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COMPLETE GENERAL RELATIVITY

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Завершенная общая относительность

Завершенная общая относительность — это теория Эйнштейна, раскрывающая глубоко скрытую калибровочную природу гравитации и включающая в себя в качестве основы принцип общей относительности и метрическое поле, с одной стороны, и принцип общей калибровочной относительности и общее калибровочное поле, с другой стороны. Общее калибровочное поле не имеет источников и с физической точки зрения представляет интерес как естественный и единственный источник гравитационного поля Эйнштейна. Показано, что синглетное состояние общего калибровочного поля представляет собой электромагнитное поле. Установлены основные уравнения завершенной общей относительности, физический смысл которых обсуждается с различных точек зрения.

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Complete General Relativity

Complete General Relativity is the Einstein theory that discloses the deeply hidden gauge nature of gravity and includes, as a basis, the Principle of General Relativity with the Einstein metric field, on the one hand, and the Principle of General Gauge Relativity with the general gauge field, on the other hand. The general gauge field is a nontrivial sourceless gauge field, which is of physical interest as the natural and the only source of the Einstein gravitational field. Its singlet state becomes apparent in the form of familiar electromagnetic field. The main equations of Complete General Relativity are established, and their physical content is discussed from different points of view.

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INTRODUCTION

The left-hand side (l.h.s.) of the Einstein equation is defined by the curvature tensor of the gravitational field and, hence, it is very beautiful from a geometrical point of view. Einstein believed that the right-hand side (r.h.s.) of his equation should also be a very beautiful expression in Complete General Relativity. That is why this problem was constantly in the sphere of Einstein's investigations [1]. His works during the last decades of his life clearly indicate that he regarded the right-hand side as not the final story, but a temporary way out.

Quoting Chen Ning [2], "I believe that the right-hand side should become a very beautiful expression and not the derivative of the garbage. But what that should be remains to be worked out. I personally believe this is a field which may have dramatic developments in the next ten years". As for the current status of the r. h. s., see [3–5].

Our goal here is to formulate the main equations of Complete General Relativity with the needed details of the right-hand side.

The paper is organized as follows. In Sec. 1, we consider the parallel displacement defined by the metric and covariant derivative associated with this displacement. The commutator of the covariant derivatives gives the curvature tensor of the gravitational field which defines the left-hand side of the Einstein equation. Following these ideas, we consider the most general parallel displacement (quite independent of the metric and anything else) and recognize that it is tightly connected with general local internal symmetry which defines a gauge covariant derivative and the Principle of General Gauge Relativity dual to the Principle of General Relativity. The commutator of gauge covariant derivatives gives the curvature tensor of the general gauge field and the needed details of the right-hand side of the Einstein equation. The notion of the ground state of the general gauge field is introduced. In Sec. 2, we consider the curvature tensor of the general gauge field in more detail. The curvature tensor of the gravitational field is traceless, but that of the general gauge field has a trace. Hence, we separate the trace part and deal with it separately. We derive equations of Complete General Relativity and demonstrate that the trace part represents a familiar electromagnetic field. In Sec. 3, it is shown how the Dirac field defines the nontrivial ground state. In Conclusions, we discuss some physical aspects of Complete General Relativity.

1. GENERAL GAUGE FIELD

The curvature tensor ${R_{ijl}}^k$ of the gravitational field $ds^2 = g_{ij}dx^i\,dx^j$ results from the parallel displacement

$$\delta V^i = dV^i + \Gamma^i_{jk}dx^j V^k = 0,$$

where the connection Γ^i_{jk} is defined by the metric as

$$\Gamma^i_{jk} = \frac{1}{2}g^{il}(\partial_i g_{jl} + \partial_j g_{il} - \partial_l g_{ij}),$$

the covariant derivative $\delta V^i = \nabla_j V^i dx^j$

$$\nabla_j V^i = \partial_j V^i + \Gamma^i_{jk}v^k$$

and the commutator of the covariant derivatives

$$(\nabla_i\nabla_j - \nabla_j\nabla_i)V^k = {R_{ijl}}^k V^l.$$

Hence,

$${R_{ijl}}^k = \partial_i\Gamma^k_{jl} - \partial_j\Gamma^k_{il} + \Gamma^k_{in}\Gamma^n_{jl} - \Gamma^k_{jn}\Gamma^n_{il}.$$

