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THE SCHRÖDINGER–CHETAEV EQUATION IN BOHMIAN MECHANICS AND DIFFUSION MECHANISM OF ALPHA DECAY, CLUSTER RADIOACTIVITY AND SPONTANEOUS FISSION

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Уравнение Шредингера–Четаева в механике Бома и диффузионный механизм альфа-распада, кластерной радиоактивности и спонтанного деления

В рамках квантовой механики Бома, дополненной теоремой Четаева об устойчивых траекториях динамики в присутствии диссипативных сил, показана принципиальная возможность классического (без туннелирования) универсального описания процессов распада тяжелых радиоактивных ядер, которые при определенных условиях порождают так называемый индуцированный шумом переход или, иначе говоря, стохастический канал альфа-распада, кластерной радиоактивности и спонтанного деления, обусловленного диффузионным механизмом Крамерса.

На основе известных экспериментальных ENSDF-данных с помощью метода динамической авторегуляризации Александрова (FORTRAN-реализация программы REGN–Dubna) найдены параметризованные решения уравнения Крамерса ланжевеновского типа, описывающие с высокой точностью зависимость периодов полураспада (вероятностей распада) тяжелых радиоактивных ядер от полной кинетической энергии дочерних продуктов распада.

Верификация решения обратной задачи в рамках крамерсовского совместного описания α-распада, кластерной радиоактивности и спонтанного деления, которая проводилась на основе новейших экспериментальных данных об α-распаде сверхтяжелых четно-четных ядер ($Z = 114$, 116, 118), показала хорошее согласие экспериментальных и теоретических значений периодов полураспада в зависимости от энергии распада α-частиц.

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The Schrödinger–Chetaev Equation in Bohmian Mechanics and Diffusion Mechanism of Alpha Decay, Cluster Radioactivity and Spontaneous Fission

In the framework of Bohmian quantum mechanics supplemented with the Chetaev theorem on stable trajectories in dynamics in the presence of dissipative forces we have shown the possibility of the classical (without tunneling) universal description of radioactive decay of heavy nuclei, in which under certain conditions the so-called noise-induced transition is generated or, in other words, the stochastic channel of alpha decay, cluster radioactivity and spontaneous fission conditioned by the Kramers diffusion mechanism.

Based on the ENSDF database we have found the parameterized solutions of the Kramers equation of Langevin type by Alexandrov dynamic auto-regularization method (FORTRAN code REGN–Dubna). These solutions describe with high accuracy the dependence of the half-life (decay probability) of heavy radioactive nuclei on total kinetic energy of daughter decay products.

The verification of inverse problem solution in the framework of the universal Kramers description of the alpha decay, cluster radioactivity and spontaneous fission, which was based on the newest experimental data for alpha decay of even–even superheavy nuclei ($Z = 114$, 116, 118), has shown good coincidence of the experimental and theoretical half-life dependence on alpha-decay energy.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

In this paper we consider the question, which can be formulated the following rather strict and paradoxical form: «Are the so-called quantization conditions that are imposed on the corresponding spectrum of a dynamical system possible in principle in classical mechanics, analogously to what is taking place in quantum mechanics?»

Surprisingly, the answer to this question is positive and has been given more than 70 years ago by the Russian mathematician N. G. Chetaev in his article «On Stable Trajectories in Dynamics» [1, 2]. The leading idea of his work and of the whole scientific ideology was the most profound personal paradigm, with which begins, by the way, his principal work [2]: «Stability, which is a fundamentally general phenomenon, has to appear somehow in the main laws of nature». Here, it seems, Chetaev states for the first time the thesis of the fundamental importance of theoretically stable motions and their relation to the motions actually taking place in mechanics. He explains it as follows: the Hamiltonian theory of holonomic mechanical systems being under the action of forces admitting the force function has well proven itself, although, as Liapunov has shown [3], arbitrarily small perturbation forces can theoretically make such stable motions unstable. And since in actual fact holonomic mechanical systems regardless of everything often maintain stability, Chetaev puts out the paradoxical idea of the existence of special type of small perturbation forces, which stabilize the real motions of such systems [2]. Finally, Chetaev come to a conclusion that these arbitrarily small perturbation forces or, more precisely, «small dissipative forces with full dissipation, which always exist in our nature, represent a guaranteeing force barrier which makes negligible the influence of nonlinear perturbation forces» [4]. Furthermore, it has turned out that this «clear stability principle of actual motions, which has splendidly proven itself in many principal problems of celestial mechanics... unexpectedly gives us a picture of almost quantum phenomena» [5].

It is interestingly to note that a similar point of view can be found in the different time and with the different extent of closeness in Dirac [6] and 't Hooft [7]. For example, in [6] a quantization procedure appears in the framework of generalized Hamiltonian dynamics which is connected with the selection of the so-called small integrable A-spaces, only in which solutions of the equations of motion, and thus, only stable motions of a physical system are possible (see Eqs. (48) in Ref. [6]). On the other hand, according to the 't Hooft idea, a classical deterministic theory (on the Planck scale) supplemented with a dissipative mechanism generates the observed quantum behavior of our world on a laboratory scale. In particular, 't Hooft has shown that there is a very important class of classical deterministic system in which Hamiltonian is positive due to dissipation mechanism, that leads to «an apparent quantization of the orbits which resemble the quantum structure seen in the real world» [7]. It is obvious that the 't Hooft idea on the verbal level is practically an adequate reflection of the crux of the Chetaev idea, since a physical essence of both ideas is based on the fundamental role of dissipation in microcosm, which can be described by nontrivial but unambiguous (on the Planck scale) thesis: there is not a dissipation — there is not a quantization!

In this paper we generalize the Chetaev theorem on stable trajectories in dynamics in the presence of dissipative forces in case when the Hamiltonian of system is explicitly time-dependent, and using obtained results we develop the alternative model of radioactive decay of heavy nuclei (alpha decay, cluster radioactivity and spontaneous fission), which relies not on the traditional quantum effect of the particle penetration or tunneling through the nuclear potential barrier, but on the classical «jump» over this barrier due to diffusion induced by a noise.

2. THE SCHRÖDINGER EQUATION AS THE CONDITION OF STABLE TRAJECTORIES IN CLASSICAL DYNAMICS IN THE PRESENCE OF DISSIPATIVE FORCES

Below we generalize the Chetaev theorem on stable trajectories in dynamics in case when the Hamiltonian of system is time-dependent [8]. For that let us consider a material system (where $q_1, \ldots, q_n$ and $p_1, \ldots, p_n$ are generalized coordinates and momenta of a holonomic system) in the field of potential forces admitting the force function of $U(q_1, \ldots, q_n)$ type.

The complete intergal of the Hamilton–Jacobi differential equation corresponding to the system under consideration has the form

$$S = f(t, q_1, \ldots, q_n; \alpha_1, \ldots, \alpha_n) + A, \tag{1}$$

where $\alpha_1, \ldots, \alpha_n$ and A are arbitrary constants, and the general solution of the mechanical problem, according to the well-known Jacobi theorem is defined by the formulas

$$\beta_i = \frac{\partial S}{\partial \alpha_i}, \quad p_i = \frac{\partial S}{\partial q_i}, \quad i = 1, \ldots, n, \tag{2}$$

where β_i are new constants of integration. The possible motions of the mechanical system are determined by the different values of constants α_i and β_i.

We will call the motion of the material system, of which the stability is going to be studied, nonperturbed motion. To begin with, let us study the stability of such a motion with respect to the variables q_i under the perturbation only of the initial values of the variables (i.e., of the values of the constants α_i and β_i) in absence of perturbation forces.

If we denote by $\xi_j = \delta q_j = q_j - q_j(t)$ and $\eta_j = \delta p_j = p_j - p_j(t)$ the variations of the coordinates q_j and the momenta p_j, and by $H(q_1, \ldots, q_n, p_1, \ldots, p_n)$ the Hamilton function, then it is easy to obtain for Hamilton's canonical equations of motion

$$\frac{dq_j}{dt} = \frac{\partial H}{\partial p_j}. \quad \frac{dp_j}{dt} = -\frac{\partial H}{\partial q_j} \tag{3}$$

differential equations (in first approximation) in Poincaré's variations [9], which have the following form:

$$\begin{aligned} \frac{d\xi_i}{dt} &= \sum_j \frac{\partial^2 H}{\partial q_j \partial p_i}\xi_j + \sum_j \frac{\partial^2 H}{\partial p_j \partial p_i}\eta_j, \\ & \qquad\qquad\qquad\qquad (i = 1, \ldots, n), \\ \frac{d\eta_i}{dt} &= -\sum_j \frac{\partial^2 H}{\partial q_j \partial q_i}\xi_j - \sum_j \frac{\partial^2 H}{\partial p_j \partial q_i}\eta_j, \end{aligned} \tag{4}$$

where the coefficients are continuous and bounded real functions of t. These equations are of essential importance in studies of the stability of motion of conservative mechanical systems. Let us show this.

Poincaré has found [9] that if ξ_s, η_s and ξ'_s, η'_s are any two particular solutions of variational equations (4), then the following quantity is invariant:

$$\sum_s (\xi_s \eta'_s - \eta_s \xi'_s) = C, \tag{5}$$

where C is a constant. The proof is just a differentiation over t.

