D17-2022-30

Yu. M. Shukrinov [1,2,*], M. Nashaat [1,3,**]

JINR COLLABORATION WITH SCIENTIFIC INSTITUTIONS OF EGYPT IN THE FIELD OF THEORETICAL RESEARCH OF SUPERCONDUCTING JOSEPHSON NANOSTRUCTURES

Submitted to “Phys. Part. Nucl.”

[1] Joint Institute for Nuclear Research, Dubna
[2] Department of Nanotechnology and New Materials, Dubna State University, Dubna, Russia
[3] Department of Physics, Faculty of Science, Cairo University, Giza, Egypt
* E-mail: shukrinv@theor.jinr.ru
** E-mail: majed@sci.cu.edu.eg

Шукринов Ю. М., Нашаат М. Д17-2022-30

Сотрудничество ОИЯИ с научными учреждениями Египта в области теоретических исследований сверхпроводящих джозефсоновских наноструктур

Представлен обзор результатов, полученных в рамках сотрудничества ОИЯИ с научными учреждениями Египта в области теоретических исследований и моделирования различных сверхпроводящих джозефсоновских наноструктур. В частности, рассмотрены системы связанных джозефсоновских переходов с различного типа шунтированием, представлены результаты, отражающие их фазовую динамику и вольт-амперные характеристики при различных резонансных явлениях. Большое внимание уделено исследованию сверхпроводниковых структур с ферромагнитными материалами, позволяющих управлять магнитными свойствами системы посредством сверхпроводящего тока. Важное место занимают совместные исследования топологических и хаотических явлений в джозефсоновских структурах.

Работа выполнена в Лаборатории теоретической физики им. Н. Н. Боголюбова ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 2022

Shukrinov Yu. M., Nashaat M. D17-2022-30

JINR Collaboration with Scientific Institutions of Egypt in the Field of Theoretical Research of Superconducting Josephson Nanostructures

An overview of the results obtained in the framework of JINR collaboration with Egyptian scientific institutions in the field of theoretical research and modeling of various superconducting Josephson nanostructures is presented. In particular, systems of coupled Josephson junctions with various types of shunting are considered, and results are presented that reflect their phase dynamics and current–voltage characteristics for various resonance phenomena. Much attention is paid to the study of superconducting structures with ferromagnetic materials, which make it possible to control the magnetic properties of the system by a superconducting current. An important place is occupied by joint studies of topological and chaotic phenomena in Josephson structures.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 2022

1. INTRODUCTION

Collaboration between JINR and scientific institutions in Egypt in the field of theoretical studies of superconducting nanostructures was started in 2010. It was initiated by JINR University Center arranging the practice for young scientists and students every year in JINR laboratories (Figs. 1–3). In the next five years, eleven young scientists and students joined the practice at BLTP on "Computer simulation of tunneling characteristics of superconducting nanostructures". Seven of them, Mahmoud Gaafar, Ahmed Marawan, Hazem AbdelHafiz, Mostafa ElDemary, Radwa Dawood, Majed Nashaat, and Ali Abouhaswa, continued their work after that in this field and contributed to collaborative publications [1–22].

Our main goal is to create a base for fundamental and applied research in the field of superconducting electronics and spintronics in both countries. Currently, intensive joint work is being carried out on the simulation of superconducting nanostructures, in particular, coupled Josephson junctions, shunted systems, and hybrid Josephson structures with various types of barriers. This makes it possible to study the phase dynamics, current–voltage characteristics and obtain new information about various resonant phenomena in these structures.

The collaborative research carried out so far has led to a number of interesting results. In particular, a resonant-type hysteresis in the

Fig. 1. Participants of Practice for Young Scientists and Students 2010, including Mahmoud Gaafar, Ahmed Marawan, Hazem Abdelhafiz, Majed Nashaat, and Ali Abouhaswa, who continued their work on superconducting nanostructures and contributed to collaborative publications

Fig. 2. Participants of Practice for Young Scientists and Students 2019, including Sara Ali Abdelmoneim who joined the project on spintronics effects in Josephson junctions

Fig. 3. Participants of Practice for Young Scientists and Students 2022, including Nayra Moussa who worked on the study of magnetization dynamics in SFS junction

current–voltage characteristic (CVC) is predicted, which occurs in layered high-temperature superconductors [1]. The appearance of an electric charge in the superconducting layers was demonstrated, and it was shown that a change in the amplitude of external electromagnetic radiation changes the length of the longitudinal plasma wave that occurs during parametric resonance [2–5].

The paper [6] reported on the occurrence of a charge density wave in a system of coupled Josephson junctions, where the transformation of a longitudinal plasma wave into a charge density wave was shown, and transitions between various types of charge density waves were demonstrated.

Ladder structures of Shapiro steps were found in the branching region of the CVC [7].

Moreover, the effects of non-stationary and stationary charge imbalance in layered superconducting structures were demonstrated [8–11]. We found that the charge imbalance effect appears at the Shapiro step, which exhibits a finite slope and deviation of the voltage magnitude from its canonical value. The values of the shift and slope depend on the relaxation time of the quasiparticles and the thickness of the superconducting layer. It opens a possibility to determine the quasiparticle relaxation time by the slope of the Shapiro step. In this connection, we have shown that a new possibility opens up for determining the relaxation time of quasiparticles from the slope of the Shapiro step. The transformation of traveling waves into standing waves in shunted systems was observed in Ref. [12]

Our collaboration allowed us to develop a series of computer programs for modeling the dynamics and CVC of Josephson structures and superconducting quantum interference devices (SQUIDs) with nontrivial barriers [13–15]. It has been shown that the current–voltage characteristics of shunted Josephson junctions with localized subgap Majorana states provide a phase-sensitive method for their detection. The appearance of additional ladder structures on the current–voltage characteristics makes it possible to develop methods for recording Majorana states in Josephson structures. In the case of SQUIDs with nontrivial barriers, the corresponding resonant branch has a voltage shift, which can also be used to experimentally detect Majorana fermions.

An important challenge in superconducting spintronics dealing with the Josephson junctions coupled to magnetic systems is the achievement of electric control over the magnetic properties by the Josephson current and its counterpart, i.e., the achievement of magnetic control over the Josephson current [23–27]. In such systems the spin–orbit coupling plays a major role in the attainment of such control [28]. An example is a full magnetization reversal in a superconductor/ferromagnet/superconductor (SFS) structure with spin–orbit coupling by adding an electric current pulse [29]. Such a reversal may be important for certain applications in quantum computing. In this field, we demonstrate interesting and important results by concentrating our work on superconducting structures with magnetic materials. An important place of joint studies is occupied by topological and chaotic phenomena in different types of Josephson structures.

It is well known that at ferromagnetic resonance (FMR), spin wave can be excited by a microwave magnetic field, when all the elementary spins precess perfectly in phase [30]. The coupling between the Josephson phase and a spin wave was studied in the series of papers [27, 31–36]. In the SFS Josephson junction, we have discovered specific ladder structures that appear under the action of a circularly polarized magnetic field [16]. In [17], we have shown that the ferromagnetic resonance linewidth and resonant frequency strongly depend on the ratio of the Josephson and magnetic energies. We have also shown that for SFS junctions on a topological insulator, the easy axis of the ferromagnet splits. Such a splitting can lead to the stabilization of an unconventional fourfold degenerate ferromagnetic state [18].

Other interesting results were obtained in the framework of our collaboration by studing manifestation of Kapitza pendulum features in SFS junctions. As is known, Kapitza's pioneering work [37] initiated the field of vibrational mechanics, and his method was used to describe periodic processes in a variety of different physical systems, like atomic physics [38–41], plasma physics, optics [42], condensed matter physics, biophysics [43] and cybernetical physics (see [44–49]). In nonlinear control theory the Kapitza pendulum is used as an example of a parametric oscillator that demonstrates the concept of "dynamic stabilization". In our collaboration work we demonstrate that the Kapitza pendulum can be considered as a mechanical analog of SFS junction, if we take into account the effective field due to supercurrent and quasiparticle current [19]. In addition, in [20], analytical formulas are obtained that determine the stable positions of the magnetic moment both with and without external periodic excitation, when the Josephson frequency is greater than the frequency of the ferromagnetic resonance. Moreover, we demonstrated the effect of external periodic excitation on the total reorientation voltage and showed that it follows the Bessel function. On the other hand, for the Josephson frequency near the ferromagnetic resonance, we demonstrate a reorientation of the easy axis of the ferromagnet associated with a change in the dynamics of the magnetization [21].

Our recent work demonstrates a unique perspective for the control and manipulation of magnetic moment in hybrid superconducting systems and the appearance of locking step in magnetization due to external electric field. This opens up the possibility to manipulate the locking steps by superconducting current [22].

Thus, our collaboration makes it possible to jointly solve urgent problems of modern superconducting electronics and spintronics. We believe that it will create the basis for their development in both countries.

2. PHYSICAL PHENOMENA IN THE SYSTEM OF COUPLED JOSEPHSON JUNCTIONS

The intrinsic Josephson effect, which consists in the tunneling of Cooper pairs between adjacent superconducting CuO_2 layers inside highly anisotropic high-temperature superconductors (HTSC), gives the background to consider HTSC as a system of coupled Josephson junctions (JJs) [50, 51]. This effect is a part of many theories of HTSC and is important for determining the IV characteristic of HTSC-based tunnel structures and the properties of the vortex lattice in these materials. At present, intrinsic tunneling is not only an interesting subject and a powerful tool for studying the nature of HTSCs, transport along a stack of superconducting layers and the physics of vortices, but it finds applications in different fields of superconducting electronics.

Observation of sufficiently powerful coherent radiation from the stack of intrinsic Josephson junctions (IJJs) presents broad prospects for various applications [52]. The system of coupled JJs is a model for studying the physical properties of IJJs in HTSCs, its nonlinear features, nonequilibrium phenomena and a variety of resonant properties.

