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BEAM STABILITY IN SYNCHROTRONS  
WITH DIGITAL FILTERS IN THE FEEDBACK LOOP  
OF A TRANSVERSE DAMPER

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Устойчивость пучка в синхротронах с цифровыми системами подавления когерентных поперечных колебаний

Приводятся результаты теоретического исследования устойчивости ионного пучка в синхротронах в зависимости от параметров цифровых фильтров в цепи обратной связи систем подавления (СП) когерентных поперечных колебаний пучка. СП применяются в синхротронах с целью создания условий для предотвращения развития когерентных поперечных неустойчивостей пучка, а также для демпфирования остаточных колебаний частиц после инжекции. Система подавления когерентных поперечных колебаний пучка обеспечивает коррекцию поперечного импульса сгустков на каждом обороте с помощью дефлектора с учетом данных о смещении центра тяжести пучка, измеренных датчиком положения. Для обеспечения оптимальных условий демпфирования поперечных колебаний сгустков используются цифровые методы обработки сигналов в цепи обратной связи между датчиком положения и дефлектором. Приводятся данные о зависимостях темпа подавления когерентных колебаний от коэффициента передачи цепи обратной связи для различных параметров цифровых фильтров (узкополосного режекторного, фазового и Гильберта).

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Beam Stability in Synchrotrons with Digital Filters in the Feedback Loop of a Transverse Damper

The stability of an ion beam in synchrotrons with digital filters in the feedback loop of a transverse damper is treated. Solving the characteristic equation allows one to calculate the achievable damping rates as a function of instability growth rate, feedback gain and parameters of the signal processing. A transverse feedback system (TFS) is required in synchrotrons to stabilize the high intensity ion beams against transverse instabilities and to damp the beam injection errors. The TFS damper kicker (DK) corrects the transverse momentum of a bunch in proportion to its displacement from the closed orbit at the location of the beam position monitor (BPM). The digital signal processing unit in the feedback loop between BPM and DK ensures a condition to achieve optimal damping. Damping rates of the feedback systems with digital notch, Hilbert and all-pass filters are analyzed in comparison with those in an ideal feedback system.

The investigation has been performed at the Veksler and Balдин Laboratory of High Energy Physics, JINR.

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## INTRODUCTION

Heavy ion beams of a high quality are required by many physicists for experimental studies. Gold ion beams are accelerated now in RHIC (BNL) [1], it is planned to accelerate lead ions in LHC [2]. Future accelerator facilities at GSI (FAIR project [3]) and JINR (NICA project [4]) are designed for acceleration of uranium beams. These facilities include a linear accelerator and several synchrotrons. For example, the CERN accelerator chain for ion beams consists of Linac – LEIR – PS – SPS – LHC. In the framework of the FAIR project the existing GSI accelerators serve as injectors for new synchrotrons SIS100 and SIS300. It is planned to build a booster as the injector for the Nuclotron operated now with a linac at JINR and to use the Nuclotron as the injector for a collider designed in the framework of the NICA project. It is clear that injection errors during the beam path from the linac to synchrotrons can lead to the undesirable growth of a beam emittance. It should be emphasized also that high intensity beams will be provided by these accelerators. The ultimate intensities after injection into the LHC will be about  $4.8 \cdot 10^{10}$  ions for the  $^{208}\text{Pb}^{82+}$  beam with an energy of 177 GeV/u. The peak intensities of particles after injection into the SIS100 will be about  $5 \cdot 10^{11}$  for the  $^{238}\text{U}^{28+}$  beam with an energy of 0.2 GeV/u. These intensities can lead to coherent transverse instabilities. Theoretical predictions for the instability rise time  $\tau_{\text{inst}}$  correspond to hundreds revolution periods  $T_{\text{rev}}$  of particles in the synchrotron. Therefore, it is necessary to cure the transverse instabilities as well as to damp the transverse oscillations of the beam due to injection errors.