It is not difficult to show that for each ξ_s, η_s there is always at least one solution ξ'_s, η'_s for which the constant C in Poincaré's invariant does not vanish. Indeed, for a nontrivial solution ξ_s, η_s one of the initial values ξ_{s0}, η_{s0} at time t_0 will be different from zero. Then the second particular solution can always be defined by the initial values ξ'_{s0}, η'_{s0} in such a way that the constant under consideration does not vanish.

Let for two solutions of the variation equations ξ_s, η_s and ξ'_s, η'_s the value of the constant C be different from zero, and λ and λ' are the characteristic functions corresponding to these solutions. If we apply to this invariant Liapunov's theory [3], then we can directly, on the one hand, conclude that the characteristic value of the left-hand side of the invariance relation (5), corresponding to the nonvanishing constant, is zero. On the other hand, this allows us to obtain the following inequality:

$$\lambda + \lambda' \leqslant 0. \tag{6}$$

If we now assume that the system of Pincaré's variation equations is correct, then using Liapunov's theorem on the stability of the systems of differential equations in first approximation [3], it is easy to show that for the stability of the nonperturbed motion of the Hamiltonian system under consideration it is necessary that all the characteristic numbers of the independent solutions in Eq. (6) be equal to zero:

$$\lambda = \lambda' = 0. \tag{7}$$

Thus, Eq. (7) represents a stability condition for the motion of the Hamiltonian system (4) with respect to the variables q_i and p_i under the perturbation of the initial values of the variables only, i.e., the values of the constants α_i and β_i. However, the determination of the characteristic numbers as functions of α_i and β_i is a very difficult problem and therefore is not practical. The problem becomes simpler, if we note that, since the nonperturbed motion of our Hamiltonian system satisfying condition (7) is stable under any perturbations of initial conditions, it has to be stable under arbitrary perturbations of the constants β_i only. In other words, the problem is reduced to the determination of the so-called conditional stability.

According to this assumption about the character of initial perturbations from the solutions of the Hamilton–Jacobi equation (2) the following relations are obtained immediately up to the terms of the second order:

$$\eta_i = \sum_j \frac{\partial^2 S}{\partial q_i \partial q_j} \xi_i, \tag{8}$$

which allows us, taking into consideration the relation

$$H = \frac{1}{2} \sum g_{ij} p_i p_j + U, \tag{9}$$

to write the first group of Eqs. (4) in the form

$$\frac{d\xi_i}{dt} = \sum_{js} \xi_s \frac{\partial}{\partial q_s} \left(g_{ij} \frac{\partial S}{\partial q_j} \right), \tag{10}$$

where coefficients g_{ij} depend on coordinates only. Here the variables q_j and the constants α_j on the right-hand sides must be replaced using their values corresponding to nonperturbed motion.

If the variation equations (10) are correct, then, according to the well-known Liapunov theorem [3] about the sum of the eigenvalues of the independent solutions and to condition (9), we can conclude that the following condition is necessary for stability (10):

$$\lambda \left\{ \exp \int L dt \right\} = 0, \quad \text{where} \quad L = \sum_{ij} \frac{\partial}{\partial q_i} \left(g_{ij} \frac{\partial S}{\partial q_j} \right), \tag{11}$$

where λ is the eigenvalue of the function in the brackets.

Furthermore, if the system of equations (10), in additions to the correctness giving the condition (11), satisfies reducibility requirements and if the corresponding linear transformation

$$x_i = \sum_j \gamma_{ij} \xi_j \tag{12}$$

has a constant determinant $\| \gamma_{ij} \| \neq 0$, then, due to the invariance of the eigenvalues of the solutions of system (10) under such a transformation and due to the well-known Ostrogradsky–Liouville theorem, it can be shown that in this case from Eq. (11) a necessary stability will follow in the form

$$L = \sum_{ij} \frac{\partial}{\partial q_i} \left(g_{ij} \frac{\partial S}{\partial q_j} \right) = 0, \tag{13}$$

which expresses the vanishing of the sum of the eigenvalues of system (11). A simple but elegant proof of condition (13) can be found in Ref. [10].

Let us now consider a more complicated problem. Let the material system in actual motion is under the action of forces with force function U, presumably taken into account by the considerations above, and some unknown perturbation (dissipative) forces, which are supposed to be potential and admitting the force function Q. Then the actual motion of the material system will take place under the influence of the forces with the joint force function $U^* = U + Q$, and thereby the actual motion of the system will not coincide with the theoretically predicted (in the absence of perturbation).

Keeping the problem setting the same as above, which concerns the stability of the actual unperturbed motions under the perturbation of initial conditions only, the necessary condition for stability in the first approximation of the type (13) will not be effective in the general case, since the new function S is unknown (just as Q). Nevertheless, it turns out that it is possible to find such stability conditions, which do not depend explicitly on the unknown action function S and potential Q.

Thus, let us start with the stability requirement of the type (13), assuming that the conditions for its existence (well-definiteness, etc.) for actual motions are fulfilled. Let us introduce in Eq. (13) instead of function S a new function ψ, defined by

$$\psi = A \exp(ikS), \tag{14}$$

where k is a constant and A is a real function on the coordinates q_i and time t only.

From this follows

$$\frac{\partial S}{\partial q_j} = \frac{1}{ik}\left(\frac{1}{\psi}\frac{\partial \psi}{\partial q_j} - \frac{1}{A}\frac{\partial A}{\partial q_j}\right) \tag{15}$$

and, therefore, Eq. (13) will become

$$\sum_{i,j} \frac{\partial}{\partial q_i}\left[g_{ij}\left(\frac{1}{\psi}\frac{\partial \psi}{\partial q_j} - \frac{1}{A}\frac{\partial A}{\partial q_j}\right)\right] = 0. \tag{16}$$

On the other hand, for the perturbed motion it is possible to write the Hamilton–Jacobi equation in the general case when Hamiltonian H is explicitly time-dependent:

$$\frac{1}{2k^2}\sum_{i,j} g_{ij}\left(\frac{1}{\psi}\frac{\partial \psi}{\partial q_i} - \frac{1}{A}\frac{\partial A}{\partial q_i}\right)\left(\frac{1}{\psi}\frac{\partial \psi}{\partial q_j} - \frac{1}{A}\frac{\partial A}{\partial q_j}\right) = \frac{\partial S}{\partial t} + U_0 + Q, \tag{17}$$

where $\partial S/\partial t$ is obtained by Eq. (14). Adding Eqs. (17) and (18) we have a necessary stability condition (in the first approximation) in this form

$$\frac{1}{2k^2\psi}\sum_{i,j}\frac{\partial}{\partial q_i}\left(g_{ij}\frac{\partial \psi}{\partial q_j}\right) - \frac{1}{2k^2 A}\sum_{i,j}\frac{\partial}{\partial q_i}\left(g_{ij}\frac{\partial A}{\partial q_j}\right) -$$
$$- \frac{1}{k^2 A}\sum_{i,j} g_{ij}\frac{\partial A}{\partial q_j}\left(\frac{1}{\psi}\frac{\partial \psi}{\partial q_i} - \frac{1}{A}\frac{\partial A}{\partial q_i}\right) - \frac{1}{ikA\psi}\left[A\frac{\partial \psi}{\partial t} - \psi\frac{\partial A}{\partial t}\right] - U - Q = 0. \tag{18}$$

Equality (18) will not contain Q, if the amplitude A is defined from the equation

$$\frac{1}{2k^2 A}\sum_{i,j}\frac{\partial}{\partial q_i}\left(g_{ij}\frac{\partial A}{\partial q_j}\right) + \frac{i}{kA}\sum_{i,j} g_{ij}\frac{\partial A}{\partial q_j}\frac{\partial S}{\partial q_i} - \frac{1}{ikA}\frac{\partial A}{\partial t} + Q = 0, \tag{19}$$

which, after the separation into the real and imaginary parts, splits into two equations

$$Q = -\frac{1}{2k^2 A}\sum_{i,j}\frac{\partial}{\partial q_i}\left(g_{ij}\frac{\partial A}{\partial q_j}\right), \quad \frac{\partial A}{\partial t} = -\sum_{i,j} g_{ij}\frac{\partial A}{\partial q_j}\frac{\partial S}{\partial q_i}, \tag{20}$$

where Q is dissipation energy.

Thus, if the perturbing forces have the structure of the type (20) satisfying the requirement formulated above, the necessary stability condition (18) will have the form of «Schrödinger's» type differential equation

$$\frac{i}{k}\frac{\partial\psi}{\partial t} = -\frac{1}{2k^2}\sum_{i,j}\frac{\partial}{\partial q_i}\left(g_{ij}\frac{\partial\psi}{\partial q_j}\right) + U\psi. \tag{21}$$

In other words, we obtain the following important result. Equation (13) corresponding to the Chetaev stability condition is equivalent to the equation of «Schrödinger's» type (21) when using the transformation (14). It is obvious that for the equations of type (21) one-valued, finite and continuous solutions for the function ψ in stationary case are admissible only for the eigenvalues of the total energy E and, consequently, the stability of actual motions considered here, can take place only for these values of total energy E. It should be noted, that Schrödinger was the first who paid attention (while mathematicians knew this long ago [11]) to such a class of differential equations, where the discreteness of spectrum displays under the natural conditions such as an intergrability of solution squared modulus and the solution finiteness at the singular points of equation [11] (unlike, for example, boundary-value problems with boundary conditions).

