rho_a}{2a_0\rho}$$

is treated as one of the non-dimensional parameters of the problem and referred to as drag number.

For completing the statement of the problem, the corresponding boundary conditions should be added. The first group of the latter follows from the definition of the characteristic scales:

$$A(0) = 1, \quad V(0) = 1, \quad T(0) = 1. \tag{4}$$

The second group describes the drawing process at take-up conditions:

$$V(1) = E, \quad T'(1) = -2\sqrt{A(1)}\left\{\mathrm{St}^* \left[T(1) - T_a\right] + H \left[T^4(1) - T_a^4\right]\right\}, \tag{5}$$

where E denotes the extension ratio. Note that the temperature condition in Eq. (5) is obtained from Eq. (1c) after neglecting the effect of the axial heat transfer in the take-up zone.

The temperature dependence of the fiber viscosity in Eq. (1b) is given in exponential form

$$\eta(T) = \exp\left\{-k(T-1)\right\},$$

where $k = \hat{k}T_0$ is referred to as non-dimensional viscosity exponential coefficient.

Finally, the full set of non-dimensional parameters of the problem consists of E, Re, Fr, We, Pe, St, D_R, k, T_a.

3. NUMERICAL RESULTS AND DISCUSSION

In order to solve non-linear boundary-value problem (1), (4), (5), an algorithm based on Continuous analog of Newton method (Puzynin et al. [11]) is used. This method has some important advantages in comparison with the classical one, and in particular a wider range of convergence. At each iteration the resulting linear boundary-value problems are solved numerically by means of spline-collocation scheme of higher order of accuracy (Boyadjiev [12]).

As far as the fiber drawing process is multi-parametric, it is not realistic to illustrate the effect of each parameter. Instead, in what follows we concentrate on the cooling effects due to the heat convection and radiative heat transfer. It should be noted that the non-dimensional parameters in our calculations are derived from the Test M1 of Glicksman [2] and read as follows:

$$\mathrm{Re} = 9.03 \times 10^{-2}, \quad \mathrm{Fr} = 9.89 \times 10^{-7}, \quad \mathrm{We} = 6.21 \times 10^{-5}, \quad \mathrm{St} = 0.55,$$
$$D_R = 0.396, \quad \mathrm{Pe} = 7890, \quad H = 3.20, \quad k = 21.7, \quad T_a = 2.04 \times 10^{-2}.$$

Where applicable they correspond to a take-up position located at $L \approx 1$ m.

In Fig. 2 the effect of cooling is illustrated by the temperature at take-up section. As expected, the higher the Stanton number, the closer the take-up temperature to the surrounding air temperature T_a. Practically for St $\geqslant 0.8$ the take-up temperature reaches its final value and then remains unchanged.

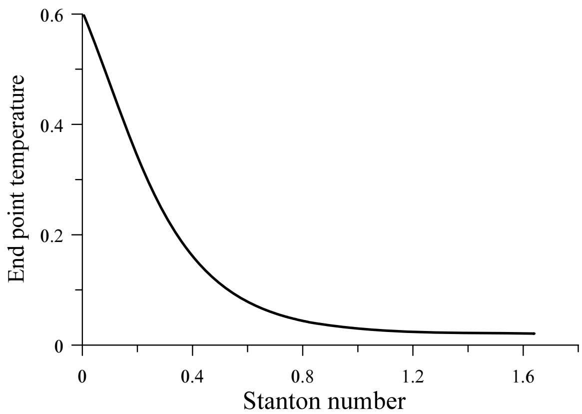

Fig. 2. Take-up temperature versus Stanton number

The full temperature profiles for selected values of the Stanton number, arranged in descending order, are shown in Fig. 3. The latter shows that decreasing Stanton number reduces the temperature gradient because of the decrease of convective cooling.

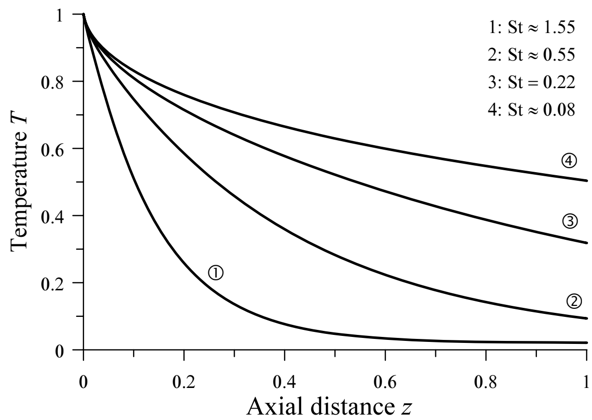

Fig. 3. Fiber temperature profiles versus axial coordinate for selected values of Stanton number

For the same Stanton numbers the corresponding axial velocity profiles are shown in Fig. 4. At high Stanton numbers (curves 1 and 2 in Fig. 4) the fiber forming zone ends far before the take-up point, resulting in constant velocity