Transverse feedback systems (TFS) are used widely in synchrotrons for damping of coherent oscillations. The damping time  $\tau_{\text{d}}$  of TFS must be shorter than the instability rise time  $\tau_{\text{inst}}$  to suppress instability:  $\tau_{\text{d}} < \tau_{\text{inst}}$ . In addition to that the damping time must be chosen to limit the emittance growth due to the beam injection errors. If  $e_{\text{inj}}$  is the maximum assumed amplitude of a beam deviation from the closed orbit due to displacement and angular errors at injection, then the relative emittance growth  $\Delta\epsilon/\epsilon$  is [5, 6]

$$\frac{\Delta\epsilon}{\epsilon} = \frac{e_{\text{inj}}^2}{2\sigma^2} F_{\text{a}}^2; \quad F_{\text{a}} = \left( 1 + \frac{\tau_{\text{dec}}}{\tau_{\text{d}}} - \frac{\tau_{\text{dec}}}{\tau_{\text{inst}}} \right)^{-1}, \quad (1)$$

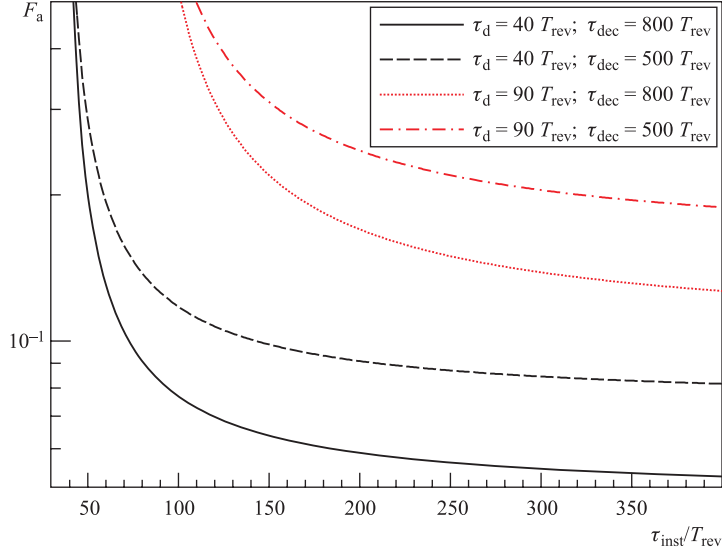


Fig. 1. Dependences of  $F_a$  on  $\tau_{\text{inst}}/T_{\text{rev}}$

where  $\sigma$  is the initial RMS beam size and  $\tau_{\text{dec}}$  is the beam decoherence time. Dependences of the form factor  $F_a$  on the instability rise time  $\tau_{\text{inst}}$  for several values of the damping time  $\tau_d$  and the beam decoherence time  $\tau_{\text{dec}}$  are shown in Fig. 1. As rule  $F_a < 0.1$  is assumed that corresponds to  $\tau_d \approx 40T_{\text{rev}}$  for  $\tau_{\text{inst}} > 100T_{\text{rev}}$  and  $\tau_{\text{dec}} > 500T_{\text{rev}}$ . The damping time  $\tau_d = 40T_{\text{rev}}$  is used commonly as the design specification of TFS for synchrotrons [7, 8].

## 1. BASIC DESCRIPTION

A classical transverse feedback system (see Fig. 2) consists of a beam position monitor (BPM), a damper kicker (DK) and an electronic feedback path with appropriate signal transmission from BPM to DK [9]. The damper kicker corrects the transverse momentum of a bunch in proportion to its displacement  $x[n, s_P]$  from the closed orbit at the BPM location  $s_P$  of the synchrotron's circumference  $C_0$  at the  $n$ th turn. The digital signal processing ensures the adjustment of the phase advance and the correction of the time of flight for optimum damping. The total delay  $\tau_{\text{delay}}$  in the signal processing of the feedback loop from BPM to DK is adjusted to be equal to  $\tau_{\text{PK}}$ , the particle flight of time from BPM to DK, plus an additional delay of  $q$  turns