Let us now come back to the problem of quantization and illustrate it on a very simple example. Let us consider a material point of mass m moving in the field of conservative forces with force function U, which in the general case is time-dependent. We will study stability of motion of this point in Cartesian coordinates x_1, x_2, x_3. Denoting the momenta coordinatewise as p_1, p_2, p_3 we obtain for the kinetic energy the well-known expression

$$T = \frac{1}{2m}(p_1^2 + p_2^2 + p_3^2). \tag{22}$$

In this case, conditions (21) for the structure of the perturbation forces admit the following relations:

$$Q = -\frac{\hbar^2}{2m}\frac{\Delta A}{A}, \quad \frac{\partial A}{\partial t} = -\sum\frac{\partial A}{\partial x_i}\frac{p_i}{m}, \quad k = \frac{1}{\hbar}, \tag{23}$$

and the differential equation (21), which is used for the determination of stable motions, becomes

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi + U\psi, \tag{24}$$

i.e., coincides with the well-known Schrödinger equation in quantum mechanics [12], which represents a relation constraining the choice of the constants of integration (of the total energy E in stationary case) of the full Hamilton–Jacobi integral. Below Eq. (24) will be named by the Schrödinger–Chetaev equation to emphasize in that way the specificity of its origin.

In the case it becomes interesting to consider the case connected with the inverse substitution of the wave function (14) in the Schrödinger equation (24), which generates an equivalent system of equations, known as the Bohm–Madelung system of equations [13–15], however, taking into account conditions (13) and (20):

$$\frac{\partial A}{\partial t} = -\frac{1}{2m}\left[A\Delta S + 2\nabla A\cdot\nabla S\right] = -\nabla A\cdot\frac{\nabla S}{m}, \tag{25}$$

$$\frac{\partial S}{\partial t} = -\left[\frac{(\nabla S)^2}{2m} + U - \frac{\hbar^2}{2m}\frac{\Delta A}{A}\right]. \tag{26}$$

It is important to note that the last term in Eq. (26), which in interpretation [13] is of a «quantum» potential of the so-called Bohm's ψ-field [14, 15] exactly coincides with the dissipation energy Q in Eq. (23). At the same time Eq. (25) is identical with condition for $\partial A/\partial t$ in Eq. (20).

If we make a substitution of $P = \psi\psi^* = A^2$ type, Eqs. (25) and (26) can be rewritten as follows:

$$\frac{\partial P}{\partial t} = -\nabla P\cdot\frac{\nabla S}{m}, \tag{27}$$

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + U - \frac{\hbar^2}{4m}\left[\frac{\Delta P}{P} - \frac{1}{2}\frac{(\nabla P)^2}{P^2}\right] = 0. \tag{28}$$

The statement that $P(x, y, z, t)$ indeed is the probability density function of particle trajectory number is substantiated as follows. Let us assume that the influence of the perturbation forces generated by the potential Q on the wave packet in an arbitrary point in the phase space is proportional to the density of the particle trajectories ($\psi\psi^* = A^2$) at this point. From where follows that the wave packet is practically not perturbed when the following condition is fulfilled:

$$\int Q\psi\psi^* dV \Rightarrow \min, \quad \text{where} \quad \int \psi\psi^* dV = 1, \tag{29}$$

where dV denotes a volume element of the phase space. And this implies in turn that, for the totality of motions in the phase space, the perturbation forces allow the absolute stability only if condition (29) is fulfilled or, in other words, when the obvious condition for the following variational problem (with respect to Q) is fulfilled:

$$\delta\int Q\psi\psi^* dV = \delta Q = 0. \tag{30}$$

This variational principle (30) is actually nothing else than the principle of least action of perturbation, which will be named below by the Chetaev variational principle [1, 4, 16].

Let us write for Q (using the previous notations and Eq. (9)) the following equality:

$$Q = -\frac{\partial S}{\partial t} - U - T = -\frac{\partial S}{\partial t} - U - \frac{1}{2}\sum_{ij} g_{ij}\frac{\partial S}{\partial q_i}\frac{\partial S}{\partial q_j}. \qquad (31)$$

On the other hand, if (14) holds, it is easy to show that

$$\frac{1}{2}\sum_{ij} g_{ij}\frac{\partial S}{\partial q_i}\frac{\partial S}{\partial q_j} = -\frac{1}{2k^2\psi^2}\sum_{ij} g_{ij}\frac{\partial \psi}{\partial q_i}\frac{\partial \psi}{\partial q_j} + \frac{1}{2k^2A^2}\sum_{ij} g_{ij}\frac{\partial A}{\partial q_i}\frac{\partial A}{\partial q_j} + \\ + ik\frac{1}{2k^2A^2}\sum_{ij} g_{ij}\frac{\partial A}{\partial q_i}\frac{\partial S}{\partial q_j}. \qquad (32)$$

Further, we need to carry out the following successive substitutions. First, for the first term on the right-hand side of Eq. (32) we substitute its value from the first stability condition (16), then we insert the obtained relation into (31) and finally put the result into the equation corresponding to the variational principle (30).

It is remarkable that as a consequence of the substitution procedure described above we obtain a relation which exactly equal to Eq. (18) and, therefore, the resulting structure expression and the necessary condition for stability coincide with (20) and (21). And this means that based on Chetaev's variational principle (30) we obtained an independent confirmation of the fact that the physical nature of $P(x, y, z, t)$ indeed reflects not simply the notion of probability density of «something» according to Bohm's equation of continuity [13–15], but plays the role appropriate to the probability density of the particle trajectory number.

Such semantic content of the probability density function $P(x, y, z, t)$ and simultaneously, the exact coincidence of the «quantum» potential of Bohm's ψ-field [13–15] in Eq. (26) and of the force function of perturbation Q in Eq. (23) leads to astonishing, but fundamental conclusions:

—- In view of Chetaev's theorem on the stable trajectories of dynamics the reality of Bohm's ψ-field is an evident and indisputable fact, which in turn leads at first glance to the paradoxical conclusion that classical and quantum mechanics are two complementary procedures of one Hamiltonian theory. In other words, classical mechanics and the quantization (stability) conditions represent, in contradiction to the correspondence principle, two complementary procedures for description of stable motions of a physical system in a potential field. In the framework of this theory Eq. (26) is an ordinary Hamilton–Jacobi equation and it differs from the analogous equation obtained from $\hbar \to 0$ ($Q \to 0$ [14]) only in so far as its solution is a priori stable. It is obvious, that exactly this difference is a cause of such phenomenon as quantum chaos characterizing, as is generally known, the peculiarities of quantum mechanics of the systems with chaotic behavior in the classic limit [17].

— Obviously, that in the light of Chetaev's theorem the sense of Heisenberg's uncertainty relations cardinally changes, because in this case the basic cause of statistical straggling characterized by variances of coordinates and momenta, are small dissipative forces (see Eq. (23) for $\partial/\partial t$), which are generated by the perturbation potential (or that is equivalently, by the quantum potential Q). At the same time it is easy to show that the variance values of coordinates and momenta are predetermined by the average quantum potential $\langle Q \rangle$. This is visually demonstrated within the framework of the mathematical notation of uncertainty relations for one-dimensional case in the following form (see Eq. (6.7.23) in Ref. [14]):

$$\left\langle (\Delta x)^2 \right\rangle \left\langle (\Delta p_x)^2 \right\rangle = \left\langle (\Delta x)^2 \right\rangle \langle Q \rangle \, 2m \geqslant \hbar^2 / 4, \tag{33}$$

where $\langle (\Delta x)^2 \rangle$ and $\langle (\Delta p_x)^2 \rangle$ are variances of coordinates and momenta, respectively.

— Based on the principle of least action of perturbation (30) it is shown that the function $P(x, y, z, t)$ semantically and syntactically is equivalent to the probability density function of particle trajectory number. It is easy to see (Eq. (27)) that the function $P(x, y, z, t)$ directly gives information about the velocity (and consequently about a coordinate) of particle, for which it is possible to write down the expression in the form more evident than Eq. (27):

$$u = \frac{dx}{dt} = -\frac{\partial P / \partial t}{\partial P / \partial x}. \tag{34}$$

This proof in combination with the new (Chetaev's) interpretation of the Heisenberg uncertainty relations directly and unambiguously shows that Bohmian quantum mechanics supplemented with Chetaev's generalized theorem does not have hidden parameters in the form of the velocity and coordinate of the particle. In other words, these parameters, which describe the particle trajectory, not only exist but are completely predetermined by the wave solution of the Schrödinger–Chetaev equation (24), or, more exactly, by the density function of particle trajectory number.

It is important to note here that all the most known alternative theories of quantum mechanics, including Bohmian mechanics, are theories with hidden variables. In the first place, it concerns the stochastic quantum mechanics of Nelson [18], the fractal quantum mechanics of Nottale [19] and the geometric quantum mechanics of Santamato [20]. They are all like that just because the probability density function $P(x, y, z, t)$ is exclusively defined by the continuity equation (i.e., without taking into account $\Delta S = 0$ in Eq. (25)), which, as stated above, only in the case of Chetaev's interpretation is transformed into a condition (27), which automatically eliminates the problem of the hidden variables.

Now let us consider the distinctions in principle of Bohmian quantum mechanics supplemented with generalized Chetaev's theorem both from Bohmian mechanics itself and from the traditional, i.e., probabilistic quantum mechanics.