The discovery of the intrinsic Josephson effect created a new line of research in superconductivity and solid state physics. The different types of coupling between junctions that occurs in different Josephson structures determine the variety of IV characteristics observed in HTSCs. The question of the relationship between IJJs in HTSCs, its nature and magnitude remains one of the fundamental problems of modern research. Particularly, the value of the capacitive coupling parameter is different in different HTSCs and organic superconductors, i.e., it is actually tunable in these systems. Therefore, it is of great interest to systematically study the dynamics of the system, with an emphasis on the dependence of the phase dynamics on the value of the coupling parameter, from weak to strong coupling. A detailed comparison of the calculated and experimental IV characteristics of IJJs in HTSCs is still missing.

Coherent electromagnetic radiation from the system of IJJs in the terahertz frequency range, which significantly exceeds previous results in terms of power, presents wide opportunities for various applications. The main directions of research here are related to the elucidation of the mechanism of this radiation and the search for new possibilities for further increasing its power, which according to the latest data, is about 1 mW at a frequency of 0.5 THz using several series-connected systems of IJJs.

The electrical and magnetic properties of IJJs in HTSCs are strongly nonlinear and determined by their phase dynamics. The phase dynamics of IJJs is used to explain the mechanism of coherent terahertz radiation [52–57]. One of the most spectacular indications of the Josephson effect in HTSCs is locking of the Josephson oscillations of each junction to the frequency of external electromagnetic radiation. This locking leads to appearance of steps in the IV characteristic (IVC) at quantized voltages $V_n = n\hbar\omega/2e$, called Shapiro steps (SS), where ω is the frequency of the applied signal, and n is integer number [58, 59].

3. MANIFESTATION OF PARAMETRIC RESONANCE IN A SYSTEM OF COUPLED JOSEPHSON JUNCTIONS

As is known, the one-dimensional models with coupling between junctions capture the main features of real IJJs, like hysteresis and branching of the IVC, and help to understand their physics. An interesting and very important fact is that the 1D models can also be used to describe the properties of a parallel array of Josephson junctions, which is often considered as a model for long Josephson junctions. In particular, the experimental data have demonstrated a series of resonances in the IVC. They were analyzed using the discrete sine-Gordon model and an extension of this model which includes a capacitive interaction between neighboring Josephson junctions [60, 61]. The parametric instabilities of one-dimensional parallel array of N identical Josephson junctions were predicted by theoretical analysis of the discrete sine-Gordon equation (also known as the Frenkel–Kontorova model) and observed experimentally in Ref. [62]. In particular, the novel resonant steps related to the parametric instability were found in the IV characteristic of discrete Josephson ring.

One of the first scientific results of the joint study with fellow scholar Mahmoud Gaafar was an observation of hysteresis in the IV characteristic for a system of coupled JJs due to parametric resonance [1]. We found that, in contrast to the McCumber–Stewart hysteresis, the value of the resonant hysteresis is inversely proportional to the McCumber parameter, and depends on the value of the coupling parameter and boundary conditions. The study of the time dependence of the electric charge in the superconducting layers shows that the origin of the resonant hysteresis is associated with different phase dynamics in the process of decreasing and increasing bias current in the resonant region. In Fig. 4, which shows time dependence of the charge along with the IV characteristic for 9 JJs in the stack, we see that the charge in the superconducting layer with increasing current (Fig. 4, *b*)

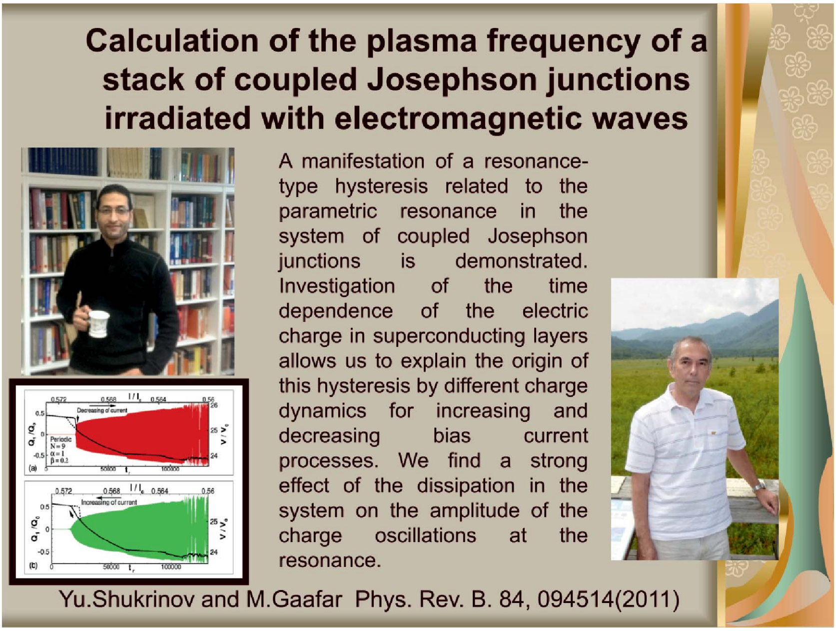

Fig. 4. The paper [1] reported on the discovery of a new type of hysteresis in a system of coupled Josephson junctions

disappears at a different value, in contrast to the case of decreasing current (Fig. 4, *a*).

An interesting feature of IJJs is a longitudinal plasma wave (LPW) propagating along the c axis [63, 64]. It follows from the fact that the thickness of the S layers is comparable to the Debye screening length r_D, and so there is no complete screening of electric charge in a separate S layer. The frequency of Josephson oscillations ω_J is determined by the voltage in the junction, and at $\omega_J = 2\omega_{\mathrm{LPW}}$ (ω_{LPW} is LPW frequency) the parametric resonance (PR) is realized: the Josephson oscillations excite the LPW by their periodical actions. IV characteristics of IJJs demonstrate the multiple branch structure [1, 65–67] and have a breakpoint (BP) and some breakpoint region (BPR) in the outermost branch before transition to the inner branch.

The next interesting results with fellow scholar Mahmoud Gaafar were found in the study of phase dynamics in HTSCs in the presence of external electromagnetic radiation [4]. We have observed the appearance of an electric charge in superconducting layers in the range of the bias current corresponding to the Shapiro step. The possibility of changing the length of a longitudinal plasma wave under external electromagnetic radiation at parametric resonance was demonstrated. It was shown that the double resonance of Josephson oscillations with external radiation and plasma oscillations leads to an additional parametric resonance and a Bessel dependence of the Shapiro step width on the frequency and amplitude of the external radiation.

One of the most important results related to the effect of external radiation is presented in Fig. 5. It shows three IV characteristics of a stack with 10 coupled JJs: without radiation (curve *1*); under external radiation with frequency $\omega = 2$ and amplitude $A = 0.1$ (curve *2*); with amplitude $A = 0.5$ at the same frequency (curve *3*). We call the parametric resonance in the absence of external radiation a fundamental parametric resonance (fPR). As the radiation amplitude increases, the region of parametric resonance shifts upward along the voltage axis. According to the external radiation frequency $\omega = 2$, the first Shapiro step is observed at a voltage $V = \omega_J N = 20$. The dotted line emphasizes this fact. The filled arrows indicate the position of fPR. Hollow arrows indicate the region of parametric resonance due to radiation (rrPR).

We have demonstrated the influence of the external radiation amplitude on the length of the longitudinal plasma wave in the fPR region. A change of the longitudinal plasma wavelength that arises at parametric resonance was demonstrated. This effect at $\omega = 2$ is shown in Figs. 5, *a*–*d*. In the absence of radiation, there is no charge in the S layers before resonance (Fig. 5, *a*). At resonance (Fig. 5, *b*) a longitudinal plasma wave with wave number $k = \pi$ ($\lambda = 2d$) is formed. At $A = 0.14$ the length of the longitudinal plasma wave changes. Figure 5, *c* illustrates the charge distribution along the stack forming the wave with $\lambda = 10d$. At $A = 0.23$, the length of the longitudinal plasma wave becomes equal to $\lambda = 5d$, as shown in Fig. 5, *d*.

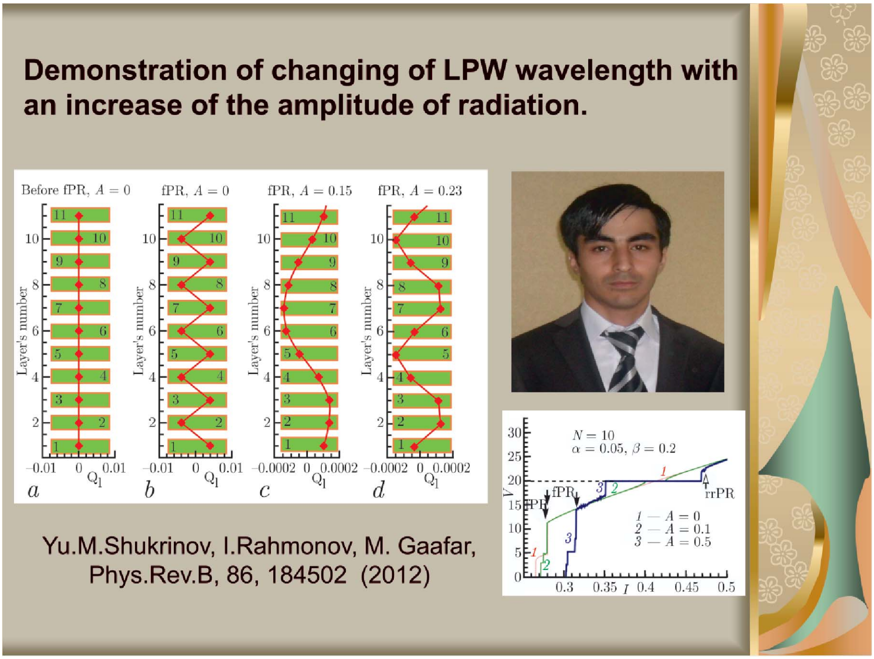

Fig. 5. In Ref. [4] the possibility to change the wavelength of longitudinal plasma wave by external radiation is demonstrated. The figure presented an example of such variation with increasing amplitude of the external radiation. It shows IV characteristics of a stack with 10 coupled JJs without and with radiation at different amplitude