The antisymmetric tensor (trace) ${R_{ijk}}^k$ of the curvature tensor of the gravitational field is trivial, since $\partial_i\Gamma^k_{jk} - \partial_j\Gamma^k_{ik} = 0$. But from ${R_{ijl}}^k$ we can create the symmetric tensor $R_{jl} = {R_{kjl}}^k$ and the scalar $R = R_{jl}g^{jl}$ and thus define absolutely the l.h.s. of the Einstein equation

$$R_{ij} - \frac{1}{2}\,g_{ij} = T_{ij}.$$

With this sequence of ideas in mind, it is natural to put forward an assumption that a curvature tensor, which defines the r.h.s. of the Einstein equation, results from the general parallel displacement

$$\delta V^i = dV^i + P^i_{jk}dx^j V^k = 0,$$

where the connection P^i_{jk} is considered as a primary entity. However, here we need to stop and look for an intuitively clear expansion of the Principle of General Relativity.

To define the Principle of General Gauge Relativity, we start from the consideration of the linear operators in the space of vector fields V^i. Nondegenerate linear transformation has the form

$$\overline{V}^i = S^i_j V^j, \quad \mathrm{Det}\,(S^i_j) \neq 0,$$

where S^i_j is a tensor field of the second rank. These local internal transformations form a general gauge group with an associative binary operation $P^i_j = S^i_k T^k_j$.

The parallel displacement of the vector fields $\overline{V}^i$ and V^i can be produced only by a pair of connections $\overline{P}^i_{jk}$ and P^i_{jk}. From the law of parallel displacement we have

$$\overline{P}^i_{jk} = S^i_m P^m_{jn} {S^{-1}}^n_k + S^i_m \partial_j {S^{-1}}^m_k,$$

where ${S^{-1}}^k_j$ — the components of the operator S^{-1} inverse to the operator S, $S^i_k {S^{-1}}^k_j = \delta^i_j$. Hence, the Principle of General Gauge Relativity states here that a physical configuration is not a given potential P^i_{jk}, but rather a class of gauge equivalent potentials defined above. This principle essentially uniquely defines the dynamics of the general gauge field and the r.h.s. of the Einstein equation.

For the gauge covariant derivative $\delta V^i = \mathrm{D}_j V^i dx^j$

$$\mathrm{D}_j V^i = \partial_j V^i + P^i_{jk} V^k,$$

we have

$$\overline{\mathrm{D}}_j \overline{V}^i = S^i_k \mathrm{D}_j V^k.$$

The commutator of the gauge covariant derivatives

$$(\mathrm{D}_i \mathrm{D}_j - \mathrm{D}_j \mathrm{D}_i) V^k = [\mathrm{D}_i, \mathrm{D}_j] V^k = H_{ijl}{}^k V^l,$$

where

$$H_{ijl}{}^k = \partial_i P^k_{jl} - \partial_j P^k_{il} + P^k_{in} P^n_{jl} - P^k_{jn} P^n_{il},$$

gives the curvature tensor (the strength tensor) of the general gauge field with the potential P^i_{jk}, since $H_{ijl}{}^k$ is a natural derivative of P^i_{jk} (a generalization of the trivial $\varphi, \partial_i \varphi$).

