E3-2011-36

A. M. Sukhovoj, V. A. Khitrov

MODIFIED MODEL OF NEUTRON RESONANCE WIDTHS DISTRIBUTION. RESULTS OF TOTAL GAMMA-WIDTHS APPROXIMATION

Суховой А. М., Хитров В. А. E3-2011-36

Модифицированная модель распределений ширин нейтронных резонансов.
Результаты аппроксимации полных гамма-ширин

Для анализа экспериментальных данных по полным радиационным ширинам нейтронных резонансов использованы функциональные зависимости вероятности наблюдения заданного значения Γ_n^0 и алгоритмы определения наиболее вероятных значений параметров модифицированной модели их распределения. Как и для нейтронных ширин, для прецизионного описания распределений Γ_γ необходима суперпозиция трех или более распределений вероятности квадратов случайных нормально распределенных величин с ненулевым средним и неединичной дисперсией. Этот результат подтверждает предварительный вывод, полученный ранее при анализе Γ_n^0, о том, что практически во всех 56 тестируемых наборах полных гамма-ширин есть несколько групп, заметно отличающихся друг от друга по структуре своей волновой функции. Дополнительно установлено, что радиационные ширины намного чувствительнее, чем нейтронные, к структуре волновых функций резонансов. Также проведен анализ параметров распределений 157 наборов приведенных нейтронных ширин резонансов в области масс ядер $35 \leqslant A \leqslant 249$. Показано, что экспериментальные значения нейтронных ширин с высокой вероятностью соответствуют суперпозиции нескольких независимых распределений с ненулевым средним и неединичной дисперсией.

Работа выполнена в Лаборатории нейтронной физики им. И. М. Франка ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна, 2011

Sukhovoj A. M., Khitrov V. A. E3-2011-36

Modified Model of Neutron Resonance Widths Distribution.
Results of Total Gamma-Widths Approximation

Functional dependences of probability to observe given Γ_n^0 value and algorithms for determination of the most probable magnitudes of the modified model of resonance parameter distributions were used for analysis of the experimental data on the total radiative widths of neutron resonances. As in the case of neutron widths, precise description of the Γ_γ spectra requires a superposition of three and more probability distributions for squares of the random normally distributed values with different nonzero average and nonunit dispersion. This result confirms the preliminary conclusion obtained earlier at analysis of Γ_n^0 that practically in all 56 tested sets of total gamma widths there are several groups noticeably differing from each other by the structure of their wave functions. In addition, it was determined that radiative widths are much more sensitive than the neutron ones to resonance wave functions structure. Analysis of early obtained neutron reduced widths distribution parameters for 157 resonance sets in the mass region of nuclei $35 \leqslant A \leqslant 249$ was also performed. It was shown that the experimental values of widths can correspond with high probability to superposition of several expected independent distributions with their nonzero mean values and nonunit dispersion.

The investigation has been performed at the Frank Laboratory of Neutron Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna, 2011

1. INTRODUCTION

In 1936, N. Bohr suggested the hypothesis [1] on extremely complicated structure of high-lying levels of compound nucleus. After this, the properties of neutron resonances are described in the framework of statistical approach. But the experience of the science shows that the real picture of the phenomenon under study is usually much more complicated than any hypotheses and notions of it. Most probably, the hypothesis [1] is not an exception as well.

Estimation of its precision can be performed only on the basis of the modern experimental data and theoretical developments of existing nuclear models. So, the realized at FLNP JINR idea of obtaining the direct and reliable experimental information on such nuclear parameters as the level density and radiative strength functions [2], and interpretation of the obtained data [3, 4] show that structure of a nucleus below the neutron binding energy B_n undergoes cyclic change with a step of about $2\Delta_n$. By this, the correlation function of the Cooper pair of nucleons in heated nucleus below B_n insignificantly differs from the analogous value Δ_0 for cold nucleus (although, most probably, decreases at increase of excitation energy). A degree of fragmentation of nuclear structures like n-quasi-particles $\otimes$ m-phonons for these states in region B_n according to theoretical analysis by L. A. Malov and V. G. Soloviev [5], cannot be the same, i.e., one can expect that in the wave functions of neutron resonances at change of their energy can appear available for observation changes.

It is absolutely necessary for their revealing to execute two conditions:

a) to use the algorithm of analysis for any experimental data with the lowest possible quantity of model notions and

b) to perform quantitative comparison of a few variants of approximation of the tested resonance parameter distributions.

The more essential is the second condition — it just determines the vector of required changes in the existing notions of nuclear properties in the studied region of excitation. Unfortunately, the variants of analysis of neutron resonance parameters performed by now did not take into account these circumstances. But, both conditions were to a full degree realized in [6].

2. CONDITIONS OF Γ_γ ANALYSIS

The intensity of the primary gamma transitions of following decay of neutron resonance depends on the same components of their wave functions as neutron ones. Therefore, in the distributions of partial radiative widths the peculiarities must appear which are analogous to those appearing in distributions of reduced neutron widths. First of all, in [7] there is observed the discrepancy with the Porter–Thomas distribution [8] of partial radiative widths in any form. The indirect answer on this question can be obtained from the analysis of the distributions of cumulative sums of the relative Γ_γ values in maximally wide interval of nuclear mass.

For analysis of form of distribution of the random Γ_γ values were used the same algorithm and programs which were prepared for analysis of the reduced neutron widths distribution. The independent variable of analysis $X_\gamma = \Gamma_\gamma/\langle\Gamma_\gamma\rangle$ corresponds to the ratio of total radiative width of given resonance to the mean experimental value of the tested set. Naturally, all events with $X \equiv 1$ (used by experimentalists for determination of Γ_n^0 for a part of resonances) were excluded from analysis. This selection is really nonessential because corresponding portion of cumulative sum can be approximated with good precision by value $\sigma < 0.01$. The analysis was performed by analogy with the analysis of reduced neutron widths for two hypotheses. The first — the distribution of the total radiative widths of resonances corresponds to distribution of squares of the normally distributed random values with one and the same dispersion and mean value ($k = 1$). The second one used the same distributions with several set ($k = 4$) of different parameters. Practical basis for this variant is obvious asymmetry of Γ_γ cumulative sums of distributions of the experimental X_γ values for many nuclei. Unfortunately, the use of relative values of radiative widths inevitably shifts obtained values of analysis parameters b_k and σ_k as for $k = 1$, and for $k > 1$.