The results of a detailed study for $\omega = 2$ in the amplitude range $(0, 0.35)$ are summarized in Fig. 6, *a*. Thus, in the case of fundamental parametric resonance, we have registered the following changes in the length of the longitudinal plasma wave: $\lambda = 2d \Rrightarrow \lambda = 10d \Rrightarrow \lambda = 5d \Rrightarrow \lambda = 3d \Rrightarrow \lambda = 2d$. An increase in A also changes the length of the longitudinal plasma wave in the region of parametric resonance due to the radiation: $\lambda = 10d \Rrightarrow \lambda = 5d \Rrightarrow \lambda = 3d$ as A increases from zero to 0.35. As noted above, external radiation can also lead to the appearance of an electric charge in the superconducting layers in the range of the bias current corresponding to the Shapiro step in the IV characteristic. In the region of double resonance, when the frequency of Josephson oscillations coincides with the frequencies of external radiation and longitudinal plasma oscillations, an additional resonance arises in the system with a specific dependence of the width of the Shapiro steps on the amplitude of external radiation. Figure 6, *b* demonstrates the appearance of the charge in the layer at $\omega = 1.155$. Inset (3) shows that the charge oscillations correspond to the π mode of the longitudinal plasma wave. Charge oscillations in the region of fundamental parametric resonance also

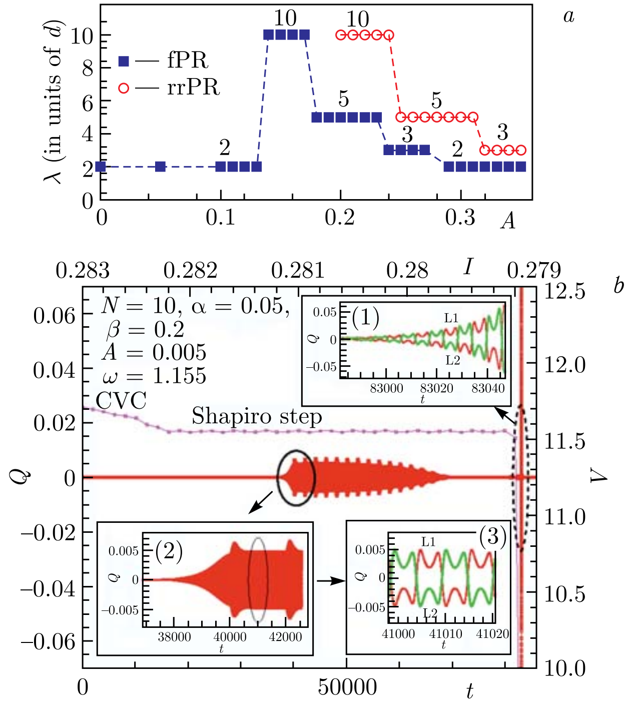

Fig. 6. Change in the length of the longitudinal plasma wave at fundamental parametric resonance and demonstration of the appearance of a charge in the superconducting layer of high-temperature superconductor

correspond to the π mode. However, in principle, there are no restrictions in the creation of a longitudinal plasma wave with a different wave number for other system parameters [4].

4. CHARGE DENSITY WAVES

As we mentioned above, one of the interesting objects in Josephson nanostructures are plasma waves. In a joint stydy with fellow scholar Hazem Abdelhafiz (Fig. 7), we have demonstrated the occurrence of charge density waves along a stack of coupled Josephson junctions in layered superconductors [6]. The transformation of a longitudinal plasma wave into a charge density wave (CDW), as well as transitions between CDWs of various types, was found. The effect of external electromagnetic radiation on the states corresponding to CDWs is fundamentally different from the case of a single JJ. We have shown that the voltage values of the Shapiro steps in the JJ along the stack do not directly reflect the frequency of the external radiation, but correspond to the distribution of rotating and oscillating Josephson junctions in the system. In Fig. 8 we show the distribution of charge along the stack

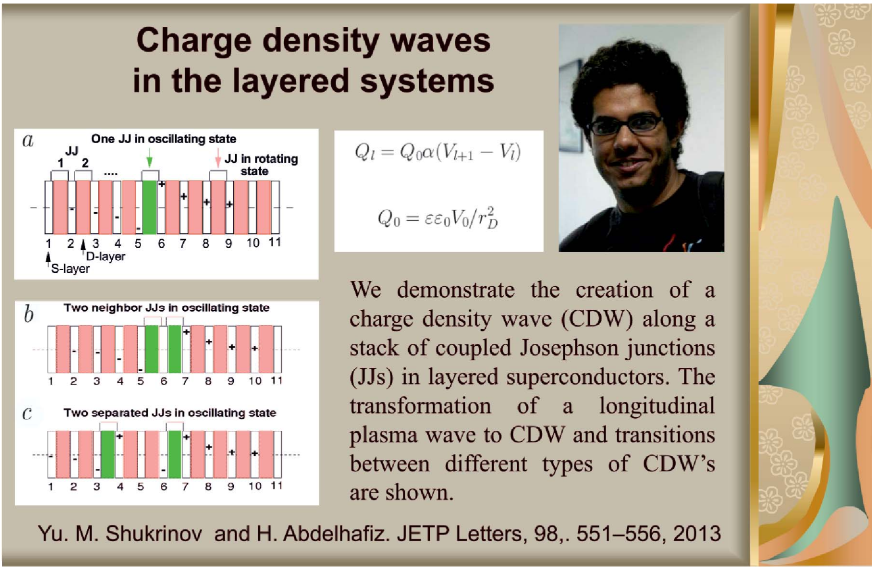

Fig. 7. Demonstration of charge distribution along the stack with rotating and oscillating Josephson junctions

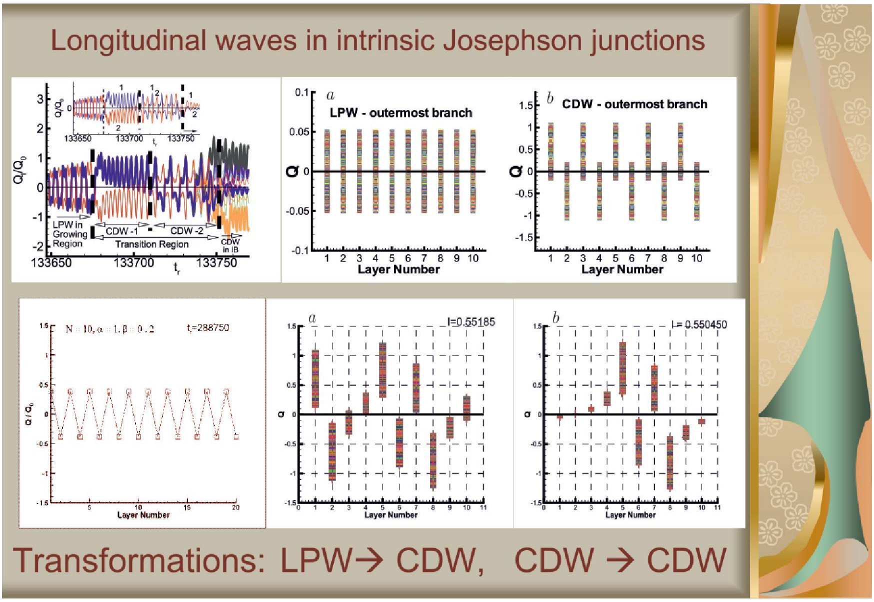

Fig. 8. The paper [6] reported the possibility of mutual transformation of longitudinal plasma (LPW) and charge density waves (CDW) in the stack with 10 coupled Josephson junctions

presented demonstrating the LPWs and CDWs. Our findings demonstrate rich physics of CDWs in IJJs. The transition from the outermost (with all junctions in the rotating state) to the inner branch is caused by the transformation of an LPW to a breathing CDW.

5. INVESTIGATION OF NONEQUILIBRIUM PHENOMENA IN A SYSTEM OF COUPLED JOSEPHSON JUNCTIONS

Nonequilibrium effects in layered materials created by stationary current injection have been studied very intensively in the last years [68–73]. A layered system of $N+1$ superconducting layers forms a stack of Josephson junctions. Since the 0th and Nth layers are in contact with normal metal, their thicknesses d_s^0 and d_s^N are different from the thickness of the other S layers d_s^l inside the stack due to the proximity effect. Actually, a system of IJJs in high-temperature superconductors cannot be in the equilibrium state at any value of the electrical current [63, 74]. The study of nonequilibrium phenomena in a system of coupled Josephson junctions is one of the most complicated sections of the theory of high-temperature superconductivity, the final form of which has not yet been developed. The influence of charge coupling on Josephson plasma oscillations has been stressed in Refs. [63, 71–75]. In the joint work with fellow scholar Majed Nashaat, we have investigated non-stationary and stationary charge imbalance effects in stack of JJs. Precise numerical study was done for the IV characteristic of capacitively coupled Josephson junctions under external radiation. The influence of charge imbalance on the Shapiro step is observed. The Shapiro step shows a finite slope and shift from its canonical value. The shift and slope values depend on the relaxation time of the quasiparticles and the thickness of the superconducting layer. We showed that one can find the relaxation time of the quasiparticles from the slope of the Shapiro step. We showed that the coupling between junctions results in a distribution of the magnitude of the slope along the stack. The IV characteristic also shows a shift of the Shapiro step from its canonical position, which is determined by the frequency of the external radiation. This fact makes ambiguous the interpretation, presented by P. Müller, of the experimentally found shift of the Shapiro step due to the charge imbalance effect [75].