$$\tau_{\text{delay}} = \tau_{\text{PK}} + qT_{\text{rev}}. \quad (2)$$

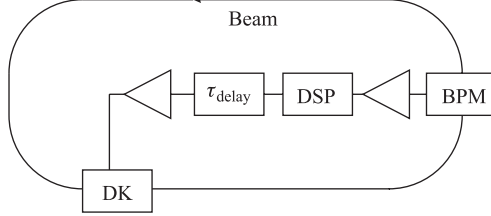


Fig. 2. Layout of a classical transverse feedback system

For the practical realization in a particle accelerator, we note that  $q = 0$  or  $q = 1$  are used [8].

Following the matrix description of the free oscillation of a particle in synchrotrons, the matrix equation for the bunch states at the BPM location  $s_P$  at the  $(n + 1)$  and  $n$ th turns after a small kick by the DK is given by [10]

$$\widehat{X}[n + 1, s_P] = \widehat{X}[n, s_P + C_0] = \widehat{M}_0 \widehat{X}[n, s_P] + \widehat{B} \widehat{T} \Delta \widehat{X}[n, s_K], \quad (3)$$

where elements of the column matrix  $\widehat{X}[n, s]$  are the bunch displacement  $x[n, s]$  and the angle  $x'[n, s]$  of its trajectory,  $\widehat{M}_0$  is the revolution matrix,  $\widehat{B}$  is an ordinary transfer matrix from the point  $[n, s_K]$  on the closed orbit at the DK location to the point  $[n, s_P + C_0]$  at the BPM position at the  $n$ th turn,  $\widehat{T}$  is the  $2 \times 2$  matrix in which  $T_{21} = 1$  and the other elements are zero. The first element of the column matrix  $\Delta \widehat{X}$  in Eq. (3) is equal to the kick value  $\Delta x'$

$$\Delta x'[n, s_K] = S_K V_{\text{out}}[n], \quad (4)$$

where  $S_K$  is the transfer characteristic of the damper kicker. The second element of the column matrix  $\Delta \widehat{X}$  can be an arbitrary value due to the form of the matrix  $\widehat{T}$ . The output voltage  $V_{\text{out}}[n]$  of the feedback loop depends on the input voltage  $V_{\text{in}}[n, s_P]$  at the BPM. In the general case of linear systems the output voltage can be written as follows:

$$V_{\text{out}}[n] = u[n - q] \sum_{m=-\infty}^{n-q} h[n - m] V_{\text{in}}[m, s_P], \quad (5)$$

where elements  $h[m]$  are determined by the electronics in the feedback loop,  $u[n]$  is the Heaviside step function and  $q$  corresponds to the number of turns for delay (see Eq. (2)). For a bunch injected at  $n = 0$ , the input voltage  $V_{\text{in}}[n, s_P]$  depends on the bunch displacement at the BPM location:

$$V_{\text{in}}[n, s_P] = S_P u[n] (x[n, s_P] + \delta x), \quad (6)$$

where  $S_P$  is the BPM sensitivity and  $\delta x$  is a deviation of the BPM electric centre from a closed orbit. It should be emphasized that Eqs. (3), (4), (5) and (6) correspond to the bunch-by-bunch feedback where the correction kick for a given bunch is computed based only on the motion of that bunch. Applying the bilateral  $Z$ -transform (see [11])

$$\mathbf{y}(z) = \sum_{n=-\infty}^{\infty} y[n]z^{-n}$$

in Eqs. (3), (4), (5) and (6) we obtain from (3):

$$\widehat{\mathbf{X}}(z) = \frac{z\widehat{I} - \widehat{\mathbf{M}}^{-1} \det \widehat{\mathbf{M}}}{\det(z\widehat{I} - \widehat{\mathbf{M}})} \left( z \begin{pmatrix} x[0, s_P] \\ x'[0, s_P] \end{pmatrix} + \frac{z^{-q}\mathbf{K}(z)}{\sqrt{\hat{\beta}_P\hat{\beta}_K}} \widehat{B}\widehat{T} \begin{pmatrix} \delta x/(1-z^{-1}) \\ 0 \end{pmatrix} \right), \quad (7)$$