3. RESULTS OF ANALYSIS AND THEIR INTERPRETATION

In Fig. 1 are presented model distributions of squares of the random $X = ((\xi + b)/\sigma)^2$ values for parameters $b = 0.5, 1, 2$; $\sigma = 0.01, 0.03, 0.10$ for ξ — standard normally distributed random variable. Cumulative sums were normalized, naturally, to the average $\langle X_\gamma \rangle$. In Fig. 2 is presented approximation of the experimental distributions of Γ_γ for ^{151}Eu and ^{235}U. These target nuclei essentially differ only by parity of proton number. But difference of the mean spacing D_0 between resonances, neutron binding energy B_n and spin of target I are practically invariable.

In the Table are presented quantitative results of the relative Γ_γ values of cumulative sums distribution approximations for some nuclei differing by their

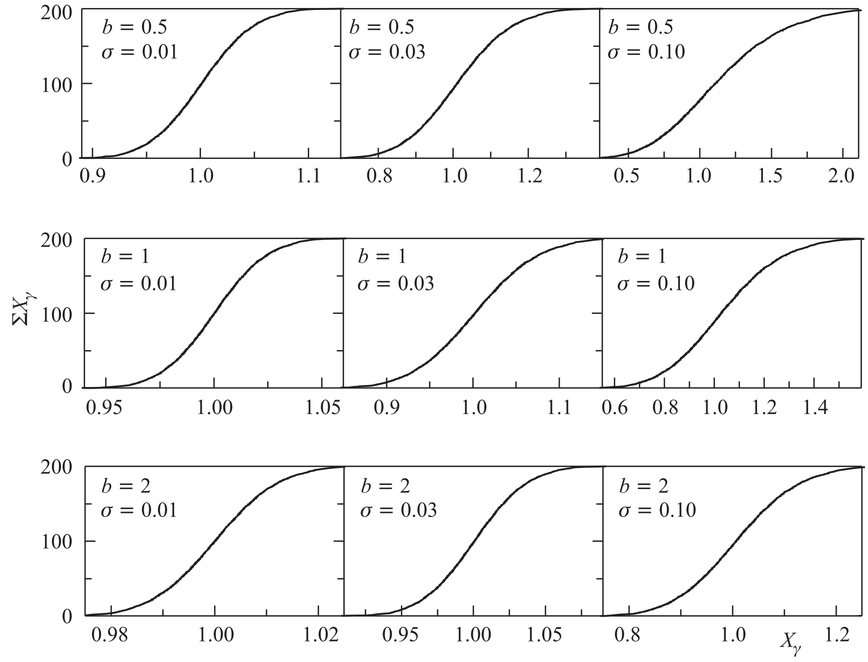

Fig. 1. The expected distribution of cumulative sums of relative values Γ_γ of the total radiative widths of resonances. The dispersion σ and mean value b are also given

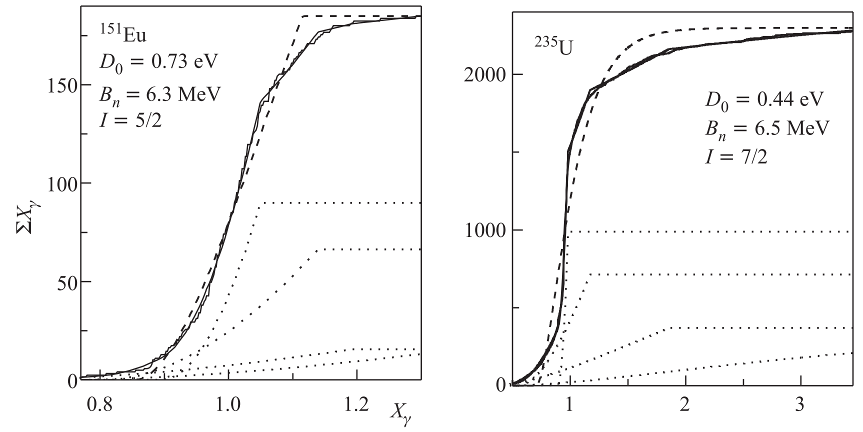

Fig. 2. The result of Γ_γ distribution approximation for ^{151}Eu and ^{235}U. Histogram — experiment, dashed line — $k = 1$, solid — $k = 4$, dotted lines — variant of decomposition of the last into four «partial» functions

The main approximated parameters of nuclei with the largest values of number N_r of experimentally determined values Γ_γ. $R = \chi^2(k = 4)/\chi^2(k = 1)$ is the ratio of the best fit parameters of both variants of analysis; S_k — the portion of two functions with maximal contribution in cumulative sum; σ and b — dispersion and their most probable mean value

Nucleus	N_r	R	$\chi^2(k=4)/N_\gamma$	S_1	σ_1	b_1	S_2	σ_2	b_2
^{60}Ni, $l = 1$	173	0.27	0.015	0.45	0.08	0.85	0.34	0.07	0.49
^{151}Eu	185	0.073	0.044	0.49	0.008	1.	0.36	0.02	0.95
^{151}Sm	525	0.068	0.012	0.44	0.06	0.68	0.38	0.06	0.87
^{235}U	2297	0.033	0.042	0.43	0.006	0.97	0.31	0.05	0.76

parameters with maximal number of their existing experimental values. Most probably, by any non-principle difference for part of the Table data, the part of cumulative sum of two most essential functions of superposition conserves with high precision. There is the sufficient argument in favour of conclusion that the experimental data on neutron resonance parameters correspond to several sets of noticeably differing by their wave functions structure.