The stationary charge imbalance effect in a system of coupled Josephson junctions was studied in [11] (see Fig. 9). We showed that it leads to a decrease in the Josephson frequency in the junctions of the stack. The formed difference in frequencies leads to a nonuniform switch of junctions along the stack to the Shapiro step regime in the presence of external electromagnetic radiation. On the other hand, such nonuniform switching leads to the appearance of voltage spikes on the IV characteristic of the stack. It was shown that the imbalance of the stationary charge is the reason for the tilt of the Shapiro step due to the difference in nonequilibrium potentials at the edges of the step. The results obtained were compared with Josephson junctions associated with voltage bias and experimental results.

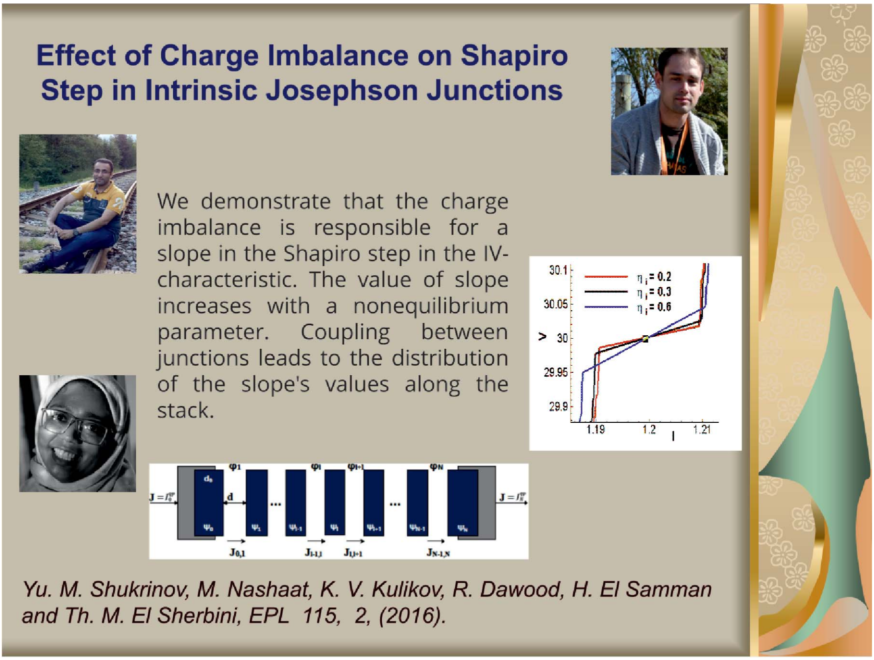

Fig. 9. Demonstration of the Shapiro step slope at different nonequilibrium parameters described in Ref. [11] for a system of five coupled JJs under external electromagnetic radiation of amplitude $A = 0.1$ and frequency $\omega = 0.5$

6. MANIFESTATIONS OF MAJORANA STATES IN THE IV CHARACTERISTIC OF THE JJ AND IN SQUID CHARACTERISTICS

Interesting properties of a Josephson junction with 2π- and 4π-periodic superconducting current components are found in the joint work with Radwa Dawood. In the range of low voltages, such junctions has a 4π periodicity of the phase difference at a Majorana current amplitude much smaller than the Josephson current, which makes it possible to observe Josephson current oscillations with a fractional period at small dissipations $\beta < 1$ in the hysteresis region. The effect of the 4π-periodic Majorana component of the current also manifests itself in a change in the sequence of steps of the ladder structure that appears in the IV characteristic of the JJ. We have demonstrated that the range of amplitudes of external electromagnetic radiation is determined, in which the manifestation of the fractional Josephson effect on the IV characteristic is most significant.

Figure 10 shows the ladder structures appearing in the IV characteristic for various junctions. The ladder structure for a conventional Josephson junction with amplitude $A = 0.8$ and frequency $\omega = 0.5$ is shown in Fig. 10, *a*.

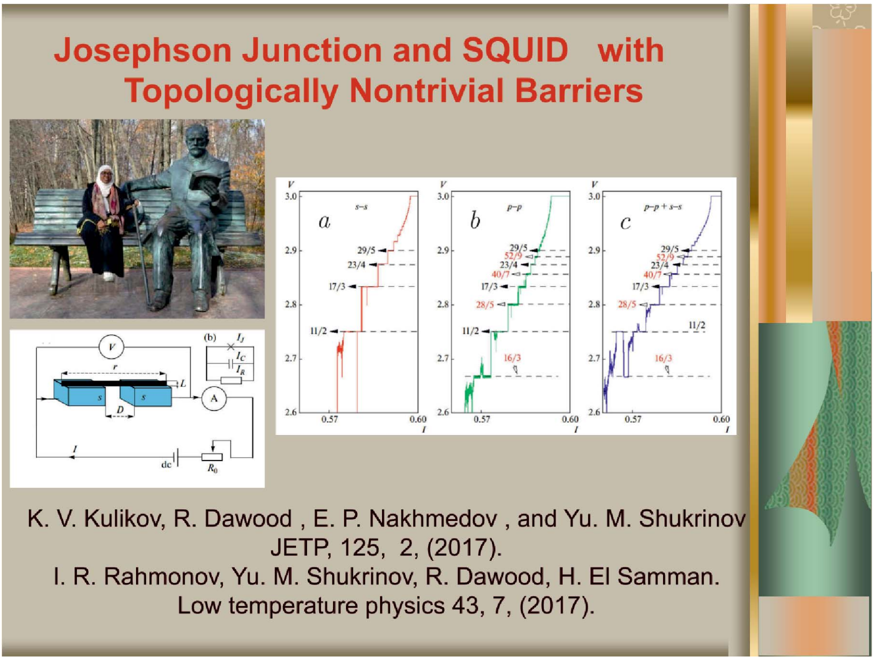

Fig. 10. Ladder structures in the IV curves of Josephson junction with semiconductor wire under external radiation with frequency $\omega = 0.5$ found in Ref. [14]

The order of formation of steps in such a structure can be described by continued fraction $V = (N + 1/n)\omega$. Conversely, the sequence appearing in a junction sustaining the Majorana bound states is described by relation $V = (N + 2/n)\omega$ (see Fig. 10, *b*). As noted in [76], the changes appearing in ladder structures can be treated as a manifestation of Majorana states in the junction. It can be seen from Fig. 10, *c* that a junction with two superconducting current components demonstrates a sequence of steps, which is described by continued fraction $V = (N + 2/n)\omega$. Consequently, in spite of the small value of the Majorana component amplitude $\gamma = 0.316$, the ladder structure of such a junction reflects the 4π periodicity. We came to the conclusion that the sequence of steps in the IV characteristic is a universal method for detecting the Majorana fermions in the capacitively shunted Josephson junction.

In the case of superconductor quantum interference device (SQUID) with nontrivial barriers, the corresponding resonant branch has a voltage shift, which can also be used to detect Majorana fermions. The results of a numerical study of the phase dynamics of a DC SQUID with topologically trivial and nontrivial barriers are presented. In our calculations, we take into account two components of the superconducting current, currents of Cooper pairs ($2p$-periodic) and Majorana fermions ($4p$-periodic). It was shown that in the case of a two-component superconducting current, there is a periodicity in the dependence of the reverse current on the magnetic field displaced by Cooper pairs and the Majorana fermion ratio with respect to the magnetic field. This effect makes it possible to experimentally determine the ratio of the currents of Cooper pairs and Majorana fermions.

7. RESONANCE PHENOMENA IN SHUNTED JOSEPHSON STRUCTURES

Shunting of Josephson structures leads to the appearance of additional resonances and opens up new possibilities in controlling their properties. In a joint study [12] with Dr. A. S. Abouhaswa, we have investigated the current–voltage characteristics and phase dynamics of intrinsic Josephson junctions shunted with inductive and capacitive elements (Fig. 11). Double and triple resonances are demonstrated and their influence on the appearance of an electric charge in superconducting layers was shown. For a larger number of junctions, shunting causes the appearance of a charge in the states corresponding to the upper and resonant branches of the current–voltage characteristic. A transformation of a traveling wave into a standing longitudinal plasma wave was observed in the system.

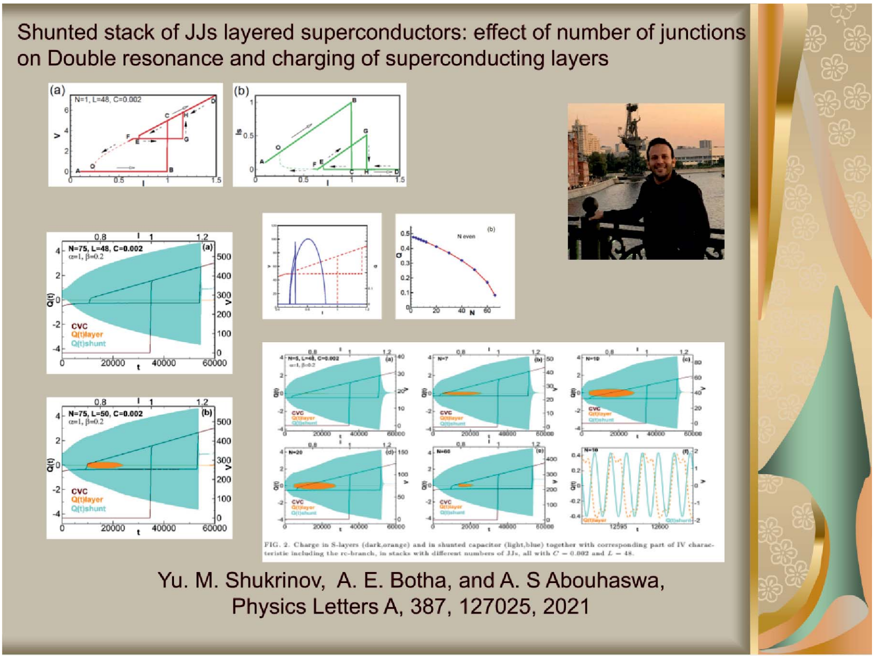

Fig. 11. Demonstration of electric charge appearance in superconducting layers in LC-shunted stack of coupled Josephson junctions

8. INVESTIGATION OF SUPERCONDUCTOR–FERROMAGNET–SUPERCONDUCTOR STRUCTURES

Josephson junction with ferromagnet layer (F) is widely considered to be the place where spintronics and superconductivity fields interact [77]. In these junctions the supercurrent induces magnetization dynamics due to the coupling between the Josephson and magnetic subsystems. The possibility of achieving electric control over the magnetic properties of the magnet via Josephson current and its counterpart, i.e., achieving magnetic control over Josephson current, has recently attracted a lot of attention [23, 77–83]. The current–phase relation in SFS junctions is very sensitive to the mutual orientation of the magnetizations in the F layer [84, 85].