Analysis of Eq. (24) naturally raises the extremely deep and fundamental question of the physical nature of really existent (as is shown above) small perturbation forces, or «small dissipative forces with total dissipation» according to Chetaev [2]. We consider that we deal with the perturbation waves of de Broglie type whose action describes by Bohmian ψ-field. Such conclusion is caused, first of all, by the fact that de Broglie's «an embryonic theory of waves and particles union» [21] was developed just on the basis of identity of least action principle and the Fermat principle, that very exactly and clearly reflects a physical essence of Chetaev's theorem on the stable trajectories in dynamics (see Eq. (30)). Moreover, it is possible to conclude that within the framework of Bohmian mechanics supplemented with Chetaev's generalized theorem the reality or, more exactly, the observability of de Broglie wave, firstly, is provided by the reality of Bohmian ψ-field, which has sense of dissipation energies Q in Eq. (23), and, secondly, it is the direct consequence of the absence of hidden variables. In other words, in the Bohm–Chetaev quantum mechanics the reality of quantum potential and the absence of the hidden variables are the necessary and sufficient condition for the reality of de Broglie wave.

Thus, it is possible to conclude that, on the one hand, the Bohm–Chetaev quantum mechanics and probabilistic quantum mechanics are the theories without hidden variables and, on the other hand, in virtue of Eq. (33) they are nonlocal theories. And because of indicated reasons the physical (non)observability of de Broglie wave is the main and essential distinction between these two theories. If a wave is physically unobserved, the probabilistic interpretation of quantum mechanics is true. If the real existence of de Broglie wave will experimentally be proved, Chetaev's interpretation of quantum mechanics will be true in this case.

Note that the question of the true existence of de Broglie wave has a long history, and today it is not something exotic or metaphysical. In this sense, today it is clear that the new fundamental experiments should apply another principle, for example, the direct determination of real wave function by the ultrasensitive detection of electromagnetic perturbation interference (i.e., ψ trajectories) [18, 22], which accompanies the electron diffraction, using the low intensity source as in the Tonomura experiment [24].

This question becomes especially topical due to the new experimental data of Catillon et al. [23], who studying the channeling of electrons in the thin silicon crystal have observed the anomalous scattering. This can testify to the possible existence of electron stable motion of *zitterbewegung* type [14, 24]. Obviously, if this result will be reliably confirmed in further experiments, it will become the direct sign of the reality of the de Broglie wave hypothesis [21].

At the same time, we consider necessary to present another, indirect but not less effective method for clearing the true interpretation of quantum mechanics. First of all, such a method is predetermined by the fact that the Bohm–Chetaev trajectory dynamics due to the real existence of dissipative forces and, consequently, due to the real existence of the de Broglie waves makes it possible to describe the decays of the radioactive decay of heavy nuclei by alternative model, which relies not on the traditional quantum effect of the particle penetration or tunneling through the nuclear potential barrier, but on the classical «jump» over this barrier due to diffusion induced by a noise. Below we show how an idea of such a description naturally and clear is formalized in the language of Bohmian quantum mechanicse, supplemented with the Chetaev generalized theorem on stable trajectories in classical dynamics in the presence of dissipative forces.

3. DIFFUSION MECHANISM OF ALPHA DECAY, CLUSTER RADIOACTIVITY AND SPONTANEOUS FISSION

As is well known, the dissipation in general case is defined by friction and random Langevin force with zero average value on the corresponding space-time sampling. Since the quantum potential Q of these forces is dissipative in the framework of the Bohm–Chetaev mechanics, the one-dimensional balance equation for dissipative forces has the form:

$$\frac{\partial Q}{\partial x} = F_{\text{frict}}(x, \dot{x}) + F_L(x, t), \quad \text{where} \quad \langle F_L \rangle = 0. \tag{35}$$

Note, that this equation is a consequence of the fact that force of friction F_{frict}, as well as the Langevin force F_L are generated by the same source, i.e., by interaction between particle and environment, for example, by interaction with heat reservoir. Then Hamiltonian of the system is not explicitly time-dependent, differentiating Eq. (26) with respect to coordinate x and taking into account Eq. (35) we obtain the well-known Langevin equation [25, 26] for one-dimensional case

$$m\ddot{x} = -dU_x - F_{\text{frict}}(x, \dot{x}) + F_L(x, t). \tag{36}$$

It is possible to show [27–29] that the Langevin equation is in general case reduced to the Fokker–Planck equation

$$\frac{\partial W}{\partial t} = -\frac{p}{m}\frac{\partial W}{\partial x} + \frac{\partial U}{\partial x}\frac{\partial W}{\partial p} + \gamma\frac{\partial}{\partial p}\left[p + mD(T)\frac{\partial}{\partial p}\right]W, \tag{37}$$

where $W = W(x, p, t)$ is the probability density distribution in phase space $\{x, p\}$.

This equation first received within the framework of the well-known Kramers diffusion problem of stochastic transitions over the potential barrier makes it possible to obtain the exact expression for transition rate, which in general case looks like [30, 31]

$$w_K = \frac{\omega_{\min}}{2\pi}\left\{\left[1+\left(\frac{\beta}{2\omega_{\max}}\right)^2\right]^{1/2} - \frac{\beta}{2\omega_{\max}}\right\}\exp\left(-\frac{\Delta U}{\langle\varepsilon\rangle}\right), \tag{38}$$

$$\beta = \frac{\gamma}{m} \geqslant \frac{\omega_{\max}}{10},$$

where the average energy of the harmonic oscillator (oscillating particle) due to thermal and «zero-order» (at $T = 0$) variations is equal [32]:

$$D(T) = \frac{\hbar\omega_{\min}}{2}\coth\left(\frac{\hbar\omega_{\min}}{2k_BT}\right) = \begin{cases} k_BT & \text{for high } T, \\ \hbar\omega_{\min}/2 & \text{when } T \to 0, \end{cases} \tag{39}$$

where $\omega_{\min}$ and $\omega_{\max}$ are the angular frequencies of potential $U(x)$ in the potential minimum and in the vertex of barrier, respectively; ΔU is potential barrier height; k_B is Boltzmann constant.

At the same time, to give the universal description of induced and spontaneous decays (at $T = 0$), we use for the calculation of Eq. (38) the relation generalizing the similar relation of Fermi-gas model [27–29]:

$$D(T) = (E^*/a)^{1/2}, \tag{40}$$

where $a = A/(8{\pm}1)$ MeV^{-1} is the parameter of one-particle level density [28, 29].

Now we are ready for the description of our subject of interest, i.e., for investigation of the Kramers stochastic transitions over the potential barrier (38) in nonlinear nuclear dynamics for α decay, cluster radioactivity and spontaneous fission.

4. THE KRAMERS CHANNEL OF α DECAY, CLUSTER RADIOACTIVITY AND SPONTANEOUS FISSION

Let us consider the general case of a potential, in which some nuclear particle, for example, α particle, cluster or spontaneously fissionable nucleus is moving (Fig. 1). This is a positive potential of Coulomb repulsion V_{Coul} out of nucleus ($r > R$) and a negative potential of nuclear attraction V_{nucl}, for example, of rectangular form, within nucleus ($r < R_{\text{nucl}}$). Note that the Kramers velocity (38) depends only on barrier height and curvature of potential in its extremes, therefore the exact shape of potential is inessential. It in full measure concerns the exact form of nuclear attraction potential. Therefore, it is important to note that obtained below results can be qualitatively applied to a wide class of bistable systems.

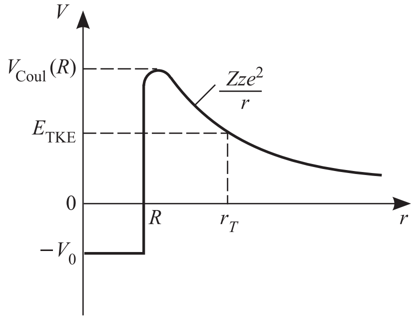

Fig. 1. The dependence of nuclear particle potential energy on distance to the nuclear center

It is well known that it is possible to determinate approximately the nuclear radii from the experimental data of α decay. The proximity means that in such experiments the distance between the centers of nucleus and α-particle, where nuclear forces cease to act, is measured. In other words, the distance R equal to the sum of nuclear radius $R_{A-4,\,Z-2}$, α-particle radius R_α and nuclear force action radius $R_{\rm nf}$ is determined. Thus, the Coulomb repulsion potential out of nucleus is acting at distances $r > R = R_{A-4,\,Z-2} + R_\alpha + R_{\rm nf}$. The same is true for the Coulomb interaction radius in case of cluster radioactivity and spontaneous fission. At the same time, the probability effective current w_K over the Coulomb potential barrier by virtue of geometry (Fig. 1) and Eq. (38) will be described by the following simplified expression:

$$w_K = \frac{b}{2\pi}\exp\left(-\frac{V_{\rm Coul} - E_{\rm TKE}}{\langle\varepsilon\rangle}\right), \tag{41}$$

where $E_{\rm TKE} = E_\alpha \approx Q_X$, $E_{\rm TKE} = E_{\rm cl}$ or $E_{\rm TKE} = E_{\rm SF}$ is the decay kinetic energy for α decay, cluster radioactivity (cl) or spontaneous fission (SF), respectively; Q_X is total decay energy.

We suppose below that the excitation heat energy E^* is some part of the decay kinetic energy $E_{\rm TKE}$:

$$E^* = \mu E_{\rm TKE}, \quad \mu \ll 1. \tag{42}$$

Physical sense and the explanation of the necessity of this condition we will give below.