In Figs. 3–8 are presented the results of approximation of the radiative width distributions for 54 sets of the data, although analysis was performed for some

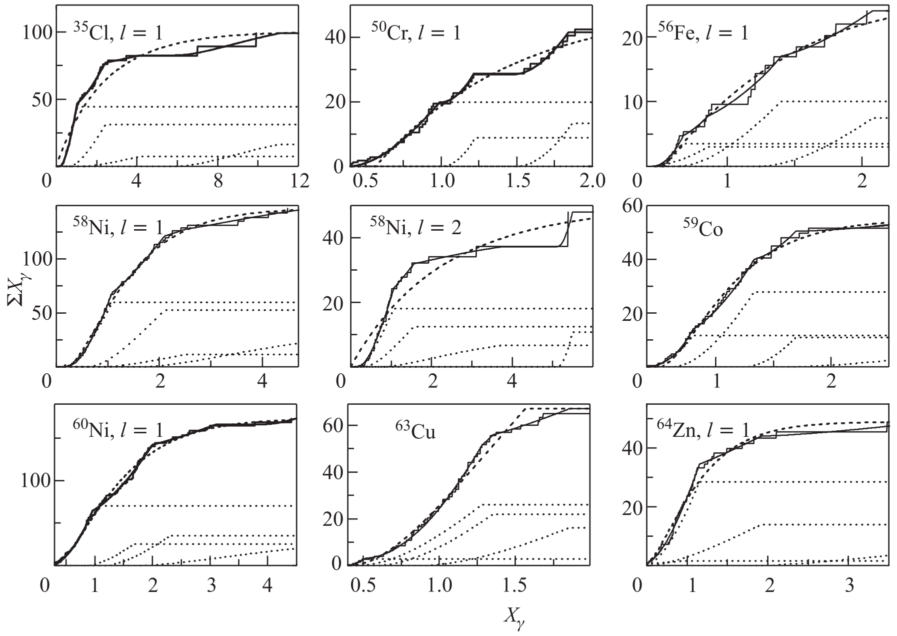

Fig. 3. The same as in Fig. 2, for ^{35}Cl, ^{50}Cr, ^{56}Fe, ^{58,60}Ni, ^{59}Co, ^{63}Cu and ^{64}Zn

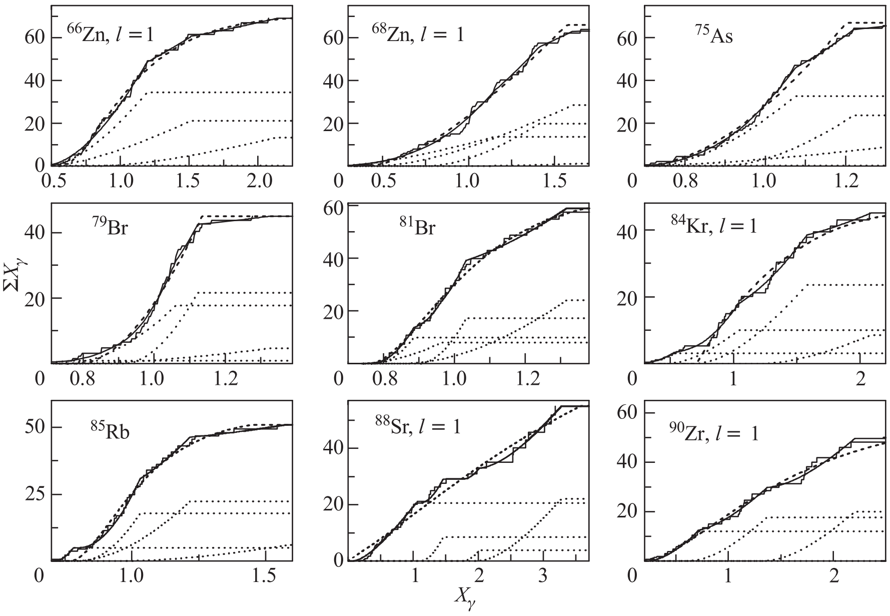

Fig. 4. The same as in Fig. 2, for ^{66,68}Zn, ^{75}As, ^{79,81}Br, ^{84}Kr, ^{85}Rb, ^{88}Sr and ^{90}Zr

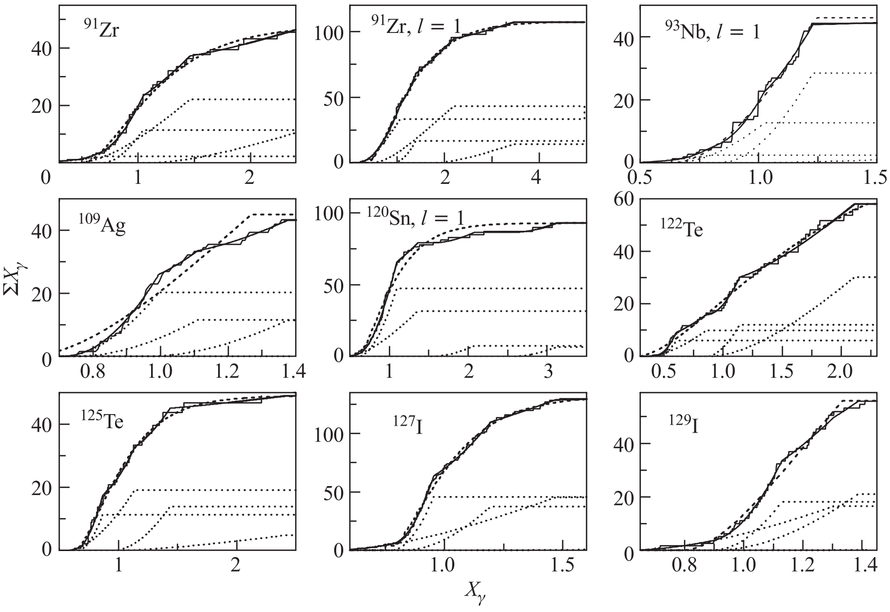

Fig. 5. The same as in Fig. 2, for ^{91}Zr, ^{93}Nb, ^{109}Ag, ^{120}Sn, ^{122,125}Te and ^{127,129}I