9. STUDY OF SPINTRONIC EFFECTS IN ANOMALOUS JOSEPHSON JUNCTIONS

In a joint work with fellow scholar Majed Nashaat we have started to investigate the spintronic effects in several superconducting Josephson junctions coupled to ferromagnet. We studied the manifestation of different types of coupling between magnetization and Josephson phase on the appearance of subharmonic steps which form the devil's staircase (DS) structure [86–89] in the IV characteristic of overdamped Josephson junction. The DS structure is a universal phenomenon and appears in a wide variety of different systems, including infinite spin chains with long-range interactions [90], frustrated quasi-two-dimensional spin-dimer systems in magnetic fields [91], and even in the fractional quantum Hall effect [92].

The coupling between the superconducting current and magnetization in the SFS Josephson junction in a circularly polarized magnetic field is studied. Particularly, we demonstrate the manifestation of ferromagnetic resonance in the frequency dependence of the magnetization amplitude and the average critical current density. The subharmonic ladder structures were found in the IV characteristic for the first time. This is due to the influence of magnetization dynamics on the phase difference in the Josephson junction, which follow the continued fraction algorithm. The dynamics of the system was described by the generalized resistively and capacitively shunted Josephson junctions model together with the Landau–Lifshitz–Gilbert equation. Analytical formula is demonstrated for the appearance conditions of fractional steps in the IV characteristic of the SFS Josephson junction. We consider that the subharmonic ladder structures can be used in various fields of superconducting spintronics particularly for detecting Majorana states in Josephson nanostructures. In Ref. [93], the authors reported the experimental observation of half-integer Shapiro steps in the strong ferromagnetic Josephson junction (Nb–NiFe–Nb) by investigating the current–phase relation under radio-frequency microwave excitation.

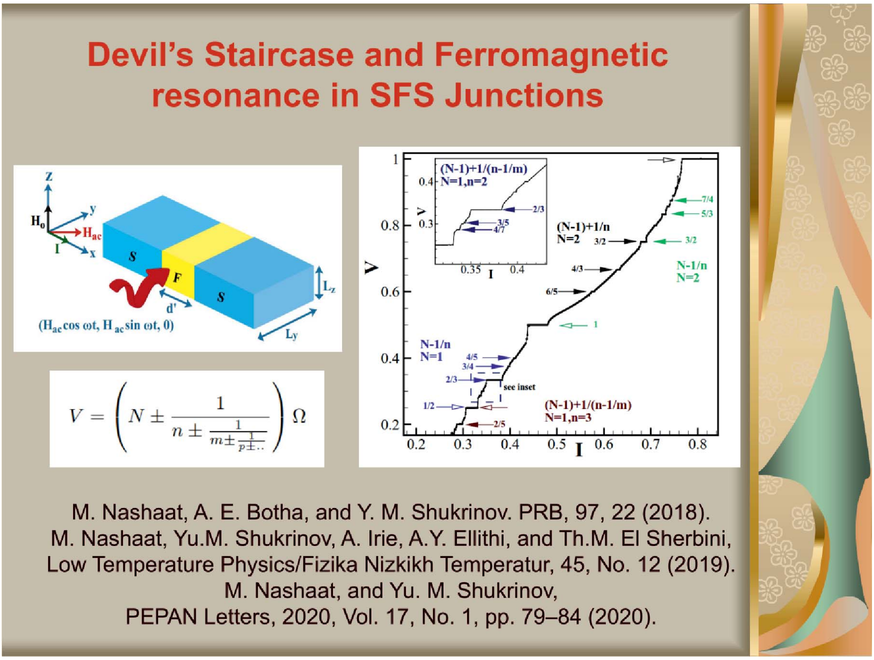

Fig. 12. Geometry of the SFS Josephson junction in uniform magnetic field H_0 and circularly polarized magnetic field H_{ac} considered in Ref. [16]. The IV characteristic demonstrates staircase structures which follow continued fraction formula

In our study we consider two superconductors separated by a ferromagnetic layer d thick, as shown in Fig. 12. The junction area is $L_y L_z$ and the bias current flows in the x direction. A uniaxial constant magnetic field H_0 is applied in the z direction, and a circularly polarized magnetic field is applied in the xy plane $\mathbf{H}_{ac} = (H_{ac} \cos \Omega t, H_{ac} \sin \Omega t, 0)$ with amplitude H_{ac} and frequency ω. The coupling between the Josephson phase and magnetization in SFS junctions plays an important role in the dynamics of this system. In the presence of this coupling, the manifestation of ferromagnetic resonance in the dynamics of magnetization and critical current is shown in both nonlinear and linearized schemes. The width of the ferromagnetic resonance curve and the position of the resonant frequency are strongly affected by the ratio of the Josephson and magnetic energies.

In Fig. 13, a the IV characteristic shows current steps at $V = m\Omega_0$, where Ω_0 is the ferromagnetic resonance frequency and m is integer, as well as some fractional steps. In the case of regular JJs (see dashed line in Fig. 13, a), the first Shapiro's step is wider than the second one. In this case, we see that the width of the first step is much less than the width of the second one. So, the width of the harmonics is different for even and odd m: large steps for even m and small steps for odd m. In our case, taking into account the

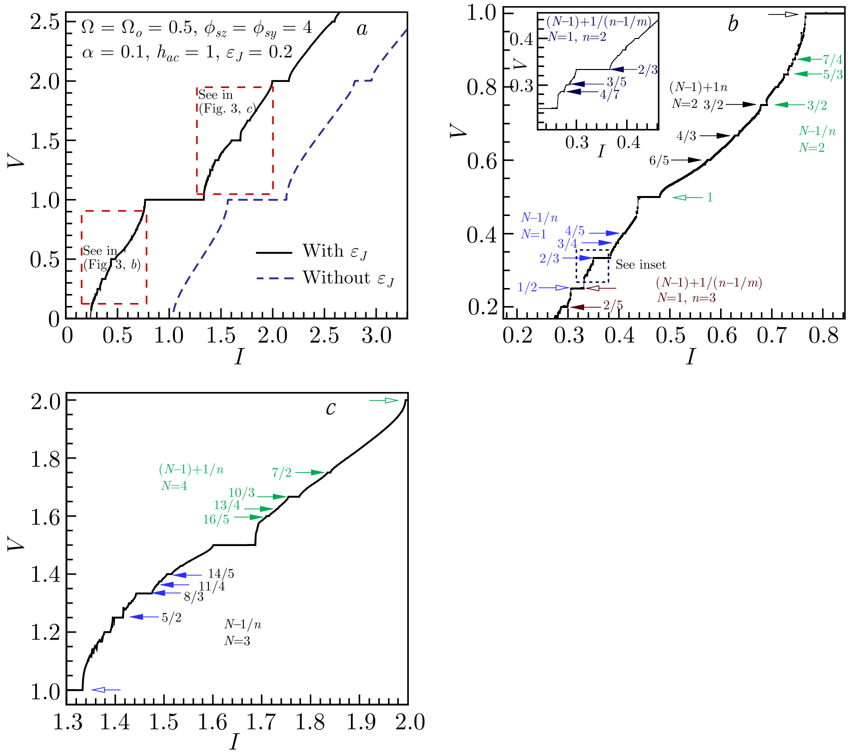

Fig. 13. Subharmonic ladder structures in various parts of the IV characteristic of the SFS junction at ferromagnetic resonance

Josephson energy in the effective field, we obtained additional Shapiro steps with odd and fractional values of m, as can be seen in Fig. 13, a. The structure of these fractional steps can be clarified by analyzing their position on the voltage scale using an algorithm based on the generalized continued fraction formula [87, 88]:

$$V = \left(N \pm \cfrac{1}{n \pm \cfrac{1}{m \pm \cfrac{1}{p \pm ..}}} \right) \Omega, \tag{1}$$

where N, n, m, p, ... are natural numbers. We demonstrate that the origin of the fractional steps are due to synchronization of Josephson oscillations and magnetic precession. Figures 13, b and 13, c show enlarged portions of the IV characteristics shown in Fig. 13, a. There are fractional Shapiro steps between $V = 0$ and $V = 0.5$, which can be described by second-level continued fractions [87] $(N-1)+1/n$ and $N-1/n$ with $N=1$ in both cases (see Fig. 13, *b*). In addition, two third-level continued fractions $(N-1)+1/(n-1/m)$ with $N=1$, $n=2$ (shown in the inset) and $n=3$ appear. Steps between $V=0.5$ and $V=1$ follow second-level continued fractions $(N-1)+1/n$ and $N-1/n$ with $N=2$ in both cases. Figure 13, *c* clearly shows the manifestation of the second-level continued fraction $N-1/n$ with $N=3$ and $(N-1)+1/n$ with $N=4$ between $V=1$ and $V=2$.