Substituting Eq. (42) into Eq. (40) and after that substituting the obtained result into Eq. (38), we have the probability effective current w_K (41) over potential barrier:

$$w_K = \frac{\langle\omega\rangle_{\rm Kramers}}{2\pi}\exp\left[-\left(\frac{A}{8\mu}\right)^{1/2}\frac{V_{\rm Coul}-E_{\rm TKE}}{\sqrt{E_{\rm TKE}}}\right]. \qquad (43)$$

Then by virtue of equality $T_K = T_{1/2} = (w_K)^{-1}$ we find Kramers's effective time

$$\lg T_{1/2} = -\lg\frac{\langle\omega\rangle_{\rm Kramers}}{2\pi} + \lg e\left(\frac{A}{8\mu}\right)^{1/2}\frac{V_{\rm Coul}-E_{\rm TKE}}{\sqrt{E_{\rm TKE}}}, \qquad (44)$$

where $T_{1/2}$ is half-life; $\langle\omega\rangle_{\rm Kramers}$ is the effective frequency of α-particle appearance on the nuclear surface of radius R;

$$V_{\rm Coul} = \frac{(Z-Z_{\rm cl})z_{\rm cl}}{R_{\rm Coul}} = \frac{(Z-Z_{\rm cl})z_{\rm cl}}{R_{A-A_{\rm cl},Z-Z_{\rm cl}}+R_{\rm cl}+R_{\rm nf}} \quad [\text{MeV}], \qquad (45)$$

where A and Z are the mass number and the charge of parent nucleus; $Z_{\rm cl}$ is the charge of outgoing particle; $(Z$–$Z_{\rm cl})$ is the charge of the daughter nucleus; $R_{\rm Coul}$ is minimal Coulomb radius, fm.

Now it is easy to explain the necessity of assumption (42). It is stipulated by the fact that just under this condition the expression for Kramers effective time (44) can be represented by the following approximate formula:

$$\lg T_{1/2} \cong \frac{C}{\sqrt{E_{\rm TKE}}} - B, \qquad (46)$$

which is one of the variants of the Geiger–Nuttall law for α decay, which was experimentally established at the earliest stage of nuclear physics. In 1989–1990 such an experimental dependence was also discovered for cluster radioactivity [33, 34].

To explain the experimental law (46) predetermining the large variations of the half-life of heavy nuclei, in 1928 the theory of quantum-mechanical tunneling of α particles through the Coulomb barrier was proposed. This mechanism in the framework of the Gamow theory [35] reduces to the following expression for the time of α-particle passage through potential barrier (the half-life $T_{1/2}$ of heavy nuclei):

$$\lg T_{1/2} = -\lg\frac{\langle\omega\rangle_{\rm Gamov}}{2\pi} + \lg e\frac{4e^2(Z-2)}{\hbar}\sqrt{\frac{2\mu_\alpha}{E_\alpha}}\left[\arccos\sqrt{x}-\sqrt{x(1-x)}\right],$$

$$x = \frac{R_{\rm Coul}}{r_T}, \qquad (47)$$

where $\mu_\alpha = (A-4)4/A$ is reduced mass; $\hbar$ is reduced Planck constant, $\langle\omega\rangle_{\rm Gamov}$ is effective frequency of α-particle appearance on nuclear surface of radius $R_{\rm Coul}$; $r_T = e^2(Z-2)2/E_\alpha$.

If the Coulomb barrier height is much greater than energy E_α, what is typical situation for all natural α-radiators (i.e., $x = R_{\rm Coul}/r_T \ll 1$), and the term in square brackets in Eq. (47) is approximately equal to $(0.5\pi - 2x^{-1/2})$, Eq. (22) for half-life $T_{1/2}$ is simplified and looks like

$$\lg T_{1/2} = -\lg \frac{\langle\omega\rangle_{\rm Gamov}}{2\pi} + \lg e \frac{\pi e^2 2(Z-2)}{\hbar} \sqrt{\frac{2\mu_\alpha}{E_\alpha}} - \lg e \frac{2}{\hbar} \sqrt{\mu_\alpha e^2 2(Z-2) R_{\rm Coul}}. \tag{48}$$

It is easy to show that Eqs. (44) and (48) characterizing the «Kramers» and «Gamow» half-life evolution, respectively, are equally well described by relation of the Geiger–Nuttall type (46). However, in both cases we have not any information about the effective frequencies ($\langle\omega\rangle_{\rm Gamov}$ and $\langle\omega\rangle_{\rm Kramers}$) of particle-cluster appearance on the nuclear surface of radius $R_{\rm Coul}$ as well as about the value of this radius. In «Kramers» case the uncertainty of value μ is else accrues (see Eq. (44)). Note that due to the uncertainty of effective frequency of α-particle appearance on nuclear surface of radius $R_{\rm Coul}$, we obtain by Eq. (48) only the order of half-life and not its exact value. The second uncertainty can be explained by the fact that, according to the measurements of quadrupole moments, the majority of α-radioactive nuclei are not spherical as it was supposed in the Gamow α-decay theory, but they have ellipsoidal form with a ratio of longer to shorter half-axes, which running up to 1.5. Since the penetrability of potential barrier of nonspherical nucleus is various in the different its parts, and it is especially high near the «ends» of a nucleus, the estimations of nuclei radius obtained by alpha-decay data give the overestimated values, which actually characterize the longitudinal radius of a nucleus and not a certain effective radius.

Thus, since the dependence of Kramers effective time on decay energy and Coulomb barrier characteristics for cluster radioactivity and spontaneous fission has the same nature as for alpha decay, the problem of indicated above uncertainties remains unsolved. It specially concerns the uncertainty of effective frequencies ($\langle\omega\rangle_{\rm Gamov}$ and $\langle\omega\rangle_{\rm Kramers}$) of cluster-particle appearance on a nuclear surface of radius $R_{\rm Coul}$, as the corresponding theoretical estimations are extremely difficult to obtain, and even if they are obtained, they are strongly approximate [36, 37].

At the same time, the energy, half-lives and decay relative probabilities for the majority of radioactive heavy nuclei are measured with good accuracy within the framework of alpha-, cluster- and fission-fragment spectroscopy. These data are collected in the well-known ENSDF nuclear data library [38] and in combination with theoretical estimation (44) make it possible to solve one of the primary problems in the nuclear spectroscopy of radioactive decay, which is formulated in the following way. Using the experimental ENSDF data, for example, $T_{1/2}^{\rm exp}$, and theoretical estimations of $T_{1/2}^{\rm theory}$ (see Eq. (44)) it is necessary to solve the inverse nonlinear problem, which can be described by the system of nonlinear equations of the following type:

$$T_{1/2}^{\rm exp} = T_{1/2}^{\rm theory}(E_{\rm TKE}\,, A,\, Z,\, A_{\rm cl},\, Z_{\rm cl},\, R_{\rm Coul},\, \omega,\, \mu) \tag{49}$$

with respect to unknown parameters $R_{\rm Coul}$, ω and μ.

The solution of the system of nonlinear algebraic equations of such a type under certain conditions allows us to obtain a set of important data on intranuclear processes. In particular, it makes possible to obtain a functional dependence of effective frequency ($\langle\omega\rangle_{\rm Kramers}$) of cluster-particle occurrence on a nuclear surface of radius $R_{\rm Coul}$ and the dependence of radius $R_{\rm Coul}$ on quantum numbers (in our case, the mass number and the charge) characterizing the parent and daughter nuclei. On the other hand, the large variations of half-lives will lead to situation, when common determinant of the system will have many «zeros», and as a whole, the system will be quasi-degenerate. This means that we have the ill-conditioned system of nonlinear equations, whose solutions can be instable to the low changes of initial data. In other words, a problem of this type belongs to a class of ill-posed problems, and to solve it we used the Alexandrov dynamic autoregularization method (FORTRAN code REGN–Dubna [39]) which is constructive development of the Tikhonov regularization method [40].

Below we show the results of the application of the Alexandrov dynamic autoregularization method for solving the inverse nonlinear problem in the framework of the Kramers universal description (19) of stochastic channels for α decay, cluster radioactivity and spontaneous fission using the well-known experimental data.

5. COMPARING THEORY WITH EXPERIMENT

In the case of heavy nucleus radioactive decay with heavy cluster emission (such as ^{14}C, ^{24}Ne, ^{28}Mg, ^{34}Si) as well as in the case of α decay, the inequality $Q_{\rm cl} < B^{\rm Coul}_{A_{\rm cl}(A-A_{\rm cl})}$ is fulfilled, where $Q_{\rm cl}$ is cluster decay energy and $B^{\rm Coul}_{A_{\rm cl}(A-A_{\rm cl})}$ is Coulomb interaction energy between daughter nucleus (of mass number A–$A_{\rm cl}$ and charge Z–$Z_{\rm cl}$) and cluster (of mass number $A_{\rm cl}$ and charge $Z_{\rm cl}$) in contact point [34]. In other words, such a process is deep-subbarrier. Tacking into account experimental facts [34, 41], which show that the kinetic energy of decay products emitted from parent nucleus (A) remains almost the same

$$E_{\rm cl}^{\rm exp} \cong Q_{\rm cl}\frac{A - A_{\rm cl}}{A}, \tag{50}$$

we can assume, that daughter nuclei and clusters are almost unexcited. Both of these arguments allow us to consider that the noticeable reorganization of parent nucleus due to decay not always takes place. Hence it is possible to assume that radioactive decay with heavy cluster emission is α-decay analogue [34]. In this sense, the experimental result on the detection of the fine structure of ^{233}Ra cluster decay [42] is very important. This result was theoretically predicted in the framework of the model [45], which use analogues with α decay. Therefore, this experiment in fact gives decisive confirmation of analogy between mechanisms of α decay and decay with heavy clusters emission [34].