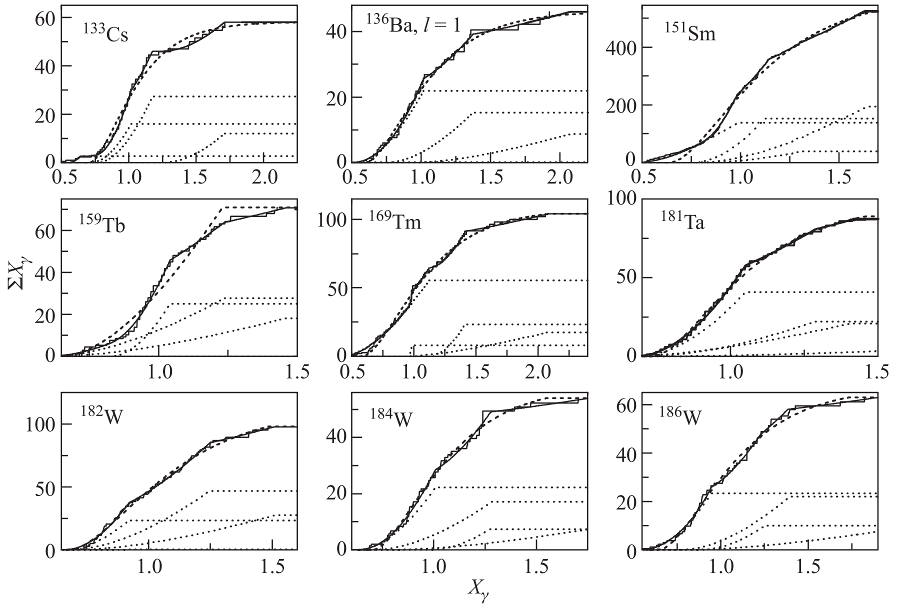

Fig. 6. The same as in Fig. 2, for ^{133}Cs, ^{136}Ba, ^{151}Sm, ^{159}Tb, ^{169}Tm, ^{181}Ta and ^{182,184,186}W

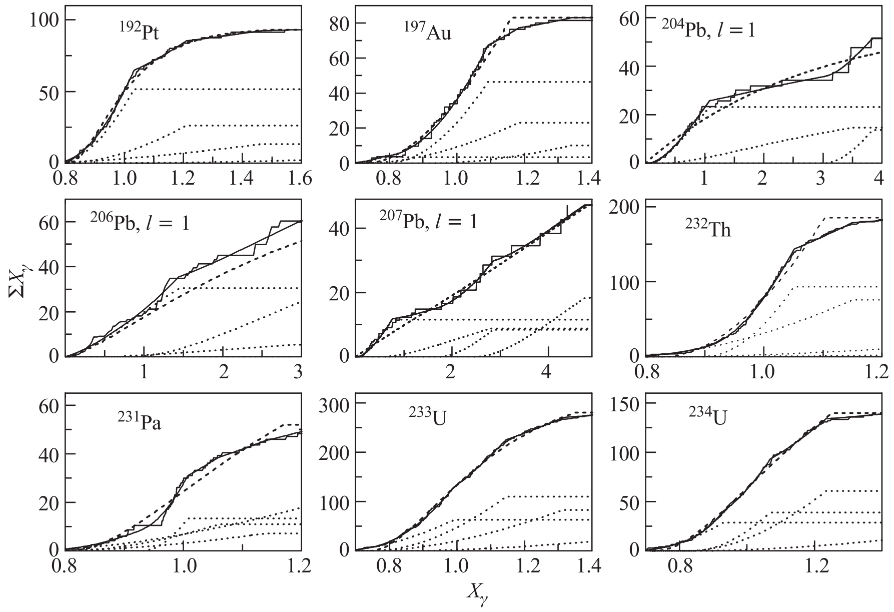

Fig. 7. The same as in Fig. 2, for ^{192}Pt, ^{197}Au, ^{204,206,207}Pb, ^{232}Th, ^{231}Pa and ^{233,234}U

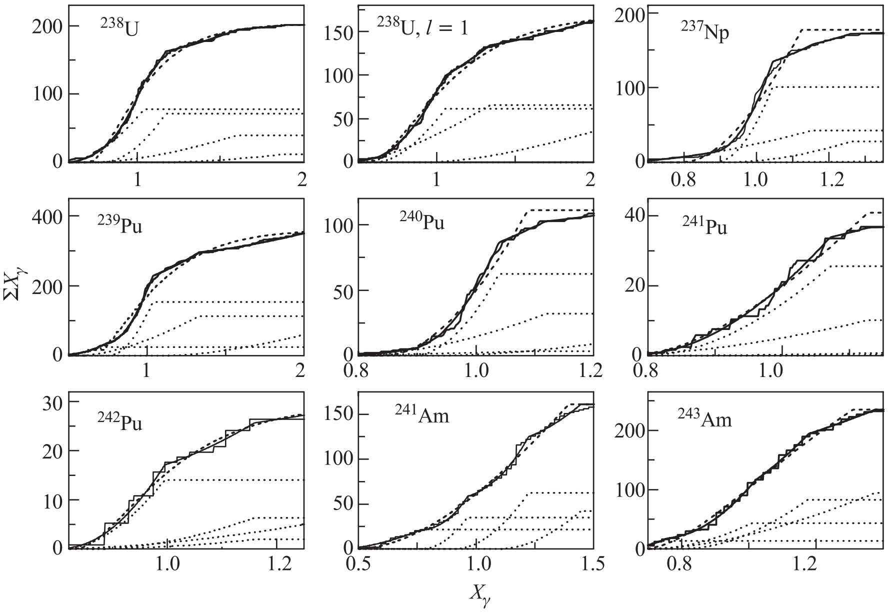

Fig. 8. The same as in Fig. 2, for ^{238}U, ^{237}Np, ^{239,240,241,242}Pu and ^{241,243}Am