The strong dependence of the Josephson energy on the orientation of the magnetization in Josephson junctions with ferromagnetic layers and spin–orbit coupling opens up the possibility of controlling the magnetization by the Josephson current or the Josephson phase. We have studied the perspectives of magnetization control in SFS Josephson junctions on the surface of a three-dimensional topological insulator containing Dirac quasiparticles (see Fig. 14) [18]. Because of the spin–momentum synchronization of these Dirac quasiparticles, the Josephson current–phase relation strongly depends on the orientation of the magnetization. It can lead to splitting of the easy axis of the ferromagnet in the voltage-bias case. Such a splitting can lead to the stabilization of an unconventional fourfold degenerate ferromagnetic state.

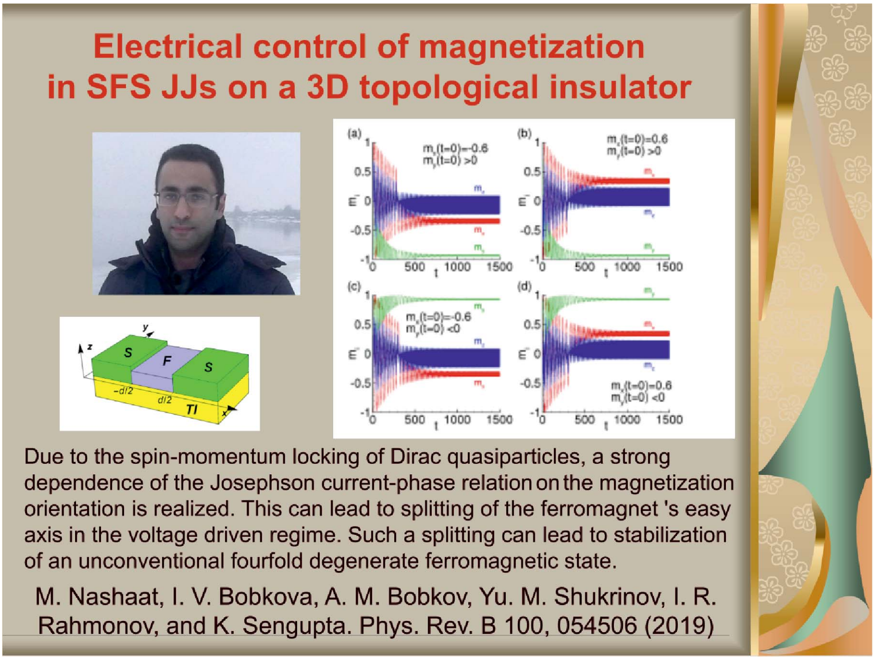

Fig. 14. Sketch of the system considered in Ref. [18]. Superconducting leads and a ferromagnetic inter-layer are deposited on top of the topological insulator. Figure shows the time evolution of the magnetization starting from different initial conditions, corresponding to four possible stable states

10. STUDIES OF THE JOSEPHSON JUNCTION/NANOMAGNET SYSTEMS

Another approach of coupling between magnetic and superconductor systems was presented in Refs. [94, 95], where the electromagnetic coupling of a nanomagnet with a weak superconducting link was considered. The authors investigated the reversal of single domain magnetic particle by an ac field. The superconducting current of a JJ coupled to nanomagnet driven by a time-dependent magnetic field was investigated in detail in Ref. [96]. The existence of Shapiro-like steps in the IV characteristic of the JJ subjected to a voltage bias for a constant or periodically varying magnetic field was demonstrated. And the effect of rotation of the magnetic field and the external ac drive on these steps was explored. As we mentioned above, P. L. Kapitsa showed the possibility of changing the equilibrium state of the pendulum by means of rapid oscillations of its suspension point. In Ref. [97], the authors realized experimentally the Kapitza pendulum at the micrometre scale using a colloidal particle suspended in water and trapped by optical tweezers. Moreover, it was analytically and experimentally demonstrated that if the oscillation direction of the pendulum suspension point changes over time, so does the pendulum

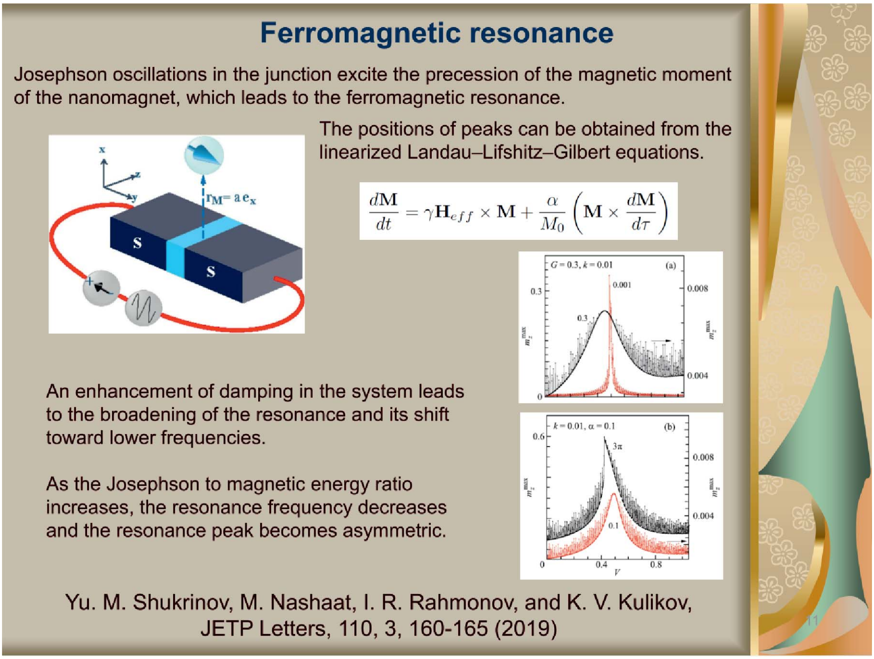

Fig. 15. Manifestation of the ferromagnetic resonance in the dependence $m_z^{\max}(V)$ at different values of Gilbert damping $\alpha = 0.001, 0.3$ and two different ratios of Josephson energy to the energy of the nanomagnet ($G = 0.1$ and 3π)

equilibrium point and active damping control can take place. The Kapitsa quantum pendulum can be stabilized in the form of quantum states near a local minimum of the effective potential energy [98].

We have demonstrated the manifestation of the Kapitza pendulum properties in the "Josephson junction and nanomagnet" system, where the direction of the easy axis of the nanomagnet [19] changes under the action of an oscillating superconducting current. Although the magnetic field induced by the superconducting current in the Josephson junction is very weak, the applied voltage can generate nonlinear dynamics of the nanomagnet, which leads to a number of interesting phenomena. We have shown that ferromagnetic resonance can arise when the frequency of Josephson oscillations (Ω_J) is equal to the natural frequency of the magnetic system (Ω_F). We have demonstrated that the easy axis of a nanomagnet is reoriented by an increase in the ratio of the Josephson energy to the magnetic one, coupling parameter and the magnetization, and the frequency of the Josephson oscillations.

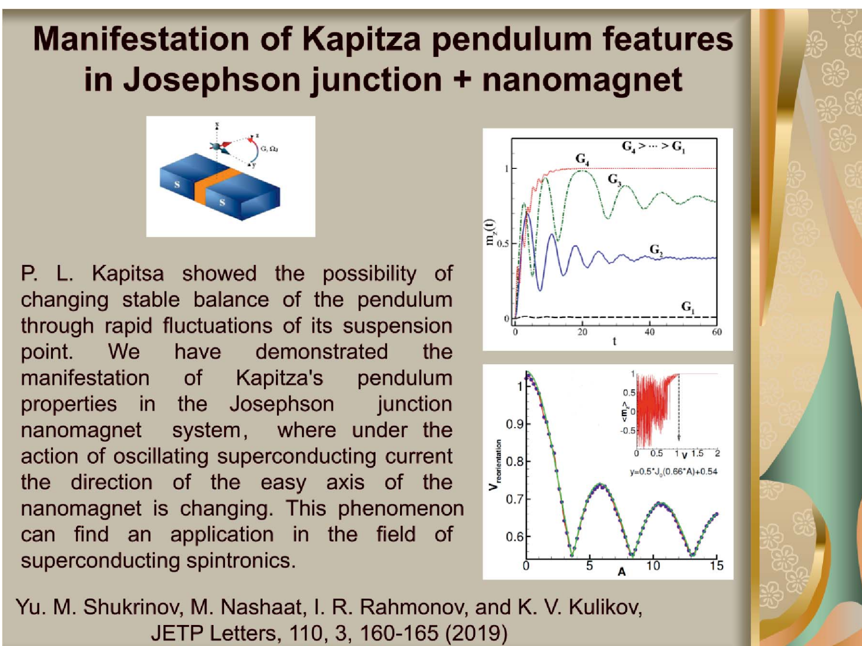

Fig. 16. Dynamics of the magnetization m_z of the nanomagnet as a function of Josephson to the magnetic energies ratio G, demonstrating its reorientation. The second figure shows the reorientation voltage as a function of the amplitude (A) of the external AC drive. The inset demonstrates the averaged component of the magnetization m_z of the nanomagnet as a function of the DC voltage

In our collaboration with Majed Nashaat, we have simulated the magnetic precession of a nanomagnet which occurs due to interaction with a superconducting current. We have considered a nanomagnet with a magnetization $\mathbf{M} = (M_x, M_y, M_z)$ located at a distance $\mathbf{r}_M = a\mathbf{e}_x$ from the center of a short Josephson junction of length l, as shown in Fig. 15. The interaction between two systems, superconducting and magnetic, was considered purely electromagnetic. The magnetic field of the nanomagnet changes the Josephson current flowing through the junction, while the magnetic flux generated by the Josephson junction affects the magnetization of the nanomagnet [94]. Figure 15 shows the calculated maximum amplitude as a function of the voltage V of the Josephson junction at $\Omega_F = 0.5$ and two Gilbert damping parameters $\alpha = 0.001$ and 0.3. In the chosen normalization $V = \Omega_J$, so the ferromagnetic resonance peak is observed at a voltage corresponding to the Josephson frequency $\Omega_J = 0.5$. The increase of Gilbert damping in the system leads to a broadening of the resonance and its shift towards lower frequencies, which can be seen in Fig. 15 for $\alpha = 0.3$. The positions of the peaks at weak damping are in good agreement with the frequencies following from the analytical formulas obtained by the linearization of the Landau–Lifshitz–Gilbert equations.