The antisymmetric tensor $F_{ij} = H_{ijk}{}^k$ (trace of the curvature tensor)

$$F_{ij} = \partial_i P^k_{jk} - \partial_j P^k_{ik}$$

is nontrivial here and should be considered separately from the irreducible (traceless) tensor of curvature

$$I_{ijl}{}^k = H_{ijl}{}^k - \frac{1}{4} H_{ijn}{}^n \delta^k_l, \quad I_{ijl}{}^l = 0.$$

For brevity, in what follows we will use the matrix notation

$$\mathbf{S} = (S^k_l), \quad \mathbf{P}_i = (P^k_{il}), \quad \mathbf{E} = (\delta^k_l), \quad \mathbf{H}_{ij} = (H_{ijl}{}^k), \quad \mathrm{Tr}\, \mathbf{H}_{ij} = H_{ijk}{}^k,$$

$$\mathbf{H}_{ij} = \partial_i \mathbf{P}_j - \partial_j \mathbf{P}_i + [\mathbf{P}_i, \mathbf{P}_j].$$

The transformations of general gauge field take the form

$$\overline{\mathbf{P}}_i = \mathbf{S}\mathbf{P}_i\mathbf{S}^{-1} + \mathbf{S}\partial_i\mathbf{S}^{-1} = \mathbf{P}_i + \mathbf{S}\mathrm{D}_i\mathbf{S}^{-1},$$

$$\overline{\mathbf{H}}_{ij} = \mathbf{S}\mathbf{H}_{ij}\mathbf{S}^{-1}, \quad \overline{\mathrm{D}}_i\overline{\mathbf{H}}_{jk} = \mathbf{S}\mathrm{D}_i\mathbf{H}_{jk}\mathbf{S}^{-1},$$

where D_i is the natural differential operator (gauge covariant derivative) associated with the general covariance and general gauge covariance, so

$$\mathrm{D}_i\mathbf{S} = \partial_i\mathbf{S} + \mathbf{P}_i\mathbf{S} - \mathbf{S}\mathbf{P}_i = \partial_i\mathbf{S} + [\mathbf{P}_i, \mathbf{S}]$$

is the tensor, $\mathrm{D}_i\mathbf{H}_{jk} = \partial_i\mathbf{H}_{jk} + [\mathbf{P}_i, \mathbf{H}_{jk}]$ is not the tensor, but $\mathrm{D}_i\mathbf{H}_{jk} + \mathrm{D}_j\mathbf{H}_{ki} + \mathrm{D}_k\mathbf{H}_{ij}$ is the tensor, and the identity

$$\mathrm{D}_i\mathbf{H}_{jk} + \mathrm{D}_j\mathbf{H}_{ki} + \mathrm{D}_k\mathbf{H}_{ij} = 0$$

is generally covariant.

The important notion of the ground state of the general gauge field is defined as a nontrivial solution of the equation

$$\mathbf{H}_{ij} = 0.$$

Let four linear independent vector fields E^i_μ be given. In this case, one can construct purely algebraic components of four covector fields E^μ_i, so that $E^i_\mu E^\mu_j = \delta^i_j$ holds valid. Setting $P^k_{il} = L^k_{il}$, where

$$L^k_{il} = E^k_\mu \partial_i E^\mu_l$$

is a linear connection of the ground state, we get a general solution of the equation $\mathbf{H}_{ij} = 0$. For the ground state we have $\mathrm{Tr}\,(\mathbf{L}_i) = \partial_i \ln|p|$, where $p = \mathrm{Det}\,(E^\mu_i)$. Thus, we can define the ground state as any quadruplet of linear independent vector fields E^i_μ associated with the connection $L^k_{il} = E^k_\mu\partial_i E^\mu_l$. The ground state is invariant with respect to the general gauge transformations. Indeed, if the quadruplet of vector fields E^i_μ represents the ground state, then $\overline{E}^i_\mu = S^i_j E^j_\mu$ is the ground state as well, since $\overline{\mathbf{L}}_i = \mathbf{S}\mathbf{L}_i\mathbf{S}^{-1} + \mathbf{S}\partial_i\mathbf{S}^{-1}$. The meaning of the notion of the ground state will be clarified in the course of the development of Complete General Relativity.