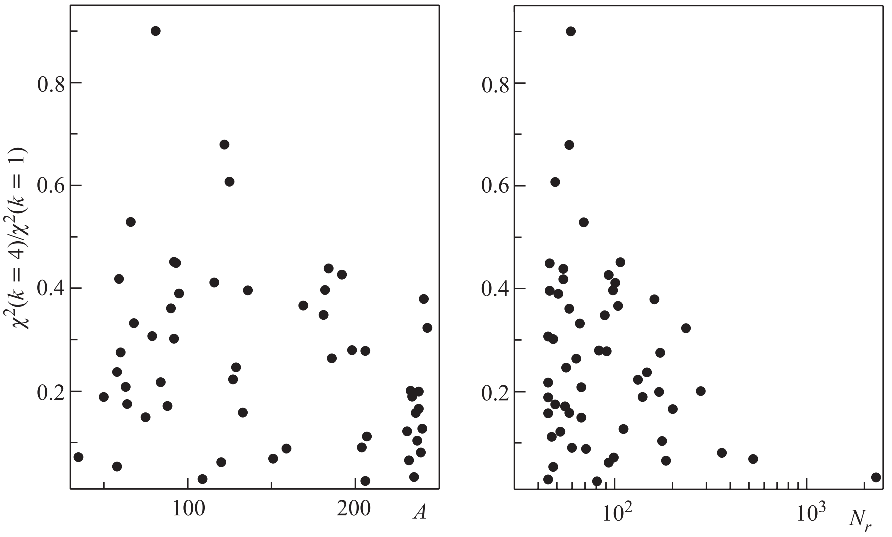

Fig. 9. The ratios of criteria of quality of best fit for two variants of analysis as a function of mass A of a nucleus or of number N_r of resonances in the set. The mean value over 56 sets equals 0.26(17)

larger number of the sets. Practical selection was done by condition that the sets of s-resonances in the figures in most cases correspond to not less than 45–50 Γ_γ values, i.e., number of points of the approximating curve for superposition from $k = 4$ fitted «partial» functions exceeds maximal number of approximation parameters by a factor ≈ 4 and more. The ratios $\chi^2(k = 4)/\chi^2(k = 1)$ for all 56 data sets are shown in Fig. 9.

4. ESTIMATION OF RELIABILITY OF MAIN RESULT OF ANALYSIS

Relation between functional measured in experiment and determining by them parameters of a nucleus, as a rule, is nonlinear. Besides, the systems of corresponding equations are badly stipulated or degenerate. Therefore, determination of unambiguous results of analysis sometimes is impossible even in principle. But, just nonlinearity of system of equations provides a possibility to get reliable enough data about nucleus even in this case. As a result, these circumstances stipulate probabilistic character of all conclusions about properties of the studied nuclei in model-free methods of analysis or bring to more or less (but, always unknown) errors in the obtained notions of nucleus under study.

Owing to the circumstances enumerated above, conclusion on the obtained experimentally parameters of resonances of superposition of k different by type of wave functions (and, respectively, different values of σ_k and b_k) can have only probabilistic character. The main problem by this — estimation of probability that mathematical expectation of the ratio $\chi^2(k = 4)/\chi^2(k = 1)$ really is less than unity. Direct use of this relation for estimation of probability to get its such or less values can give only qualitative information on this account owing to absolute absence of the data on dispersion of cumulative sums of widths in all interval of values of the variable X and nonremovable and unknown error of determination of number of degree of freedom of the best $\chi^2(k = 4)/\chi^2(k = 1)$ values. Respectively, the values $f = N_r - 4$ and $f = N_r - 13$ for $k = 1$ and $k = 4$, for any number of resonances N_r are their upper estimations (only at presence of noticeably different values of parameters b_k and σ_k in results of approximation). Nevertheless, taking into account that $N_r > 90$, it is possible to estimate from the Fisher distribution for the majority of the data [6] on Γ_n^0 that the values $R \leqslant 0.6$, most probably, are stipulated by non-random differences of the approximated and experimental cumulative sums of Γ_n^0 at $k = 1$ (Fig. 10).

Analogous analysis for the data from the Table is practically simple owing to the fact that that the interval of values of the parameter is considerably less than its maximal magnitudes in case of neutron widths. With probability of 99%, the ratio R for the data of the Table cannot be less than 0.65. Considerably more strong dependence of values Γ_γ may has simple explanation — partial radiative

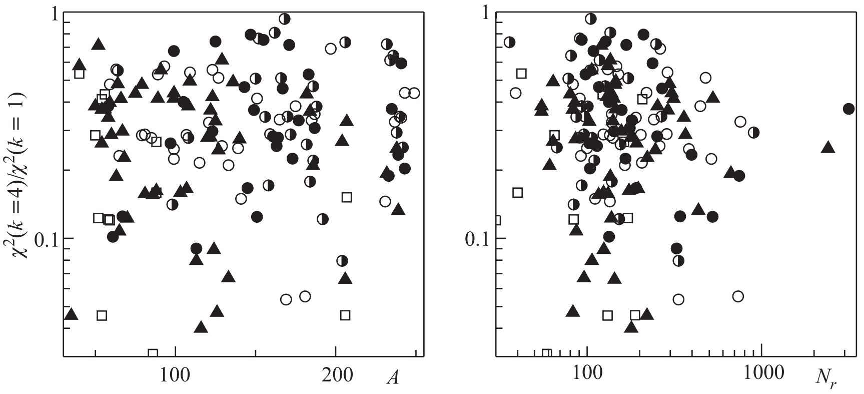

Fig. 10. The ratio of criterions χ^2 for two variants of Γ_n^0 analysis [6] as a function of nuclear mass A or number of resonances N_r. The circles: closed — even–odd, open — odd–even, semi-open — even–even target nuclei. Triangles — p-, squares — d-resonances of any nuclei

widths of the primary gamma transitions depend [9] on phonon components in wave functions of corresponding levels. Their less fragmentation [5] inevitably stipulates stronger sensitivity of Γ_γ to wave functions of neutron resonances.