Before to analyze the results of computational experiment, one important and unexpected fact has to be noted, it was found that the solution of inverse problem in the framework of the Kramers universal description of α decay and cluster radioactivity (44) is absolutely sufficient to describe the spontaneous fission without any additional adjusting parameters. This suggests that for α decay as well as for cluster radioactivity and spontaneous fission the inequality of $E_{\rm TKE} \neq Q_\alpha$ type is true. At least, it fulfils in known experiments on α decay of heavy and superheavy nuclei [44]. It can be partially explained by the fact that for α decay, where the transition in one of the excited states of finite nucleus happens or vice versa, the transition from one of the excited states of parent nucleus takes place, the energy of α particles is always less or more, respectively, than normal. Running a few steps forward, we can assume that the allowance for this strict inequality ($E_{\rm TKE} \neq Q_\alpha$) will lead to sharp decrease of the numbers of parameterization parameters for functions $R_{\rm Kramers}$, $< \omega >_{\rm Kramers}$, μ on quantum numbers A, Z, $A_{\rm cl}$, $Z_{\rm cl}$, in Eq. (49).

Now let us consider the solving of inverse problem in the framework of the Kramers (44) universal description of α decay, cluster radioactivity and spontaneous fission, where alpha particle is considered as the smallest cluster. Using the Alexandrov dynamic regularization method [39] for solving the inverse problem (44) on the set of experimental data (Tables 1–3) from the ENSDF [38], we have obtained the phenomenological functional dependences of previously unknown parameters in the framework of the Kramers (44) universal description of α decay, cluster radioactivity and spontaneous fission.

Table 1. The ENSDF experimental data for α decay of even–even nuclei and the theoretical half-lives $T_{1/2}^{\rm theory}$ obtained by our model

No.	Nuclei	A	Z	$A_{\rm cl}$	$Z_{\rm cl}$	$E_{\rm TKE}$, MeV	$Q_{\rm X}$, MeV	$T_{1/2}^{\rm theory}$, y	$T_{1/2}^{\rm exp}$, y	$\Delta T_{1/2}^{\rm exp}$, y
1	**Pt**	168	78	4	2	6.832±0.010	6.999±0.001	7.1228E-11	6.3376E-11	3.17E-12
2	**Pt**	174	78	4	2	6.038±0.004	6.184±0.001	3.6504E-8	2.8171E-08	5.39E-11
3	**Pt**	176	78	4	2	5.753±0.003	5.887±0.000	2.5613E-7	1.9963E-07	1.58E-08
4	**Pt**	178	78	4	2	5.446±0.003	5.561±0.000	9.1244E-7	6.6862E-07	1.90E-08
5	**Hg**	174	80	4	2	7.067±0.006	7.233±0.001	6.9551E-11	6.0207E-11	1.27E-12
6	**Hg**	180	80	4	2	6.119±0.004	6.258±0.000	6.9882E-8	8.1755E-08	3.17E-10
7	**Hg**	182	80	4	2	5.867±0.005	5.999±0.001	6.1335E-7	3.4318E-07	1.90E-09
8	**Pb**	186	82	4	2	6.332±0.007	6.471±0.001	7.4096E-8	1.5305E-07	1.58E-09
9	**Pb**	188	82	4	2	5.983±0.004	6.111±0.000	1.0762E-6	7.9537E-07	3.17E-10
10	**Po**	188	84	4	2	7.910±0.013	8.082±0.001	7.0991E-12	9.5064E-12	9.51E-13

Table 1 (continuation)

No.	Nuclei	A	Z	$A_{\mathbf{cl}}$	$Z_{\mathbf{cl}}$	$E_{\mathbf{TKE}}$, **MeV**	$Q_{\mathbf{X}}$, **MeV**	$T_{1/2}^{\text{theory}}$, **y**	$T_{1/2}^{\text{exp}}$, **y**	$\Delta T_{1/2}^{\text{exp}}$, **y**
11	**Po**	190	84	4	2	7.537±0.006	7.699±0.001	6.8357E-11	7.7636E-11	1.58E-12
12	**Po**	192	84	4	2	7.167±0.007	7.322±0.001	1.0062E-9	1.0520E-09	4.44E-12
13	**Po**	194	84	4	2	6.843±0.003	6.990±0.000	1.0917E-8	1.2422E-08	1.27E-14
14	**Po**	196	84	4	2	6.520±0.023	6.657±0.000	1.1203E-7	1.8157E-07	7.29E-10
15	**Po**	198	84	4	2	6.182±0.022	6.309±0.002	1.5956E-6	3.3653E-06	5.70E-09
16	**Po**	200	84	4	2	5.862±0.018	5.981±0.002	2.9636E-5	2.1865E-05	1.90E-08
17	**Po**	202	84	4	2	5.588±0.017	5.686±0.002	4.9223E-5	8.4987E-05	9.51E-08
18	**Po**	206	84	4	2	5.224±0.015	5.327±0.001	2.9137E-2	2.4093E-02	2.74E-05
19	**Po**	214	84	4	2	7.687±0.007	7.849±0.001	4.1992E-12	5.2064E-12	6.34E-15
20	**Po**	216	84	4	2	6.778±0.005	6.906±0.001	2.0487E-9	4.5948E-09	6.34E-12
21	**Po**	218	84	4	2	6.002±0.009	6.115±0.001	3.9984E-6	5.8940E-06	3.80E-09
22	**Rn**	198	86	4	2	7.205±0.005	7.349±0.000	1.9900E-9	2.0597E-09	9.51E-12
23	**Rn**	200	86	4	2	6.902±0.003	7.043±0.000	2.8424E-8	3.0421E-08	9.51E-10
24	**Rn**	202	86	4	2	6.640±0.019	6.774±0.002	2.1262E-7	3.1688E-07	9.51E-10
25	**Rn**	208	86	4	2	6.140±0.017	6.271±0.002	7.0718E-5	4.6296E-05	2.66E-08
26	**Rn**	218	86	4	2	7.129±0.012	7.263±0.002	1.1089E-9	1.1091E-09	1.58E-11
27	**Rn**	220	86	4	2	6.288±0.010	6.405±0.001	2.2059E-6	1.7619E-06	3.17E-10
28	**Rn**	222	86	4	2	5.489±0.030	5.590±0.000	1.3687E-2	1.0468E-02	8.21E-08
29	**Ra**	204	88	4	2	7.486±0.006	7.636±0.001	2.1582E-9	1.8062E-09	3.49E-12
30	**Ra**	212	88	4	2	6.899±0.017	7.040±0.002	3.3566E-7	4.1195E-07	6.34E-09
31	**Ra**	214	88	4	2	7.137±0.003	7.283±0.000	4.7379E-8	7.7953E-08	9.51E-11
32	**Ra**	220	88	4	2	7.453±0.007	7.592±0.001	7.6376E-10	5.7039E-10	6.34E-13
33	**Ra**	222	88	4	2	6.559±0.005	6.679±0.000	1.3684E-6	1.1462E-06	3.17E-10
34	**Ra**	224	88	4	2	5.685±0.015	5.789±0.000	1.3964E-2	1.0021E-02	1.10E-05
35	**Ra**	226	88	4	2	4.784±0.025	4.871±0.000	1.9239E3	1.6000E+03	7.00E-01
36	**Th**	210	90	4	2	7.899±0.017	8.053±0.002	6.1465E-10	2.8519E-10	5.39E-11
37	**Th**	216	90	4	2	7.922±0.008	8.081±0.001	7.1259E-10	8.2389E-10	4.75E-11
38	**Th**	218	90	4	2	9.666±0.010	9.849±0.001	4.2241E-15	3.4540E-15	4.12E-17
39	**Th**	222	90	4	2	7.980±0.002	8.127±0.001	1.1147E-10	7.0981E-11	4.12E-14
40	**Th**	224	90	4	2	7.170±0.010	7.304±0.001	7.4860E-8	3.3272E-08	6.34E-11
41	**Th**	226	90	4	2	6.337±0.010	6.444±0.001	3.7302E-5	5.8122E-05	1.90E-08
42	**Th**	228	90	4	2	5.423±0.022	5.520±0.002	2.6517E0	1.9120E+00	2.00E-03
43	**Th**	230	90	4	2	4.687±0.015	4.770±0.002	6.9160E4	7.5386E+04	3.00E+02
44	**Th**	232	90	4	2	4.012±0.014	4.083±0.001	9.5977E9	1.4050E+10	6.00E+07
45	**U**	226	92	4	2	7.570±0.020	7.704±0.001	1.2414E-8	1.1091E-08	4.75E-09
46	**U**	228	92	4	2	6.680±0.010	6.796±0.001	1.9017E-5	1.7302E-05	3.80E-08
47	**U**	230	92	4	2	5.888±0.007	5.993±0.001	1.0152E-1	5.6947E-02	5.75E-04
48	**U**	232	92	4	2	5.320±0.014	5.414±0.001	9.5117E1	6.8890E+01	4.00E-02
49	**U**	234	92	4	2	4.775±0.014	4.858±0.001	1.7898E5	2.4549E+05	6.00E+01
50	**U**	236	92	4	2	4.494±0.003	4.573±0.001	2.8904E7	2.3421E+07	4.00E+03
51	**U**	238	92	4	2	4.198±0.003	4.270±0.001	4.4627E9	4.4680E+09	3.00E+05
52	**Pu**	236	94	4	2	5.768±0.008	5.867±0.001	2.6875E0	2.8580E+00	8.00E-04
53	**Pu**	238	94	4	2	5.499±0.020	5.593±0.002	8.2055E1	8.7713E+01	1.00E-02
54	**Pu**	240	94	4	2	5.168±0.015	5.256±0.001	7.9892E3	6.5610E+03	7.00E-01
55	**Pu**	242	94	4	2	4.902±0.009	4.984±0.001	3.6001E5	3.7360E+05	1.10E+02
56	**Pu**	244	94	4	2	4.589±0.001	4.666±0.001	8.0242E7	8.0012E+07	9.00E+04
57	**Cm**	238	96	4	2	6.520±0.050	6.608±0.004	2.4091E-4	2.7379E-04	1.14E-06
58	**Cm**	240	96	4	2	6.291±0.005	6.398±0.001	7.1683E-2	7.3922E-02	2.74E-04
59	**Cm**	242	96	4	2	6.113±0.008	6.216±0.000	4.9037E-1	4.4617E-01	1.64E-05
60	**Cm**	244	96	4	2	5.805±0.005	5.902±0.000	1.8000E1	1.8100E+01	1.00E-02