A transition from the ground state to the excited one is characterized by the tensor of transition

$$T^i_{jk} = P^i_{jk} - L^i_{jk}$$

with a simple (homogeneous) law of transformation

$$\overline{T}^i_{jk} = S^i_m T^m_{jn} {S^{-1}}^k_j, \quad \overline{\mathbf{T}}_i = \mathbf{S}\mathbf{T}_i\mathbf{S}^{-1},$$

and the irreducible tensor

$$Q^i_{jk} = T^i_{jk} - \frac{1}{4}T^l_{jl}\delta^i_k, \quad \mathbf{Q}_j = \mathbf{T}_j - \frac{1}{4}\,\mathrm{Tr}\,(\mathbf{T}_j)\mathbf{E}$$

with the trivial trace $\mathrm{Tr}\,(\mathbf{Q}_j) = 0$.

2. EQUATIONS OF COMPLETE GENERAL RELATIVITY

We put

$$\mathcal{L}_P = -\frac{1}{4}\,\mathrm{Tr}\,(\mathbf{I}_{ij}\mathbf{I}^{ij}) + \frac{\mu^2}{2}\,\mathrm{Tr}\,(\mathbf{Q}_i\mathbf{Q}^i), \quad \mathcal{L}_{\mathrm{em}} = -\frac{1}{4}\,F_{ij}\,F^{ij}.$$

The general covariant and general gauge covariant Lagrangian $\mathcal{L}$ of the general gauge field takes the form

$$\mathcal{L} = \alpha\mathcal{L}_P + \beta\mathcal{L}_{\mathrm{em}} = \alpha\Big[-\frac{1}{4}\,\mathrm{Tr}\,(\mathbf{I}_{ij}\mathbf{I}^{ij}) + \frac{\mu^2}{2}\,\mathrm{Tr}\,(\mathbf{Q}_i\mathbf{Q}^i)\Big] + \beta\Big[-\frac{1}{4}\,F_{ij}\,F^{ij}\Big], \quad (1)$$

where μ is a constant of dimension of cm^{-1}, and $\alpha \geqslant 0$ and $\beta \geqslant 0$ are dimensionless parameters, dimension of P^i_{jk} is equal to cm^{-1} and the action is dimensionless, $\mathbf{I}^{ij} = g^{ik}g^{jl}\mathbf{I}_{kl}$, $\mathbf{Q}^i = g^{ik}\mathbf{Q}_k$. As for the form of the Lagrangian (1), we mention that the tensor ${H_{ijl}}^i$ has sense in the framework of General Relativity, but it is not the case with respect to the general gauge symmetry.

By varying the Lagrangian $\mathcal{L}$ with respect to $\mathbf{P}_i$, the following equation holds

$$\alpha\left[\frac{1}{\sqrt{g}}\mathrm{D}_i(\sqrt{g}\,\mathbf{I}^{ij}) + \mu^2\mathbf{Q}^j\right] + \beta\left[\frac{1}{\sqrt{g}}\partial_i(\sqrt{g}\,F^{ij})\mathbf{E}\right] = 0, \quad (2)$$

where $g = -\mathrm{Det}\,(g_{ij})$. From the properties of the operator D_i it is not difficult to see that Eq. (2) is invariant with respect to the transformations of the general gauge group. The tensor character of this equation can be seen on the same grounds.