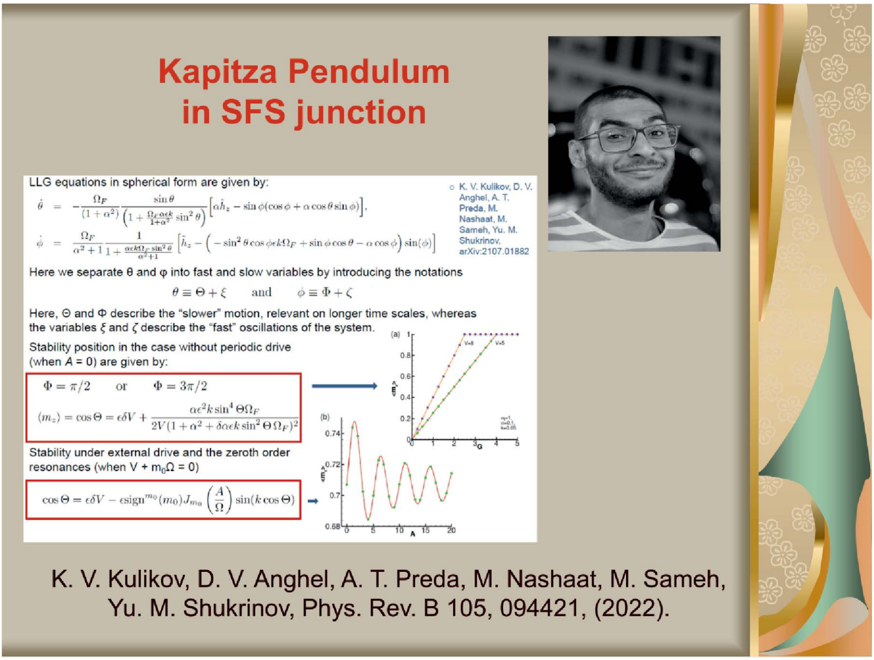

Fig. 17. In Ref. [20] analytical description of Kapitza pendulum manifestation of SFS Josephson junction was presented

The voltage applied to the Josephson junction generates a high-frequency magnetic field, which reorients the magnetization of the nanomagnet. Figure 16 shows its reorientation depending on the DC bias voltage, i.e., there is a manifestation of the Kapitza pendulum features. When V exceeds a certain value, a complete reorientation of the magnetization occurs.

Important results were obtained by analytical investigation of JJ nanomagnet system in collaboration with another fellow scholar Muhammad Sameh. We have obtained simple analytical formulas for the stable position of the magnetization both with and without external periodic drive (see Fig. 17). The effect of an external periodic drive on the total reorientation voltage is also demonstrated. Another important result was the demonstration of the magnetization reversal of the nanomagnet by a current pulse [19], which opens up new prospects for the application of this system in superconducting spintronics. Chaotic features in such a system were also studied, and the paper is submitted now for publication in "Chaos".

11. LOCKING OF MAGNETIZATION AND JOSEPHSON OSCILLATIONS AT FERROMAGNETIC RESONANCE IN φ_0 JUNCTION UNDER ELECTROMAGNETIC RADIATION

Recently our collaboration has concentrated on the locking phenomenon in φ_0 junction. In collaboration with Ms. Sara Abdelmoneim, who visited BLTP for practice, we have obtained very interesting results on locking phenomena in φ_0 JJ under external electromagnetic radiation (Fig. 18). We have demonstrated the locking of magnetization and Josephson oscillations at ferromagnetic resonance. We showed that the locking of magnetic precession happens through the locking of the Josephson oscillations. This leads to a step in the dependence of the magnetization on the bias current. The step's position is determined by the radiation frequency and the shape of the resonance curve. In junctions with a strong spin–orbit coupling, states with negative differential resistance appear in the IV characteristic, resulting in an additional locking step. We have shown that the corresponding oscillations have the same frequency as the oscillations at the first step, but they have a different amplitude and different dependence on the radiation frequency. We found an important result demonstrating the possibility to control not only the frequency, but also the amplitude of the magnetic precession in the locking region. It opens up unique perspectives for the control and manipulation of magnetic moment in hybrid superconducting systems. The important point is the possibility to manipulate the locking steps by superconducting current. We consider it will find a wide application in future like Shapiro steps or even more because of its magnetic nature. The corresponding paper is published in "Physical Review B" [22].

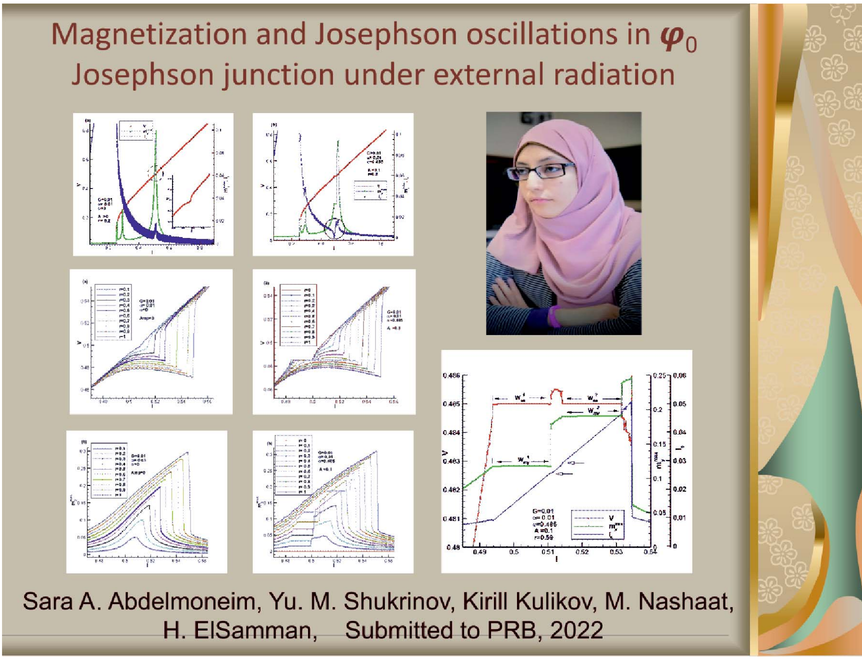

Fig. 18. Demonstration of magnetic precession locking through the locking of the Josephson oscillations in the ferromagnetic resonance region

12. CONCLUSIONS

At present, a basis has been created for conducting joint research on modeling various types of superconducting nanostructures, in particular, systems of coupled Josephson junctions with various types of shunting, which makes it possible to simulate phase dynamics, current–voltage characteristics and various resonance phenomena. Interesting and important results have been obtained in the study of superconducting structures with ferromagnetic materials, which make it possible to control the magnetic properties by superconducting current. An important place is occupied by joint studies of topological and chaotic phenomena in Josephson structures (Fig. 19).

A significant event took place in 2021 — Egypt became a full member of JINR (Fig. 20). We believe that this event will bring JINR's cooperation with scientific institutions in Egypt in the field of theoretical research of superconducting Josephson nanostructures to a new level, creating a basis for the development of applied research and various applications of superconducting electronics and spintronics in both countries.

13. ACKNOWLEDGMENTS

We would like to thank Dmitry V. Kamanin, Head of the International Cooperation Department in JINR, Mahmoud Sakr, President of the

Fig. 19. Members of JINR–Egypt collaboration in the field of superconducting electronics and spintronics

Fig. 20. A significant event took place in 2021 — Egypt became a full member of JINR

Academy of Scientific Research and Technology in Egypt (ASRT), Stanislav Z. Pakuliak, Director of the JINR University Center, and Wael Badawy, the national Egyptian representative in JINR for organizing and supporting our collaboration, as well as Anatoly E. Vasiliev, Yulia Polyakova, Elena Karpova, Yulia Rybachuk, and Elizabeth Pasca for travel and events organization.

We thank Dmitri I. Kazakov, Director of BLTP JINR, Vladimir A. Osipov, Head of the Condensed Matter Theory Department in BLTP, Tarek Hussein, Khaled Higab, Hussien El Samann, Walaa Seif, Tharwat El-Sherbini and Ali Elithi for supporting joint projects and visits of Egyptian students.

We thank the administrative staff of JINR and ASRT for the preparation of documents needed for realization of our collaboration.

14. CONFERENCE PUBLICATIONS

• *Nashaat M., Sameh M., Botha A. E., Kulikov K. V., Shukrinov Yu. M.* Bifurcations in Josephson junction coupled to the nanomagnet // XXVI Symp. “Nanophysics and Nanoelectronics”, March 14–17, 2022.

• *Shukrinov Yu. M., Rahmonov I. R., Botha A. E., Nashaat M., Abdel Moneim S. A., El Samman H.* Peculiarities of Anomalous Josephson Effect in Superconducting Nanostructures // 14th Workshop on Low Temperature Electronics, 2021.

• *Kulikov K. V., Nashaat M., Sameh M., Sengupta K., Shukrinov Yu. M.* // Intern. Symp. “Nanophysics and Nanoelectronics”, 2020.