Additional notions on degree of reliability of conclusion about presence of enough for experimental determination difference of wave functions of neutron resonances can be obtained from analysis of change of form of cumulative sums of the tested parameters in different energy intervals of neutrons.

Relatively small random fluctuations of amplitude of the width distribution for the Porter–Thomas distribution [8] can be expected for the sets of resonances with their number $N_r \approx 400$–500 and more. That is why, the approximate conservation of form of the experimental cumulative sum of widths in different energy intervals of neutron resonances of large enough width would be an additional argument in favor of hypothesis on superposition of neutron resonances of different structure in their experimentally obtained set.

Really such an analysis (although with insufficient small data set) can be performed only for s- and p-resonances of ^{235}U and ^{238}U, respectively. Although in compilation [10] and library ENDF/B-VII [11] there are given the data of spins of resonances, but at absence of the quantitative data on reliability of their determination it is preferably to use in the testing analysis the values $g\Gamma_n$. Both sets contain resonances with two possible spins. Therefore, the possibility of difference between the mean values of $g\Gamma_n$ can bring to superposition of two distributions with the expected and practically constant relation of their contributions in the total function at any neutron energies E_n, but with different parameters of their neutron amplitudes. (The evaluated data of ^{238}U for p-resonances contain «fictitious resonances» of small width, introduced by the authors [12] for reproduction of the capture cross sections of neurons. They, probably, increase the ratio R.)

The energy intervals of the studied resonances E_n for the nuclei under consideration equal 2.26 and 20.0 keV, respectively. Cumulative sums of $g\Gamma_n$ were obtained in two variants in the intervals of energy $\Delta E_n = 0.45$ and 4.0 keV for the data presented in Fig. 11 and $\Delta N_r = 450$ and 400 resonances (Fig. 12) for ^{235}U and ^{238}U, respectively. Approximation of these cumulative sums was

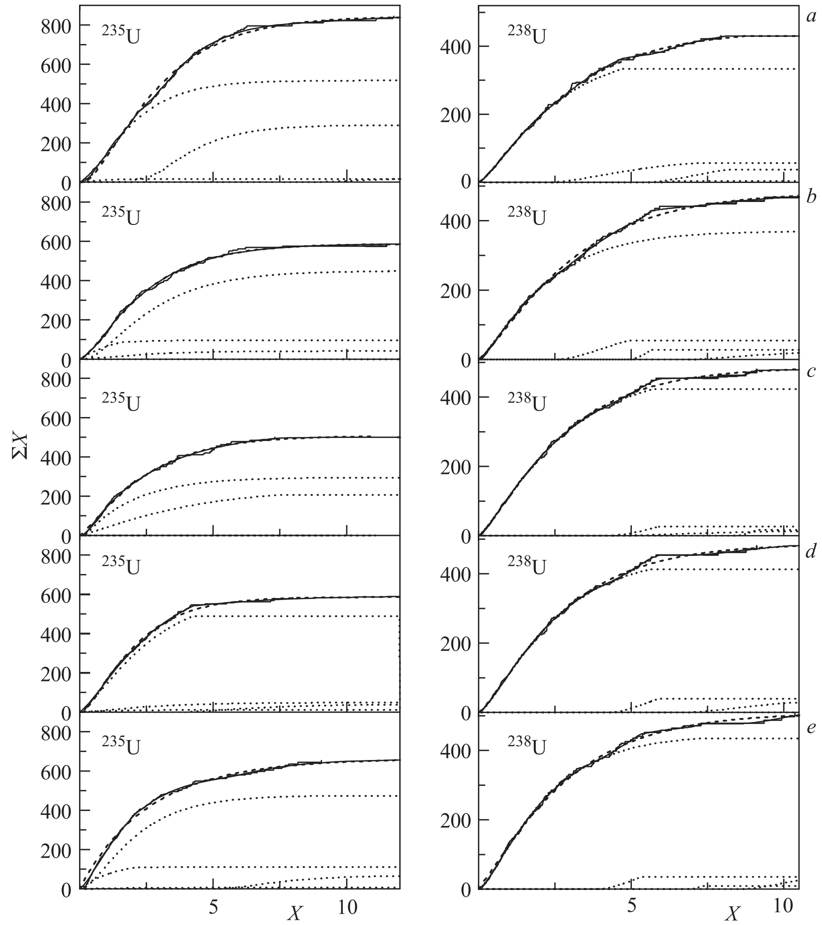

Fig. 11. Approximation of cumulative sums of the relative $X = \Gamma_n^0/\langle\Gamma_n^0\rangle$ values for five intervals of neutron energies of constant width in ^{235,238}U. Histogram — the experiment, dashed line — approximation for $k = 1$, thick line — for $k = 4$, dotted lines — the variant of decomposition of the best fit functions over partial functions