Taking trace from Eq. (2), we find that

$$\frac{1}{\sqrt{g}}\,\partial_i(\sqrt{g}\;F^{ij}) = 0, \quad (3)$$

since $\mathrm{Tr}\,(\mathbf{I}^{ij}) = \mathrm{Tr}\,(\mathbf{Q}^j) = 0$. Hence,

$$\frac{1}{\sqrt{g}}\,\mathrm{D}_i(\sqrt{g}\,\mathbf{I}^{ij}) + \mu^2\mathbf{Q}^j = 0. \quad (4)$$

For completeness, we extend this system of equations by the identities

$$\mathrm{D}_i\mathbf{I}_{jk} + \mathrm{D}_j\mathbf{I}_{ki} + \mathrm{D}_k\mathbf{I}_{ij} = 0, \quad \partial_iF_{jk} + \partial_jF_{ki} + \partial_kF_{ij} = 0.$$

From Eq. (4) it follows that $\mathbf{Q}^i$ has to satisfy the equation

$$\frac{1}{\sqrt{g}}\,\mathrm{D}_i(\sqrt{g}\,\mathbf{Q}^i) = 0, \quad (5)$$

because $\mathrm{D}_i\mathrm{D}_j(\sqrt{g}\,\mathbf{I}^{ij}) = 0$. It should be noted that the same equation appears under varying Eq. (1) with respect to E^i_μ. But as shown above, Eq. (5) represents sixteen additional gauge invariant constraints on the potential $\mathbf{P}_i$, but not equations for E^i_μ.

To make a clear and apparent conclusion from this result, we first of all mention that the strength tensor $\mathbf{I}_{ij}$ can only be written in terms of the irreducible tensor $\mathbf{Q}_i$, since

$$\mathbf{I}_{ij} = \overset{o}{\mathrm{D}}_i\ \mathbf{Q}_j - \overset{o}{\mathrm{D}}_j\ \mathbf{Q}_i + \mathbf{Q}_i\mathbf{Q}_j - \mathbf{Q}_j\mathbf{Q}_i.$$

Here $\overset{o}{\mathrm{D}}_i$ denotes the gauge covariant derivative with respect to the connection $\mathbf{L}_i$ of the ground state and, hence, $[\overset{o}{\mathrm{D}}_i, \overset{o}{\mathrm{D}}_j] = 0$. For the antisymmetric tensor F_{ij} we obtain

$$F_{ij} = \partial_i\, P^k_{jk} - \partial_j\, P^k_{ik} = \partial_i\, (L^k_{jk} + T^k_{jk}) - \partial_j\, (L^k_{ik} + T^k_{ik}) = \partial_i\, T^k_{jk} - \partial_j\, T^k_{ik},$$

since $\mathrm{Tr}\,(\mathbf{L}_i) = \partial_i \ln |p|$, $p = \mathrm{Det}\,(E^\mu_i)$. Thus, we can consider the tensor field Q^k_{il} with the constraints $Q^k_{ik} = 0$ and covariant vector field $A_i = T^k_{ik}$ as independent quantities, which obey the equations

$$\frac{1}{\sqrt{g}}\partial_i(\sqrt{g}\,F^{ij}) = 0, \quad F_{ij} = \partial_i A_j - \partial_j A_i, \tag{6}$$

$$\frac{1}{\sqrt{g}}\ \overset{o}{\mathrm{D}}_i\ (\sqrt{g}\,\mathbf{I}^{ij}) + [\mathbf{Q}_i, \mathbf{I}^{ij}] + \mu^2\mathbf{Q}^j = 0, \tag{7}$$

and

$$\frac{1}{\sqrt{g}}\ \overset{o}{\mathrm{D}}_i\ (\sqrt{g}\,\mathbf{Q}^i) + [\mathbf{Q}_i, \mathbf{Q}^i] = 0. \tag{8}$$

Since the trace of $\mathbf{I}_{ij}$ is equal to zero, it is clear why we need to consider a traceless tensor $\mathbf{Q}^i$. In our case, the trace of $\mathbf{Q}^i$ is trivial, and Eq. (7) is compatible.