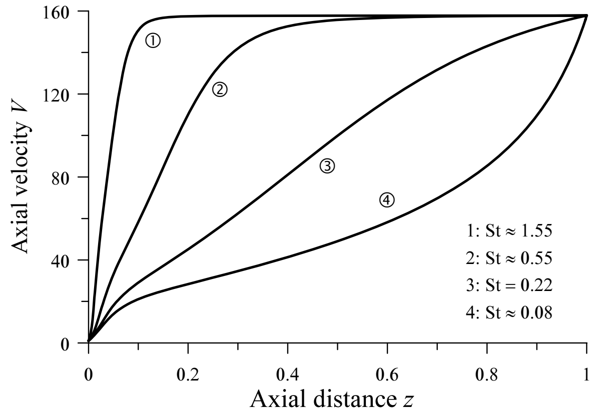

Fig. 4. Fiber velocity profiles versus axial coordinate for the same Stanton numbers as in Fig. 3. All curves are calculated for extension ratio $E = 157$ (like in Figs. 2 and 3)

(resp. in constant radius — see Fig. 5) out of the forming zone. Further decrease of the cooling (curve 3) extends the length of the forming zone and the final fiber radius (see Fig. 5) is achieved close to the take-up point. However, an

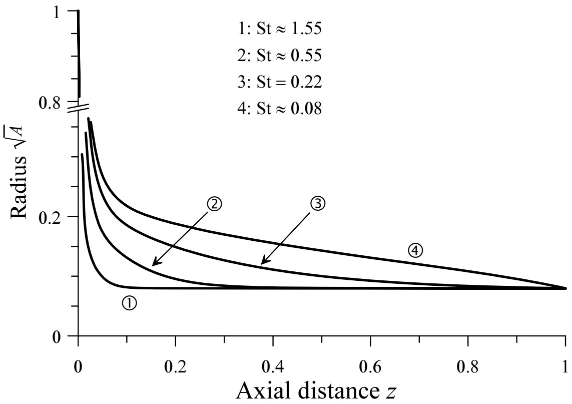

Fig. 5. Evolution of fiber radius versus axial coordinate

additional reduction of the Stanton number changes significantly the behavior of curve 4. This is due to the fact that the last portion of the forming zone is viscous dominated, resulting in an exponential variation of the velocity of the form $V(z) \sim E^z$.

In order to confirm the above conclusion, the velocity profiles in Fig. 6 are calculated for the same Stanton number $(\mathrm{St} = 0.08)$ and extension ratio $(E = 157)$ but different viscosity exponential coefficients. Increasing the latter reduces the length both of the viscous dominated zone and of the forming zone.

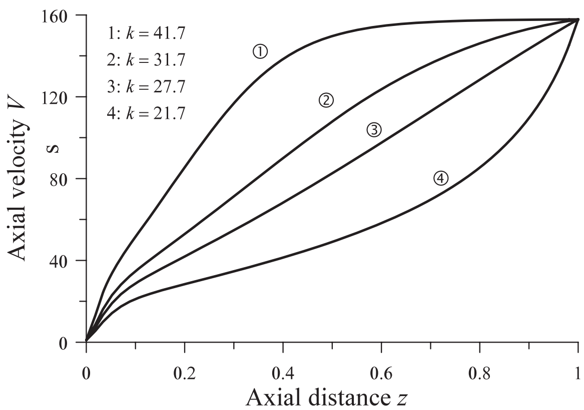

Fig. 6. Effect of the viscous exponential coefficient on the forming zone lengths

Another cooling factor acting on the forming zone is the radiation number H. The effect of the latter on the fiber radius is demonstrated in Fig. 7. It should

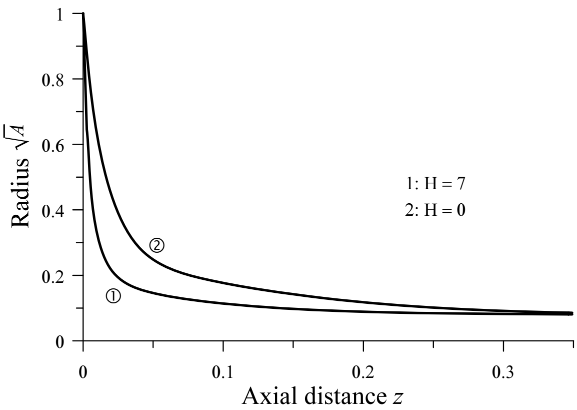

Fig. 7. Effect of the radiation number on the fiber radius

be mentioned that the cooling effect of radiation number is less important than the effect of Stanton number.

4. CONCLUSION

A spline differential numerical method is developed for studying the non-isothermal glass fiber drawing process. The method is based on one-dimensional version of the equations of motion coupled by the heat transfer equation. The effects of the temperature-dependent viscosity, gravity, surface tension and air drag, as well as those of axial heat conduction, heat convection and radiative heat transfer, are taken into account. Numerical results for the fiber radius, axial velocity and temperature are shown, illustrating the cooling effects of Stanton and radiation numbers.

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