where  $\widehat{I}$  is the identity matrix, the matrix  $\widehat{\mathbf{M}}(z)$  is given by

$$\widehat{\mathbf{M}}(z) = \widehat{M}_0 + \frac{z^{-q}\mathbf{K}(z)}{\sqrt{\hat{\beta}_P\hat{\beta}_K}} \widehat{B}\widehat{T}, \quad (8)$$

the betatron amplitude function at the point  $s_P$  of the synchrotron's circumference is  $\hat{\beta}_P = \hat{\beta}(s_P)$ , and  $\hat{\beta}_K = \hat{\beta}(s_K)$ . The transfer function  $\mathbf{K}(z)$  is determined by the system transfer function  $H(z)$  of the electronics in the feedback loop in accordance with parameters  $h[n]$  in (5):

$$\mathbf{K}(z) = \sqrt{\hat{\beta}_P\hat{\beta}_K} S_P S_K H(z), \quad H(z) = \sum_{n=-\infty}^{\infty} z^{-n} h[n]. \quad (9)$$

Consequently the bunch dynamics is determined by the poles  $z_k$  of  $\widehat{\mathbf{X}}(z)$  which are roots of the characteristic equation:

$$\det \left( z_k \widehat{I} - \widehat{\mathbf{M}}(z_k) \right) = z_k^2 - 2z_k \operatorname{Tr} \widehat{\mathbf{M}}(z_k) + \det \widehat{\mathbf{M}}(z_k) = z_k^2 - \left[ 2 \cos(2\pi\tilde{Q}) + z_k^{-q} \mathbf{K}(z_k) \sin(2\pi\tilde{Q} - \psi_{PK}) \right] z_k + 1 - z_k^{-q} \mathbf{K}(z_k) \sin \psi_{PK} = 0, \quad (10)$$

where  $\tilde{Q}$  is the beam tune,  $\psi_{PK}$  is the betatron oscillation phase advance from BPM to DK.

In the general case,  $\tilde{Q}$  is a complex function depending on  $z$  [12, 13]. The real part of  $\tilde{Q}$  is the number of betatron oscillations per turn:  $\operatorname{Re} \tilde{Q} = Q$ . The imaginary part of  $\tilde{Q}$  is determined by the increment of the transverse instability:  $2\pi \operatorname{Im} \tilde{Q} = T_{\text{rev}}/\tau_{\text{inst}}$ , where  $\tau_{\text{inst}}$  is the transverse instability rise time.

The beam is stable if eigenvalues  $z_k$  from Eq. (10) lie inside the unit circle:

$$|z_k| < 1. \quad (11)$$

Damping rates of the coherent betatron oscillations are defined by the absolute value of  $z_k$ :

$$\frac{T_{\text{rev}}}{\tau_k} = -\ln |z_k|, \quad (12)$$

where  $\tau_k$  is the time constant of the betatron oscillation amplitude decay. Fractional parts  $\{\text{Re} \tilde{Q}_k\}$  of the betatron frequency of a particle in presence of the transverse feedback system

$$\{\text{Re} \tilde{Q}_k\} = \frac{1}{2\pi} \arg(z_k) \quad (13)$$

are the fractional tunes ( $-0.5 < \{\text{Re} \tilde{Q}_k\} \leq 0.5$ ).