From the Lagrangian (1) it follows that in Complete General Relativity the r.h.s. of the Einstein equation (energy–momentum tensor) is defined by the curvature tensor of the general gauge field as

$$T_{ij} = \alpha[-\mathrm{Tr}\,(\mathbf{I}_{ik}\,\mathbf{I}_j{}^k) - g_{ij}\,\mathcal{L}_P + \\ + \mu^2\,\mathrm{Tr}\,(\mathbf{Q}_i\,\mathbf{Q}_j)] + \beta\left[-F_{il}\,F_j{}^l + \frac{1}{4}\,F_{kl}\,F^{kl}g_{ij}\right], \tag{9}$$

where $\mathbf{I}_j{}^k = \mathbf{I}_{jl}g^{kl}$. It is evident that the energy–momentum tensor (9) is invariant with respect to the transformations of the general gauge group and, hence, it is observable from the point of view of general gauge symmetry. One can show that the energy–momentum tensor (9) satisfies the equation

$$\nabla^i T_{ij} = 0, \tag{10}$$

where ∇_i denotes the covariant derivative with respect to the connection belonging to metric g_{ij}. Since $g^{ij}T_{ij} = -\mu^2\mathrm{Tr}\,(\mathbf{Q}_i\mathbf{Q}^i)$, the scale invariance is broken. The mass term $\mu^2\mathrm{Tr}\,(\mathbf{Q}_i\mathbf{Q}^i)$ is obtained by means which does not violate the general gauge symmetry, and this is important point for the renormalizability of the theory.

At last, we write the Lagrangian $\mathcal{L}_g = -(l_g/2)\, R$, where l_g is a constant of dimension cm^{-2}, and the Einstein equation of Complete General Relativity takes the form

$$l_g G_{ij} = \alpha[-\mathrm{Tr}\,(\mathbf{I}_{ik}\,\mathbf{I}_j{}^k) - g_{ij}\,\mathcal{L}_P + \mu^2 \mathrm{Tr}\,(\mathbf{Q}_i\,\mathbf{Q}_j)] + \\ + \beta\left[-F_{il}\,F_j{}^l + \frac{1}{4}\,F_{kl}\,F^{kl} g_{ij}\right], \quad (11)$$

$$\mathcal{L}_P = -\frac{1}{4}\,\mathrm{Tr}\,(\mathbf{I}_{ij}\,\mathbf{I}^{ij}) + \frac{\mu^2}{2}\mathrm{Tr}\,(\mathbf{Q}_i\,\mathbf{Q}^i).$$

When the system goes into the ground state ($\mathbf{H}_{ij} = 0$), the r.h.s. of the Einstein equation is trivial.

3. DIRAC FIELD AND GROUND STATE

In this section we pay attention to the natural mechanism of spontaneous broken symmetry inherent in the system in question.

The ground state $P^i_{jk} = L^i_{jk}$ so defined is trivial in the following sense. By the local transformation S^i_j we can reduce four covector fields E^μ_i to the form of four gradient covector fields $\partial_j \alpha^\mu = S^i_j E^\mu_i$, $\mathrm{Det}\,(\partial_j \alpha^\mu) \neq 0$. In the coordinate system $\overline{x}^i = \alpha^i(x)$, $P^i_{jk} = L^i_{jk} = 0$. It is clear from this consideration that we can define the trivial ground state simply putting $E^i_\mu = \delta^i_\mu$ and that for the nontrivial ground state the general gauge symmetry should be broken. A natural way to do this is to take into consideration the tensor fields

$$I^i_{jk} = L^i_{jk} - L^i_{kj}, \quad I_i = I^k_{ik}$$

and the Lagrangian

$$\mathcal{L}_I = \frac{l_I}{2}(I^k_{il} I^l_{jk} + a I_i I_j) g^{ij},$$

where a is a dimensionless parameter. This Lagrangian is evidently not invariant with respect to general gauge transformations. Setting $\overline{E}^i_\mu = S^i_j E^j_\mu = L^\nu_\mu E^i_\nu$, we see that if L^ν_μ is a constant matrix, then the tensor I^i_{jk} is invariant with respect to the global transformations $\overline{E}^i_\mu = L^\nu_\mu E^i_\nu$.