Table 1 (continuation)

No.	Nuclei	A	Z	$A_{\mathbf{cl}}$	$Z_{\mathbf{cl}}$	$E_{\mathbf{TKE}}$, MeV	$Q_{\mathbf{X}}$, MeV	$T_{1/2}^{\mathrm{theory}}$, y	$T_{1/2}^{\mathrm{exp}}$, y	$\Delta T_{1/2}^{\mathrm{exp}}$, y
61	**Cm**	246	96	4	2	5.387±0.010	5.475±0.001	3.0607E3	4.7596E+03	4.00E+00
62	**Cm**	248	96	4	2	5.078±0.025	5.162±0.003	4.0888E5	3.4800E+05	6.00E+02
63	**Cf**	240	98	4	2	7.590±0.010	7.719±0.001	2.9232E-6	1.8252E-06	2.85E-07
64	**Cf**	244	98	4	2	7.209±0.004	7.329±0.002	5.9464E-5	3.6885E-05	1.14E-07
65	**Cf**	246	98	4	2	6.750±0.010	6.862±0.001	4.6022E-3	4.0726E-03	5.70E-06
66	**Cf**	248	98	4	2	6.258±0.005	6.361±0.001	7.9071E-1	9.1293E-01	7.67E-04
67	**Cf**	250	98	4	2	6.030±0.020	6.128±0.002	1.0019E1	1.3081E+01	9.00E-03
68	**Cf**	252	98	4	2	6.118±0.004	6.217±0.000	3.7474E0	2.6450E+00	8.00E-04
69	**Fm**	246	100	4	2	8.237±0.015	8.361±0.002	2.8942E-8	3.4857E-08	6.34E-10
70	**Fm**	248	100	4	2	7.870±0.020	8.000±0.001	1.4652E-6	1.1408E-06	9.51E-09
71	**Fm**	250	100	4	2	7.436±0.012	7.558±0.001	5.3393E-5	6.2742E-05	5.70E-07
72	**Fm**	252	100	4	2	7.039±0.002	7.155±0.002	2.3293E-3	2.8964E-03	5.70E-06
73	**Fm**	254	100	4	2	7.192±0.002	7.308±0.002	3.8158E-4	3.6961E-04	2.28E-07
74	**№**	252	102	4	2	8.415±0.001	8.549±0.001	9.3074E-8	7.1932E-08	4.44E-10
75	**№**	256	102	4	2	8.448±0.001	8.581±0.001	4.8203E-8	9.2212E-08	1.58E-10
76	**Sg**	260	106	4	2	9.770±0.003	9.912±0.003	1.2100E-10	1.1408E-10	2.85E-12
77*	–	294	118	4	2	11.650±0.060	11.838±0.060	2.8198E-11	2.8202E-11	+3.39E-11 -8.17E-12
78*	–	292	116	4	2	10.660±0.070	10.809±0.070	4.9622E-10	5.7039E-10	+5.07E-10 -2.14E-10
79*	–	290	116	4	2	10.840±0.080	10.990±0.080	1.8945E-10	2.2499E-10	+1.01E-10 -1.20E-10
80*	–	288	114	4	2	9.940±0.060	10.091±0.060	2.6095E-8	2.5350E-08	+8.56E-9 -1.50E-8
81*	–	286	114	4	2	10.190±0.060	10.339±0.060	3.8586E-9	4.1195E-09	+1.27E-09 -2.06E-9

*Data are given by Yu. Ts. Oganessian.

Table 2. The ENSDF experimental data for cluster radioactivity of even–even nuclei and the theoretical half-lives $T_{1/2}^{\mathbf{theory}}$ obtained by our model

No.	Nuclei	A	Z	$A_{\mathbf{cl}}$	$Z_{\mathbf{cl}}$	$E_{\mathbf{TKE}}$, MeV	$Q_{\mathbf{X}}$, MeV	$T_{1/2}^{\mathrm{theory}}$, y	$T_{1/2}^{\mathrm{exp}}$, y	$\Delta T_{1/2}^{\mathrm{exp}}$, y
82	**Ra**	226	88	14	6	26.46±1.00	28.79±1.00	6.315E13	6.32E13	3.7E13
83	**Ra**	224	88	14	6	28.63±1.00	30.53±1.00	1.833E8	2.52E8	8.01E7
84	**Ra**	222	88	14	6	30.97±1.00	33.05±1.00	8.369E3	3.17E3	469.00
85	**Th**	230	90	24	10	51.98±1.00	57.68±1.00	1.208E17	1.26E17	2.21E16
86	**U**	232	92	24	10	55.86±1.00	62.31±1.00	1.370E13	1.00E13	7.17E11
87	**U**	234	92	28	12	65.26±1.00	74.13±1.00	3.097E18	1.59E18	7.48E16
88	**Pu**	236	94	28	12	70.22±1.00	79.60±1.00	1.139E14	1.59E14	1.58E14
89	**Cm**	242	96	34	14	82.88±1.00	96.43±1.00	9.977E13	1.00E14	5.86E13
90	**Th**	228	90	20	8	40.44±1.00*	44.73±1.00	1.792E13	1.68E13	–
91	**U**	234	92	24	10	51.80±1.00*	58.84±1.00	1.869E18	2.52E18	–

*Predicted values.

Table 3. The ENSDF experimental data for spontaneous fission of even–even nuclei and the theoretical half-lives $T_{1/2}^{\mathbf{theory}}$ obtained by our model

No.	Nuclei	A	Z	$A_{\mathbf{cl}}$	$Z_{\mathbf{cl}}$	$E_{\mathbf{TKE}}$, MeV	$Q_{\mathbf{X}}$, MeV	$T_{1/2}^{\mathrm{theory}}$, y	$T_{1/2}^{\mathrm{exp}}$, y	$\Delta T_{1/2}^{\mathrm{exp}}$, y
92	**U**	236	92	94	37	165.0±1.0	181.49±1.00	2.5014E+16	2.00E+16	1.00E+15
93	**U**	236	92	93	37	164.0±1.0	185.31±1.00	1.6000E+16	1.60E+16	2.00E+15
94	**Pu**	240	94	96	38	172.0±1.0	194.88±1.00	1.3400E+11	1.34E+11	2.00E+09
95	**Cm**	244	96	97	38	185.5±1.0	200.76±1.00	8.5405E+06	1.34E+7	2.00E+05
96	**Cm**	250	96	100	38	182.3±1.0	198.18±1.00	1.9154E+04	2.00E+04	5.00E+02
97	**Cf**	254	98	102	39	186.1±1.0	206.72±1.00	1.8165E-01	1.78E-01	5.48E-04
98	**Cf**	252	98	101	39	186.5±1.0	207.73±1.00	8.5490E+01	8.55E+01	1.00E+00
99	**Cf**	246	98	98	39	195.6±1.0	209.03±1.00	2.3353E+03	2.00E+03	2.00E+02
100	**Fm**	258	100	103	40	200.3±1.0	220.90±1.00	3.8000E-11	3.80E-11	6.34E-13