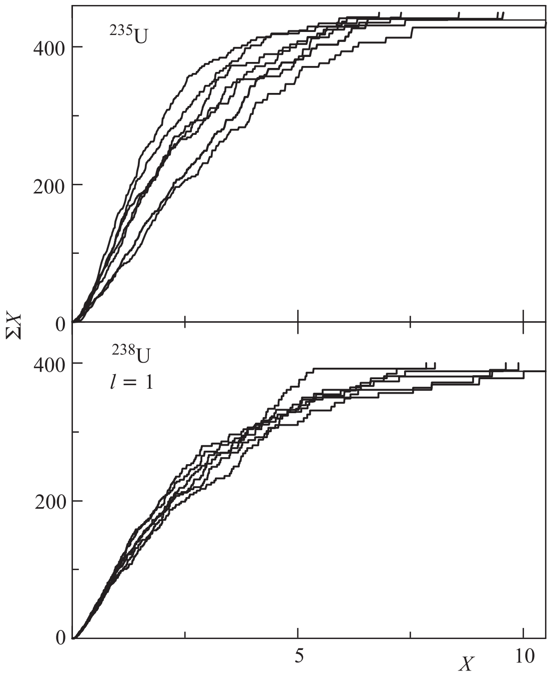

Fig. 12. Cumulative sums of the X values for the same number of resonances in each of 5 intervals of the E_n values of ^{235,238}U nuclei

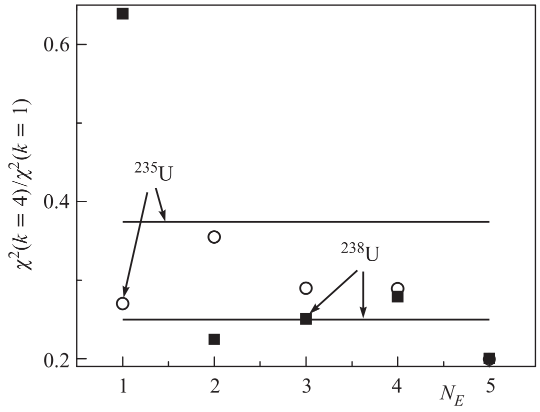

Fig. 13. Points — the ratio of criterions of quality of fitting for interval number N_E for two variants of analysis of the data of Fig. 11. Lines — the value for the total set of resonances [6]

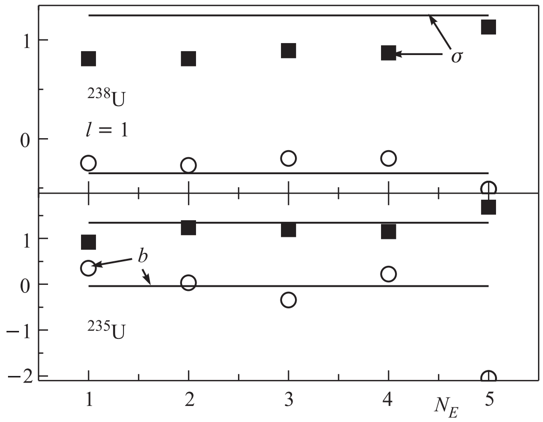

Fig. 14. The values of parameters b and σ for approximation of the data of Fig. 11 (variant $k = 1$). The notations are analogous to Fig. 13

performed completely by analogy with [6], i.e., by singular distribution $(k = 1)$ with fitted mean value of neutron amplitude b and its dispersion σ. The second variant with superposition of four such distributions was used for comparison of the obtained results. The obtained ratios $\chi^2(k = 4)/\chi^2(k = 1)$ for each interval are shown in Fig. 13, and approximated parameters b and σ — in Fig. 14.

As is seen from Fig. 12, cumulative sums for ^{235}U change from interval to interval more strongly than for ^{238}U. In correspondence with the experimental data [6] and theoretical analysis [5], one can expect, from the one hand, noticeable change of structure of resonances in ^{235}U just inside of the accessible to the experiment by the time-of-flight method region of neutron energies. On the other hand, one cannot exclude and possibility of resulting influence of omission of s-resonances and increase of portion of the mistakenly identified p-resonances at increase of E_n.

The comparison of the values of ratio $\chi^2(k = 4)/\chi^2(k = 1)$ (Fig. 13) for different intervals of neutron energies with the values from [6] (Fig. 10) allows one to conclude that the set of the experimental widths corresponds to superposition of several distributions, but it is not the result of random grouping of the widths at some their values. Also, the b and σ parameters undoubtedly change with change of E_n (as a mass of a nucleus), as it follows from the V. G. Soloviev and L.A. Malov theoretical analysis [5] of main principles of fragmentation of the complicated nuclear states. Making more precise reliability of this conclusion or its refutation requires the data on some thousands of resonances for many nuclei with different parity of nucleons and from different diapasons of their masses.

In Fig. 15 are compared the best b and σ parameters of distribution of neutron amplitudes of all 157 nuclei in the variant of approximation $k = 1$ with anal-

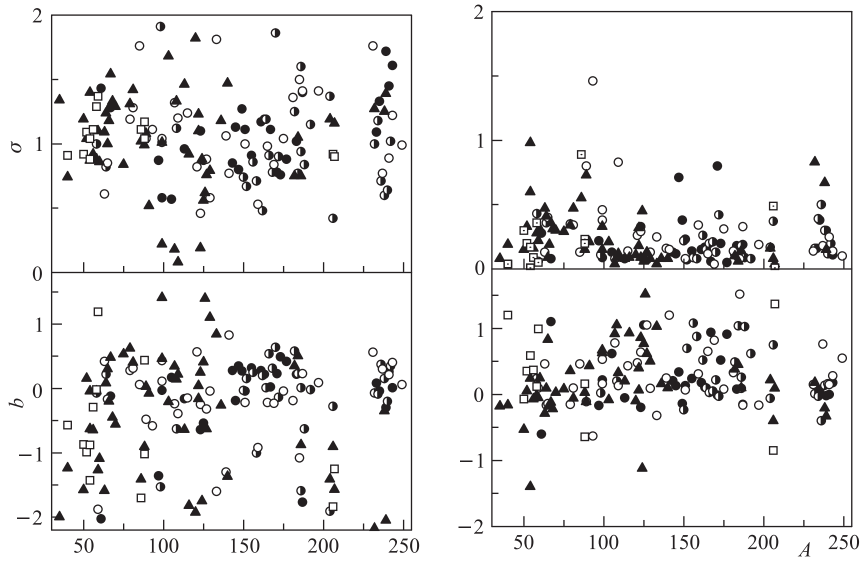

Fig. 15. Left column — the dependence of the best fit [6] values of parameters b and σ on nuclear mass A for variant $k = 1$. Right column — the same, but only for partial functions with maximal contribution in the total distribution of variant $k = 4$

ogous values of the partial distribution, which gives the largest contribution in approximation of the experimental cumulative sum. Noticeably lesser scattering of the latter is indirect confirmation of conclusion [6] on presence in any nucleus of levels with different structure and above the neutron binding energy.