It is important to explain how the Dirac field comes into Complete General Relativity and defines the nontrivial ground state. Let us consider the general covariant Lagrangian [6]

$$\mathcal{L}_D = \frac{i}{2} E^k_\mu (\overline{\psi}\gamma^\mu P_k \psi - P_k \overline{\psi}\gamma^\mu \psi) - m\overline{\psi}\psi,$$

where

$$P_k \psi = (\partial_k - i\alpha A_k)\psi, \quad P_k \overline{\psi} = (\partial_k + i\alpha A_k)\overline{\psi}$$

and α is the fine-structure constant. Setting $p = \mathrm{Det}\,(E^\mu_k)$, we have

$$\frac{1}{p}\partial_k p = E^i_\mu \partial_k E^\mu_i.$$

Varying the action

$$A = \int \mathcal{L}_D p d^4 x,$$

we derive the Dirac equations

$$iE^k_\mu \gamma^\mu (P_k + \frac{1}{2} I_k)\psi = m\psi, \tag{12}$$

$$iE^k_\mu (P_k + \frac{1}{2} I_k)\overline{\psi}\gamma^\mu = -m\overline{\psi}, \tag{13}$$

where $I_k = I^l_{kl}$ was introduced above. The trace $I_k = I^l_{kl}$ plays an important role under the proof that the current

$$J^k = E^k_\mu \overline{\psi}\gamma^\mu \psi$$

is divergenceless. If we multiply Eq. (12) by $\overline{\psi}$, and Eq. (13) by ψ and put together, then

$$E^k_\mu \partial_k (E^\mu_l J^l) + I_k J^k = 0.$$

Considering this and equation

$$I_k = \frac{1}{p}\,\partial_k p - E^l_\mu \partial_l E^\mu_k,$$

we conclude that the current J^k is conserved

$$\frac{1}{p}\,\partial_k (pJ^k) = 0.$$

Now let us put

$$W^\mu_k = \frac{i}{2}\,(\overline{\psi}\gamma^\mu\, D_k\psi - D_k\,\overline{\psi}\gamma^\mu\psi).$$

Then $\mathcal{L}_D = E^k_\mu W^\mu_k - m\overline{\psi}\psi$ and, hence, from the action

$$A = \int \mathcal{L}_D\, p\, d^4 x + \int \mathcal{L}_I \sqrt{g}\ d^4 x, \quad g = -\mathrm{Det}\,(g_{ij})$$

we have the equation for the ground state E^k_μ which can be written in the form

$$l_I \frac{1}{\sqrt{g}} \partial_j (\sqrt{g}\, H^{j\mu}_\nu) + \varepsilon W^\mu_\nu = 0,$$

where

$$W^\mu_\nu = E^l_\nu W^\mu_l, \quad \varepsilon = p/\sqrt{g}\,,$$

$$H^{j\mu}_\nu = H^{jl}_k E^\mu_l E^k_\nu, \quad H^{jl}_k = g^{ij}(I^l_{ik} + aI_i\delta^l_k) - g^{il}(I^j_{ik} + aI_i\delta^j_k).$$

A weak impact of the nontrivial ground state on gravitational interactions is defined by the energy–momentum tensor

$$T_{ij} = g_{ij} L_I - l_I (I^k_{il} I^l_{jk} + aI_i I_j).$$

We can see the indirect influence of matter with spin on the gravitational effects in the framework of the Dirac theory. In this sense, unification of General Relativity and quantum mechanics is trivial. The Maxwell equation (6) in this case reads as

$$\frac{1}{\sqrt{g}}\,\partial_i(\sqrt{g}\,F^{ij}) + \alpha\varepsilon J^l = 0, \quad J^l = E^l_\mu \overline{\psi}\gamma^\mu\psi.$$

In brief, the Dirac field participates in gravitational interactions indirectly. It follows from our consideration that the Dirac field can interact with the gravitating physical system in question only through the channel of the ground state. In the case of interaction, the canonical energy–momentum tensor of the Dirac field is a source of the field that describes the ground state of the general gauge field (in this case, nontrivial).