In this case, the system of nonlinear equations (49) looks like

$$\lg T_{1/2}^{\mathrm{exp}} = \lg T_{1/2}^{\mathrm{Kramers}}(E_{\mathrm{cl}},\, A,\, Z,\, Z_{\mathrm{cl}},\, R_{\mathrm{Kramers}}(A,\, Z,\, A_{\mathrm{cl}},\, Z_{\mathrm{cl}}), \\ \omega_{\mathrm{Kramers}}(R_{\mathrm{Kramers}}), \mu(Z,\, A_{\mathrm{cl}},\, Z_{\mathrm{cl}})), \quad (51)$$

for which we have applied the parameterization of functions R_{Kramers}, $\omega_{\mathrm{Kramers}}$, μ with respect to quantum numbers A, Z, A_{cl}, Z_{cl} (which determine the mass numbers and charges of parent nucleus and cluster) and energies E_{TKE}, Q_{cl} (which determine the kinetic and total energy of decay) in the following form:

$$\lg \frac{\langle\omega\rangle_{\mathrm{Kramers}}}{2\pi} = a_{20} + \frac{1}{R_{\mathrm{Kramers}}}, \quad (52)$$

$$\mu = \exp\left[a_1 + a_2\frac{(A-2Z)^2}{A^2} + a_3\frac{A-A_{\mathrm{cl}}}{A}\left(1-\frac{E_{\mathrm{TKE}}}{Q_{\mathrm{cl}}}\right) + \right. \\ \left. + \left(a_4\frac{A-A_{\mathrm{cl}}}{A} + a_5\frac{1}{Z_{\mathrm{cl}}}\right)\left(1-\frac{1}{Z_{\mathrm{cl}}}\right)\right], \quad (53)$$

$$R_{\mathrm{Kramers}} = \left[B_1(A-Z_{\mathrm{cl}})^{1/3} + B_1 A_{\mathrm{cl}}^{1/3} - 1\right] B_2 \quad [\mathrm{fm}], \quad (54)$$

$$B_1 = \exp\left[a_6\left(\frac{A-2Z}{A}\right)^2 + a_7\frac{Z}{A} + \left(a_8 + a_9\frac{A-A_{\mathrm{cl}}}{A} + a_{10}\frac{1}{A_{\mathrm{cl}}}\right)\times \right. \\ \left. \times\left(1-\frac{E_{\mathrm{TKE}}}{Q_{\mathrm{cl}}}\right) + \left(a_{11} + a_{12}\frac{A-A_{\mathrm{cl}}}{A} + a_{13}\frac{1}{Z_{\mathrm{cl}}}\right)\left(1-\frac{1}{Z_{\mathrm{cl}}}\right)\right], \quad (55)$$

$$B_2 = \exp\left[a_{14}\frac{1}{Z} + a_{15}\left(\frac{A-2Z}{A}\right)^2 + a_{16}\frac{Z}{A} + a_{17}\frac{A-A_{\mathrm{cl}}}{A}\left(1-\frac{E_{\mathrm{TKE}}}{Q_{\mathrm{cl}}}\right) + \right. \\ \left. + \left(a_{18} + a_{19}\frac{Z-Z_{\mathrm{cl}}}{Z}\right)\left(1-\frac{1}{Z_{\mathrm{cl}}}\right)\right]. \quad (56)$$

Note, one of ways for finding the hidden dependences of parameters on characteristic variables (in our case, on the quantum numbers A, Z, $A_{\rm cl}$, $Z_{\rm cl}$ and energies $E_{\rm TKE}$, $Q_{\rm cl}$), which determine the state of investigated system, is briefly described in [45].

The solutions of inverse nonlinear problem of the Kramers type (51) for the ENSDF experimental data set (Table 1), which are presented as the values of parameters a_i and their relative errors $\Delta a_i/a_i$, are collected in Table 4. Data in Tables 1–3 and in Fig. 2 show good coincidence ($\chi^2/\text{NDF} = 82.5/72$) of the experimental and theoretical half-lives for α decay, cluster (^{14}C, ^{24}Ne, ^{28}Mg, ^{34}Si) radioactivity and spontaneous fission depending on decay total kinetic energy $E_{\rm TKE}$.

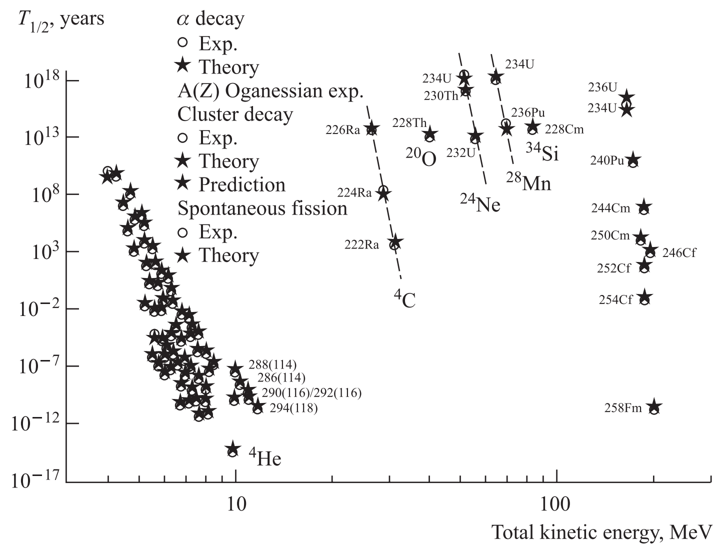

Fig. 2. The theoretical and experimental values of half-life for even–even nuclei as function of the fission total kinetic energy $E_{\rm TKE}$ for α decay, cluster radioactivity and spontaneous fission

To verificate the obtained solution of inverse problem in the framework of the Kramers universal description (44) of α decay, cluster radioactivity and spontaneous fission, whose parameters are given in Table 4, we have used the experimental data of α decay for superheavy nucleus, which were kindly given by Yu. Ts. Oganessian (JINR, Dubna, Russia) [44]. Figure 3 shows good accordance between the experimental and theoretical (Kramers's) half-lives for alpha decay, depending on the decay energy E_α. We consider that some lack of the coincidence of the theoretical and experimental data in Fig. 3 (see Table 4) is caused by low number of measurements (due to understandable reasons), which did not exceed 25 measurements for each decay type [44].

Table 4. The values of parameters a_i and their relative errors $\Delta a_i/a_i$

i	$\boldsymbol{a}_i$	$\Delta\boldsymbol{a}_i$, %
1	–0.5786501537235E+01	2.90
2	–0.2096263480335E+02	1.90
3	–0.3814591516659E+02	2.70
4	0.6900587198207E+01	2.70
5	0.7660345598675E+01	5.00
6	0.1908313257301E+02	1.40
7	0.1826397833295E+02	1.30
8	0.5919551276390E+01	1.60
9	–0.1028171722816E+02	1.50
10	–0.4411225968202E+02	3.50
11	–0.1131089043128E+02	1.00
12	0.1314247777365E+01	1.20
13	0.2865440882262E+01	2.60
14	0.4240393211738E+01	18.00
15	–0.1229614115313E+02	2.20
16	–0.1772081454140E+02	1.20
17	0.1689691120764E+02	0.75
18	0.1666949024191E+02	0.82
19	–0.8643631861055E+01	0.78
20	0.2749864919484E+02	1.20

Based on the solution of inverse problem in the framework of the Kramers (19) universal description of α decay, cluster radioactivity and spontaneous fission, whose parameters are shown in Table 4, we give predictable value of energy E_{TKE} for ^{234}U and ^{228}Th nuclei, which inclined to ^{20}O and ^{24}Ne cluster-radioactivity, respectively (see Table 2 and Fig. 2). These data can be of interest for future experiments.

Finally, it is possible to conclude that received results are an indirect confirmation of the applicability of the Langevin fluctuation-dissipative dynamics and, in particular, of the Kramers diffusion mechanism [30, 31] for the effective description of collective motions in nuclei, which generate the stochastic channel of α decay, cluster radioactivity and spontaneous fission. Although a situation is, at first sight, complicated by the fact that dissipation in nucleus (or more exactly, the nuclear friction) is experimentally unobserved magnitude [46], but, it turned out, there is the obvious possibility of the unambiguous proof of its real existence. For example, it is well known, that the introduction of the external periodic signal into the Langevin equation (36) must result in the observation of stochastic resonance [47]. In other words, the experiments on the search of nuclear stochastic resonance in α decay, which cannot in principle take place within the framework of probabilistic interpretation of quantum mechanics but must be

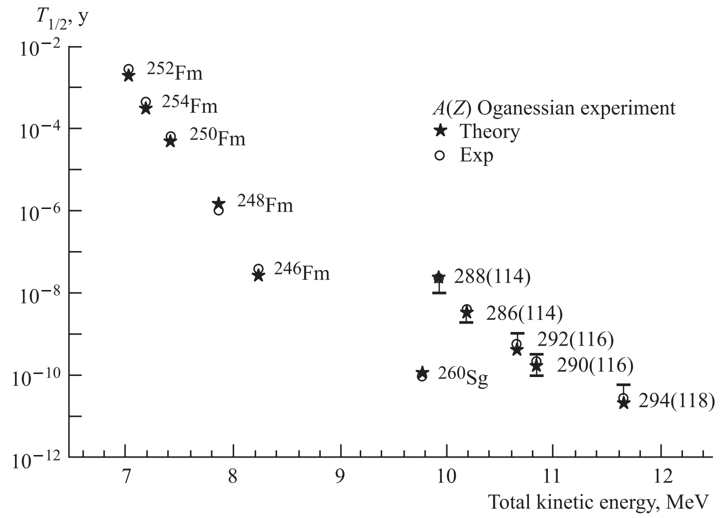

Fig. 3. The theoretical and experimental values of the half-life of even–even nuclei as function of the fission total kinetic energy $E_{\rm TKE}$ for α decay of superheavy nuclei with $Z = 114, 116, 118$

observed within the framework of Bohmian mechanics supplemented with the Chetaev generalized theorem [8, 48], can become the determinative factor for the revelation of fundamental role of dissipation not only in the nuclear dynamics but generally in quantum physics.

At last, note that, when the first research UV-lasers of frequency about 10^{18}–$10^{20}\,{\rm s}^{-1}$ [49] will appear in the near future, the problem of the stochastic mode excitation in atomic nucleus under action of periodic external field will become actual not only in respect to the direct study of dissipation and, consequently, of self-organization and quantum chaos in it, but fundamental in view of possible break-through of «probabilistic smokescreen» [14] to the holistic understanding of causal interpretation of quantum physics.

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