As can be seen from Fig. 15, considerable fluctuations of parameters b and σ point to presence in the tested sets of the reduced neutron widths of noticeable systematical errors, as a minimum. And, as a maximum, — on presence of evident deviations of these parameters from assumptions [8]. But the available data do not allow one to make the undoubted final choice between the variants $k = 1$ and $k \geqslant 2$.

Unfortunately, such a conclusion for case $k > 1$ can be mistaken if systematical errors of Γ_n^0 and Γ_γ are caused by the strong enough unknown and determinate by only experiment condition at different resonance energy, its neutron widths, and so on. For example, by the larger, as compared with the mean, probability of omission of resonances not only with small Γ_n^0, but also with small Γ_γ. Or in the case if in the experiment was revealed only a very small part (for instance, from several to 10–20%) of really existing levels of compound nucleus with fixed spin above B_n. Such a possibility directly follows from the attempt [13] of approximation of the experimental distributions of reduced neutron widths of actinides and following its extrapolation to the $\Gamma_n^0 = 0$ value in the framework of the modified model of neutron widths distribution. (The Porter–Thomas distribution [8] is its particular case.)

5. CONCLUSION

Practically, the described here model-free analysis of the distributions of the total radiative widths of neutron resonances confirms (with not small probability) the determined specific of the existing experimental data:

a) the absence of uniformity of the Γ_γ distributions for different nuclei,

b) significantly better correspondence of the experimental data to the hypothesis of superposition in the observed experimental data of the combination of resonances with noticeably differing structure, than to the assumption on practical (in the framework of modern status of nuclear experiment) constancy of their structure,

c) considerably higher sensitivity of radiative widths than of neutron ones to differences in structure of wave functions of resonances.

Probable presence of groups of resonances with the different mean values $\langle\Gamma_\gamma\rangle$ also corresponds to the conclusion [14] on difference of the radiative strength functions of the primary transitions which exceeds the limits of the expected random fluctuations. This conclusion explains well the difference of the strength functions, measured in the thermal point, with the data for ^{60}Ni obtained [15] from reanalysis of the data on intensities of gamma-cascades following proton capture in several tens of ^{59}Co proton resonances [16].

Final conclusion concerning this matter can be obtained after observation of corresponding differences in the spectra of the primary transitions in a number of neutron resonances of the same nucleus. Modern state of nucleus quasiparticle–phonon model development does not exclude [9] possibility of qualitative observation of such a dependence.

REFERENCES

1. *Bohr N.* // Nature. 1936. V. 137. P. 344.
2. *Sukhovoj A. M., Khitrov V. A.* // Phys. Part. Nucl. 2005. V. 36. P. 359.
3. *Sukhovoj A. M., Khitrov V. A.* // Phys. Part. Nucl. 2006. V. 37. P. 899.
4. *Sukhovoj A. M., Khitrov V. A.* // Phys. At. Nucl. 2010. V. 73. P. 1507.
5. *Malov L. A., Soloviev V. G.* // Sov. J. Nucl. Phys. 1977. V. 26. P. 384.
6. *Sukhovoj A. M., Khitrov V. A.* JINR Commun. E3-2011-33. Dubna, 2011;
   Sukhovoj A. M., Khitrov V. A. // Proc. of the XVIII Intern. Seminar on Interaction of Neutrons with Nuclei, Dubna, May 2010. E3-2011-26. Dubna, 2011. P. 199; http://isinn.jinr.ru/18/contents.html.
7. *Sukhovoj A. M.* // Phys. At. Nucl. 2008. V. 71. P. 1907.

8. *Porter C. F., Thomas R. G.* // Phys. Rev. 1956. V. 104. P. 483.
9. *Soloviev V. G.* // Sov. J. Part. Nucl. 1972. V. 3. P. 390.
10. http://www.springermaterials.com/navigation/#l_1_197716.
11. http://www-nds.iaea.org.
12. *Derrien H., Leal L. C., Larson N. M., Courcelle A.* ORNL/TM-2005/241. Oak Rodge National Laboratory, 2005.
13. *Sukhovoj A. M., Khitrov V. A.* // Proc. of the XVIII Intern. Seminar on Interaction of Neutrons with Nuclei, Dubna, May 2010. E3-2011-26. Dubna, 2011. P. 208; http://isinn.jinr.ru/18/contents.html.
14. *Sukhovoj A. M., Furman W. I., Khitrov V. A.* // Phys. At. Nucl. 2008. V. 71. P. 982.
15. *Sukhovoj A. M., Khitrov V. A.* // Proc. of the XVIII Intern. Seminar on Interaction of Neutrons with Nuclei, Dubna, May 2010. E3-2011-26. Dubna, 2011. P. 180; http://isinn.jinr.ru/18/contents.html.
16. *Voinov A. et al.* // Phys. Rev. C. 2010. V. 81. P. 024319;
    Voinov A. et al. nuc-ex/0907.1819.

Received on April 11, 2011.

Корректор *Т. Е. Попеко*

Подписано в печать 20.06.2011.
Формат 60 × 90/16. Бумага офсетная. Печать офсетная.
Усл. печ. л. 1,18. Уч.-изд. л. 1,59. Тираж 250 экз. Заказ № 57355.

Издательский отдел Объединенного института ядерных исследований
141980, г. Дубна, Московская обл., ул. Жолио-Кюри, 6.
E-mail: publish@jinr.ru
www.jinr.ru/publish/