CONCLUSIONS

Thus, the general gauge field P^k_{jl} has two states: the familiar electromagnetic field $A_i = T^k_{jk}$ (which should be considered as its singlet state) and more general state Q^k_{il}, $Q^k_{ik} = 0$, that can be called the general electromagnetic field. As is known, we put particle called photon into correspondence to definite state of the proper electromagnetic field. With this in mind, we will call a massive particle that corresponds to a definite state of the general electromagnetic field "mphoton". These particles are the only source of the gravitational field in the framework of the General Gauge Relativity. Hence, the nature of gravity is disclosed, and we can say that the universe is arranged as a system of particles with the Bose–Einstein statistics. Some region of space filled with the gravitational and general electromagnetic fields looks like absolute darkness.

From Eq. (11) it follows that the interactions between photons and mphotons are realized by a graviton exchange. This interaction can be characterized by an angle of mixing. In accordance with Eq. (11), we can put $\sin\varphi = \alpha/\sqrt{\alpha^2+\beta^2}$, $\cos\varphi = \beta/\sqrt{\alpha^2+\beta^2}$ and redefine l_g.

It is important to pay attention to the following analogy between gravity and electromagnetism. In the electron theory of Lorentz [7], the right-hand side of the Maxwell equations was presented with continuous phenomenological distributions of charge and current. With the discovery of quantum mechanics or, more exactly, the Schrödinger and Dirac equations, the details of the right-hand side in this case were clarified. But the physical content of the Maxwell–Dirac equations was disclosed only in the framework of quantum electrodynamics. We see the same situation in the Einstein theory of the gravitational field. It is clear that the investigation of Complete General Relativity as a closed gravitating system in the framework of quantum field theory is an urgent problem. Hence, we need to look for hidden possibilities to solve the renormalization problem in the gravitating system in question.

Whilst the need for invisible matter was established almost a century ago, only its gravitational interaction has been conformed so far. Hence, it is natural to suppose that mphotons can represent the invisible matter, and there is no reason for a plethora of models for this matter often called dark matter [8]. On dark matter search, see review [9].

Thus, from the observations we can conclude that in the framework of Complete General Relativity, the photons represent the Cosmic Microwave Background (CMB), and the mphotons correspond to the so-called Weakly Interacting Massive Particles (WIMPs). But from Eq. (11) it follows that the Cosmic Microwave Background and invisible matter are tightly connected and, hence, the investigations of CMB can provide the discovery of hidden properties of mphotons (or WIMPs).

We believe that Eqs. (6)–(11) provide a justified basis for discovering realistic cosmological models.

Since we now know the source of the gravitational field, the new status of the gravitational waves should be considered as well.

REFERENCES

1. *Einstein A.* Relativistic Theory of the Non-Symmetric Field. The Meaning of Relativity. Fifth edition. Princeton, 1955.
2. *Yang C. N.* Lectures on Frontiers in Physics. Seoul, Korea, 1980.
3. *Weinberg S.* Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity. New York: Wiley, 1972.
4. *Misner C. W., Torne K. S., Wheeler J. A.* Gravitation. San Francisco: W. H. Freeman and Company, 1973.
5. *'t Hooft G.* Introduction to General Relativity. Princeton: Rinton Press Inc., 2001.
6. *Wigner E.* // Zeitschrift für Physik. 1929. Bd. 53. S. 592–603.
7. *Lorentz H. A.* The Theory of Electrons. Leipzig: B. G. Teubner, 1916.
8. *Particle Data Group* // Phys. Lett. B. 1990. V. 239. III.3.
9. *Bednyakov V. A.* // Part. Nucl. 2016. V. 47, No. 5. P. 1315–1427.

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