If  $|\mathbf{K}(z)| = 0$ , then the solution of Eq. (10)

$$z_{\pm}^{(0)} = \exp(\pm j 2\pi \tilde{Q}) \quad (14)$$

corresponds to the solution for frequencies of the betatron motion equation of a particle in synchrotrons. Let us assume that for small values of  $|\mathbf{K}(z)|$  we can write:

$$z^{-q} \mathbf{K}(z) = g \exp(\mp j \varphi) \exp(\mp j 2\pi q \tilde{Q}), \quad (15)$$

where the gain  $|g| \ll 1$  and the phase shift

$$\varphi = \arg\left(H(z_{-}^{(0)})\right) \quad (16)$$

of the feedback loop depend weakly on  $z$ , so that we can neglect dependences of  $g$  and  $\varphi$  on  $z$  in Eq. (10), and zero approximation from (14) can be used for  $g$  and  $\varphi$  at betatron frequencies. Let us assume also that the fractional part of the tune is not close to 0 or 0.5. In this case the solutions of Eq. (10) in the linear approximation with  $|g| \ll 1$  are expressed by the formula:

$$z_{\pm} \approx \left(1 - \frac{g}{2} \exp(\pm j(\frac{\pi}{2} - \tilde{\Psi}_{\text{PK}}))\right) \exp(\pm j 2\pi \tilde{Q}), \quad (17)$$

where

$$\tilde{\Psi}_{\text{PK}} = \psi_{\text{PK}} + 2\pi q \tilde{Q} + \arg\left(H(z = \exp(-j 2\pi \tilde{Q}))\right). \quad (18)$$

Using definitions (12) and (13) the damping rates follow as

$$\frac{T_{\text{rev}}}{\tau_{\pm}} \approx \frac{g \exp(\pm \text{Im} \tilde{\Psi}_{\text{PK}})}{2} \sin(\text{Re} \tilde{\Psi}_{\text{PK}}) \pm 2\pi \text{Im} \tilde{Q}, \quad (19)$$

and the fractional parts of tunes are

$$\{\text{Re} \tilde{Q}_{\pm}\} \approx \pm\{Q\} \mp \frac{g \exp(\pm \text{Im} \tilde{\Psi}_{\text{PK}})}{4\pi} \cos(\text{Re} \tilde{\Psi}_{\text{PK}}). \quad (20)$$

Therefore, the best damping of coherent transverse oscillations is achieved by optimally choosing the positions of BPM and DK yielding a phase advance of  $\text{Re } \tilde{\Psi}_{\text{PK}}$  equal to an odd multiple of  $\pi/2$ :

$$\text{Re } \tilde{\Psi}_{\text{PK}} = \frac{\pi}{2}(2k + 1), \quad (21)$$

where  $k$  is an integer. Hence, the overall damping rate is

$$\frac{T_{\text{rev}}}{\tau} \approx \frac{g \exp(-\text{Im } \tilde{\Psi}_{\text{PK}})}{2} \cos(\pi k) - 2\pi \text{Im } \tilde{Q} = \frac{T_{\text{rev}}}{\tau_{\text{d}}} - \frac{T_{\text{rev}}}{\tau_{\text{inst}}},$$

where  $\tau_{\text{d}}$  is the damping time constant of the TFS without instability.

In the following transverse feedback systems satisfying the optimal conditions (2) and (21) are considered. We call the special case with  $\varphi = 0$  and  $q = 0$  hereafter the *ideal* transverse feedback system.

If  $\tilde{Q}$  depends weakly on  $z$ , then the characteristic equation (10) with the feedback transfer function

$$z^{-q}\mathbf{K}(z) = ga_0z^{-q}H(z)$$

can be converted to a polynomial. It can be solved with the use of a root-finding algorithm or analytically for a polynomial of degree less than five. However, it is clear from (20) that  $\{\text{Re } \tilde{Q}_k\} \approx \{Q\}$  for  $|g| \ll 1$  in the case of (21). Therefore, dependences of damping rates  $|z_k|$  on gain  $g$  for the TFS with digital filters can be compared with those for the ideal TFS if  $a_0$  is defined for  $z_Q = \exp(-j2\pi Q)$  such that

$$|a_0z_Q^{-q}H(z_Q)| = 1, \quad a_0 \sin\left(\arg\left(z_Q^{-q}H(z_Q)\right) + \text{Re } \psi_{\text{PK}}\right) > 0. \quad (22)$$

Hence, the damping regime corresponds to  $g > 0$ . The calibration condition (22) will be used hereafter for all dependences of TFS damping parameters on gain  $g$ .

## 2. DIGITAL FEEDBACK SYSTEMS

Taking into account the final value theorem [11] and the solution (7) for  $\tilde{\mathbf{X}}(z)$  we can conclude that

$$\hat{X}[\infty, s_P] = \lim_{z \rightarrow 1} (1 - z^{-1})\tilde{\mathbf{X}}(z) = 0, \quad \text{if } \mathbf{K}(z = 1) = 0. \quad (23)$$

Therefore, as minimum a notch filter to suppress all the revolution harmonics (DC included) is required in the feedback loop. The magnitude of the difference signal from the BPM electrodes, after passing through the notch filter, is proportional



to the bunch deviation from the closed orbit. The system transfer function of the notch filter is [11]

$$H(z) = H_{\text{NF}}(z) = 1 - z^{-1}. \quad (24)$$

It is clear from (24) that the notch filter changes the gain  $g$  and phase  $\varphi$  of the open loop transfer characteristics. For example, if  $Q = 6.73$ , then  $\{Q\} = -0.27$  and in accordance with (16) the phase  $\varphi$  is  $\arg(H_{\text{NF}}(z_Q)) = \varphi_{\text{NF}} = 41.4^\circ$ . The gain  $|H_{\text{NF}}| = 2|\sin(\{Q\}\pi)| = 1.5$  can be adjusted by an amplifier  $a_0$  in the feedback loop in accordance with (22). However, according to the approximation formula (19), the damping rates for the TFS with the notch filter still change due to the phase shift  $\varphi_{\text{NF}}$  resulting in slower damping than for the case of the ideal TFS.

The unwanted phase shift  $\varphi_{\text{NF}}$  due to the notch filter can be compensated by a Hilbert filter [14] with the system transfer function

$$H_{\text{HF}}(z) = h_0 z^{-3} + h_1 z^{-2}(1 - z^{-2}) + h_3(1 - z^{-6}), \quad (25)$$

where

$$h_0 = \cos(\Delta\varphi), \quad h_1 = -\frac{2}{\pi} \sin(\Delta\varphi), \quad h_3 = -\frac{2}{3\pi} \sin(\Delta\varphi)$$

are the Hilbert transform impulse response coefficients.

The electric circuit of a feedback loop with the notch and Hilbert filters is shown in Fig. 3. The difference signal  $V_{\text{in}}$  from the electrodes of the beam position monitor (BPM) is amplified by front electronics with the gain  $g_{\text{in}}$ . Then the signal proceeds through the notch filter and the Hilbert filter. The synchronization needed is adjusted by the digital delay  $\tau_{\text{dd}}$ . The output voltage  $V_{\text{out}}$  on the damper kicker (DK) is supplied by the high power amplifier with the gain  $g_{\text{out}}$ . The notch filter has the standard configuration. It includes a one-turn delay  $T_{\text{rev}}$ , an inverter and a summator. The Hilbert filter includes six one-turn delays, four

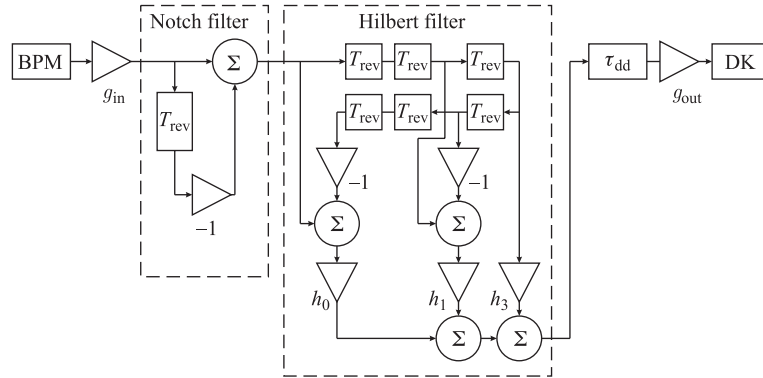


Fig. 3. Block diagram of feedback loop with the notch and Hilbert filters

summaters, two inverting amplifiers and three amplifiers  $h_0, h_1, h_3$ . For example, the phase shift needed for compensation of  $\varphi_{\text{NF}} = 41.4^\circ$  is obtained by using the Hilbert filter with  $\Delta\varphi = -72.8^\circ$ .

The unwanted phase shift  $\varphi_{\text{NF}}$  due to the notch filter can be compensated also by an all-pass filter [11] with a frequency-response magnitude that is constant but a phase advance which is variable and adjustable. The notch and Hilbert filters are FIR (finite impulse response) filters but the all-pass filter is IIR (infinite impulse response) filter. The transfer function of the first order all-pass filter is

$$H_{\text{AF}}(z) = \frac{z^{-1} - a^*}{1 - az^{-1}}, \quad (26)$$

where  $a$  is a free filter parameter for the adjustment of the phase, and  $a^*$  denotes its complex conjugate. For example, the phase shift needed for compensation of  $\varphi_{\text{NF}} = 41.4^\circ$  is obtained by using the all-phase filter with  $a = -0.501$ .

The electric circuit of a feedback loop with the notch and all-pass filters is shown in Fig. 4. The all-pass filter includes a one-turn delay  $T_{\text{rev}}$ , an inverting amplifier ( $-1/a^*$ ) in the non-recursive electric circuit, an amplifier  $a$  in the recursive electric circuit and two summaters. An additional inverting amplifier ( $-a^*$ ) in the output electric circuit is ensuring  $|H_{\text{AF}}| = 1$  for all frequencies independently of the filter parameter  $a$ . It allows one to adjust phase shifts in the feedback loop by varying the parameter  $a$  but keeping the gain of the TFS constant.

Dependences of damping rates  $|z_k|$  on gain  $g$  for the ideal TFS, the TFS with notch and the TFS with notch and all-pass filters are shown in Fig. 5 (the tune of  $Q = 6.73$  was used [15]). In case of the feedback loop with notch filter only Eq. (10) is a characteristic polynomial of the third degree. The characteristic equation (10) is a characteristic polynomial of the fourth degree in case of the TFS with notch and all-pass filters. Therefore, all dependences in Fig. 5 correspond to analytical solutions of Eq. (10). It is clear from Fig. 5 that the damping rates of the TFS with the notch filter are worse than those of the ideal TFS for all magnitudes

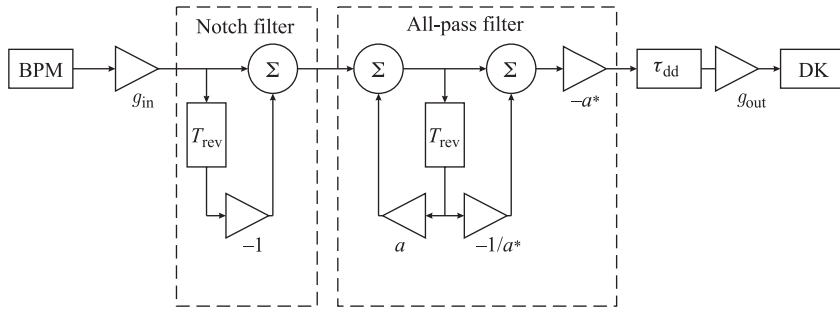


Fig. 4. Block diagram of feedback loop with the notch and all-pass filters

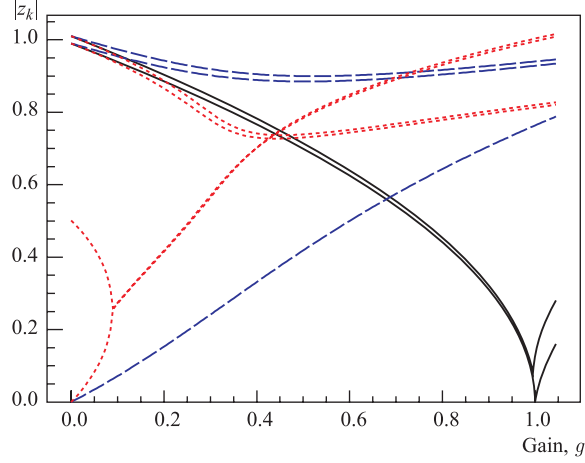


Fig. 5. Dependences of damping rates  $|z_k|$  on gain  $g$  for the ideal TFS (solid curves), for the TFS with the notch filter (dashed curves) and for the TFS with the notch and all-pass filters (dotted curves), parameter  $a = -0.501$ ; shown is the case of the tune of  $Q = 6.73$  and an assumed instability rise time of  $\tau_{\text{inst}} = 100T_{\text{rev}}$

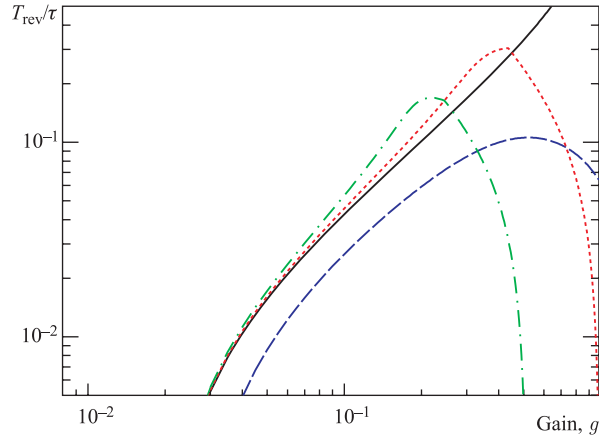


Fig. 6. Dependences of overall damping rates  $T_{\text{rev}}/\tau$  on gain  $g$  for feedbacks: the ideal TFS (solid curve), with the notch filter (dashed curve), with the notch and all-pass filters for  $a = -0.501$  (dotted curve), with the notch and Hilbert filters for  $\Delta\varphi = -72.8^\circ$  (dash-dotted curve); shown is the case of the tune of  $Q = 6.73$  and an assumed instability rise time of  $\tau_{\text{inst}} = 100T_{\text{rev}}$

of the feedback gains. However, for small gains  $g \ll 1$  the characteristics of the TFS with the notch and all-pass filters coincide with the corresponding parameters

of the ideal transverse feedback system if the phase shift of the notch filter was compensated by the all-pass filter with the parameter  $a = -0.501$ .

Dependences of overall damping rates  $T_{\text{rev}}/\tau$  on gain  $g$  for the ideal TFS and for feedback systems with digital notch, all-pass and Hilbert filters are shown in Fig.6 in cases of optimal values for  $a$  and  $\Delta\varphi$ . Therefore, the damping parameters of the ideal TFS can be obtained in the TFS with notch and all-pass or Hilbert filters for small gains. However, the stability range is wider for TFS with the notch and all-pass filters. The gain  $g$  of TFS with the notch filter only must be  $\approx 1.3$  times higher in the case of  $\tau_d = 40T_{\text{rev}}$  than for TFS with the all-pass or Hilbert filter.

## CONCLUSION

Following the analysis presented in this paper we can conclude that for small gains of the feedback loop the optimum damping characteristics of the ideal TFS can be restored in presence of a notch filter using a first order all-pass filter or a six order Hilbert filter with optimized parameters. Tuning the phase transfer characteristic of the all-pass or Hilbert filters in order to compensate the phase shift in the feedback loop caused by the notch filter we can obtain the optimal beam damping time. This possibility of tuning is an interesting feature and constitutes an advantage over a transverse damping system with notch filter only.

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