• *Nahsaat M., Shukrinov Yu. M., Botha A. E., Plecenik A., Rahmonov I. R., Kulikov K.* Numerical Simulations for Spintronic Effects in Josephson Junctions Coupled to Ferromagnet // Intern. Conf. “Mathematical Modeling and Computational Physics”, 2019, Slovakia.

• *Kulikov K. V., Medvedeva S. Yu., Dawood R., Shukrinov Yu. M.* *P–P* Josephson Junction in the Presence of Localized Majorana Bound States // Proc. of the XX Intern. Symp. “Nanophysics and Nanoelectronics”. 2016. V. 1. P. 71.

• *Rahmonov I. R., Shukrinov Yu. M., Dawood R., Nashaat M.* Phase dynamics of superconducting quantum devices with topologically nontrivial barriers // Proc. of the XX Intern. Symp. “Nanophysics and Nanoelectronics”. 2016. V. 1. P. 103.

• *Benecha E. M., Dawood R., Rahmonov I. R., Shukrinov Yu. M., Botha A. E.* Simulations of DC SQUIDs with topologically trivial and nontrivial barriers // Proc. of JINR Symp. “Few to Many Body Systems: Models and Methods and Applications”. 2016. P.İ55.

• *Shukrinov Yu. M., Rahmonov I. R., Kulikov K. V., Botha A., Gaafar M., El Samman H., Dawood R., Nashaat M., El Sherbini T.* Intrinsic Josephson Junctions for Superconducting Electronics and Quantum Computation // Proc. of the 15th Intern. Superconductive Electronics Conf. (ISEC 2015), Japan, Nagoya, July 6–9, 2015, DP-P02.

• *Rahmonov I., Shukrinov Yu., Dawood R.* Peculiarities of phase dynamics of DC-SQUID with nontrivial barriers // Abstract Book of the 10th Intern.

Symp. on Intrinsic Josephson Effects and Plasma Oscillations in High-T_c Superconductors (Plasma 2016). P. 119.

• *Rahmonov I. R., Dawood R., Shukrinov Yu. M.* Ratio of Majorana fermions and Cooper pairs in DC-SQUID with nontrivial barriers // Intern. Conf. "Superconducting Hybrid Nanostructures: Physics and Application", 2016.

• *Rahmonov I., Shukrinov Yu., Dawood R.* DC-SQUIDs with topologically trivial and nontrivial barriers: A comparative analysis // The 5th Intern. Workshop on Numerical Modelling of High Temperature Superconductors, 2016.

• *Rahmonov I., Shukrinov Yu. M., Dawood R.* Majorana fermions detectors based on DC-SQUID // Workshop "Superconductor-Based Sensors and Quantum Technologies", 2016.

• *Shukrinov Yu. M., Rahmonov I. R., Kulikov K. V., Nashaat M.* Modelling of Intrinsic Josephson Junctions in HTSC // Abstract Book of the Brazil–JINR Forum "Frontiers in Elementary Particle, Nuclear and Condensed Matter Physics", Dubna, June 15–19, 2015. P. 50.

• *Rahmonov I. R., Shukrinov Yu. M., Sengupta K., Dawood R.* Peculiarities of DC-SQUIDs with topologically nontrivial barrier // Abstract Book of the Intern. Conf. "Interaction of Superconductivity and Magnetism in Nanosystems", Moscow, September 2–4, 2015. P. 44–45.

• *Shukrinov Yu., Nashaat M., Kulikov K., Dawood R., El Samman H., El Sherbini Th. M.* Charge imbalance in a stack of Intrinsic Josephson Junctions under external radiation // Abstract Book of the Intern. Conf. " Interaction of Superconductivity and Magnetism in Nanosystems", Moscow, September 2–4, 2015. P. 42.

• *Benecha E., Rahmonov I. R., Shukrinov Yu. M., Dawood R.* Simulation of topologically nontrivial DC-SQUID // Abstract Book of the 4th South Africa–JINR Symp. "Few to Many Body Systems: Models and Methods and Applications", Dubna, September 21–25, 2015. P. 50–51.

• *Shukrinov Yu. M., Nashaat M., Kulikov K., Dawood R., El Samman H., El Sherbini Th. M.* Shapiro Step at Nonequilibrium Condition // Abstract Book of III Nat. Conf. of Applied Superconductivity, Moscow, November 25–26, 2015. P. 30.

• *Shukrinov Yu. M., Rahmonov I. R., Kulikov K., Gaafar M., Dawood R., Nashaat M., El Sherbini Th. M., El Samman H.* Physical phenomena in a system of coupled Josephson junctions and their application in superconducting electronics and quantum computers // Abstract Book of the III Nat. Conf. of Applied Superconductivity, Moscow, November 25–26, 2015. P. 28.

REFERENCES

1. *Shukrinov Yu. M., Gaafar M. A.* Charging of superconducting layers and resonance-related hysteresis in the current–voltage characteristics of coupled Josephson junctions // Phys. Rev. B. 2011. V. 84. P. 094514.

2. *Gaafar M. A., Shukrinov Yu. M., Foda A.* Shapiro and Parametric Resonances in Coupled Josephson Junctions // J. Phys. CS. 2012. V. 393. P. 012021.
3. *Gaafar M., Shukrinov Yu. M., El Samman H., Maize S.* Simulation of Shapiro steps in current–voltage characteristics of intrinsic Josephson junctions in high temperature superconductors // Lect. Notes Comput. Sci. 2012. V. 7125. P. 221–226.
4. *Shukrinov Yu. M., Rahmonov I. R., Gaafar M. A.* Calculation of the plasma frequency of a stack of coupled Josephson junctions irradiated with electromagnetic waves // Phys. Rev. B. 2012. V. 86. P. 184502.
5. *Gaafar M. A., Shukrinov Yu. M.* Effect of Microwave Irradiation on Parametric Resonance in Intrinsic Josephson Junctions // Physica C. 2013. V. 491. P. 56–58.
6. *Shukrinov Yu. M., Abdelhafiz H.* Breathing charge density waves in intrinsic Josephson junctions // JETP Lett. 2013. V. 98. P. 551–556.
7. *Shukrinov Yu. M., Rahmonov I. R., Nashaat M.* Staircase structure of Shapiro steps // JETP Letters. 2015. V. 102. P.803–806.
8. *Nashaat M., Shukrinov Yu. M., Kulikov K. V., Dawood R., El Samman H., El Sherbini Th. M.* Charge Imbalance in a Layered Structure of High Temperature Superconductors // Egypt. J. Phys. 2016. V. 44. P. 75–84.
9. *Shukrinov Yu. M., Nashaat M., Kulikov K. V., Dawood R., El Samman H., El Sherbini Th. M.* Shapiro step at nonequilibrium conditions // EPL. 2016. V. 115. P. 20003.
10. *Kulikov K. V., Shukrinov Yu. M., Nashaat M., Irie A.* Shift of Shapiro step in high critical temperature superconductors // EPJ Web Conf. 2018. V. 173. P. 03015.
11. *Kulikov K. V., Nashaat M., Shukrinov Yu. M.* Stationary charge imbalance effect in a system of coupled Josephson junctions // EPL. 2019. V. 127. P. 67004.
12. *Shukrinov Yu. M., Botha A. E., Abouhaswa A. S.* Double and triple resonance behavior in large systems of LC-shunted intrinsic Josephson junctions // Phys. Lett. A. 2021. V. 387. P. 127025.
13. *Rahmonov I. R., Shukrinov Yu. M., Dawood R.* Dynamics of SQUIDs with topologically nontrivial barriers // JETP Lett. 2016. V. 103. P. 444.
14. *Kulikov K. V., Dawood R., Nakhmedov E. P., Shukrinov Yu. M.* Josephson Junction with Two Superconducting Current Components // JETP. 2017. V. 125. P. 333.
15. *Rahmonov I. R., Shukrinov Yu. M., Dawood R., Samman H. E.* Determination of Cooper pairs and Majorana fermions currents ratio in DC-SQUID with topologically nontrivial barriers // Fiz. Nizk. Temp. (Low Temp. Phys.). 2017. V. 43. P. 824–828.
16. *Nashaat M., Botha A. E., Shukrinov Yu. M.* Devil's staircases in the IV characteristics of superconductor/ferromagnet/superconductor Josephson junctions // Phys. Rev. B. 2018. V. 97. P. 224514.
17. *Nashaat M., Shukrinov Yu. M.* Ferromagnetic Resonance and effect of supercurrent on the magnetization dynamics in S/F/S Junctions under circularly polarized magnetic field // Phys. Part. Nucl. 2020. V. 17. P. 7984.
18. *Nashaat M., Bobkova I. V., Bobkov A. M., Shukrinov Yu. M., Rahmonov I. R., Sengupta K.* Electrical control of magnetization in superconductor/ferromagnet/superconductor junctions on a three-dimensional topological insulator // Phys. Rev. B. 2019. V. 100. P. 054506.

19. *Shukrinov Yu. M., Nashaat M., Kulikov K. V., Rahmonov I. R.* Ferromagnetic Resonance and the dynamics of the magnetic moment in a "Josephson Junction–Nanomagnet" system // JETP Lett. 2019. V. 110. P. 149–154.
20. *Kulikov K. V., Anghel D. V., Preda A. T., Nashaat M., Sameh M., Shukrinov Yu. M.* Kapitza pendulum effects in a Josephson junction coupled to a nanomagnet under external periodic drive // Phys. Rev. B. 2022. V. 105. P. 094421.
21. *Nashaat M., Sameh M., Botha A. E., Kulikov K. V., Shukrinov Yu. M.* Bifurcation structure and chaos in nanomagnet coupled to Josephson junction. arXiv:2111.12659. 2022.
22. *Abdelmoneim S. A., Shukrinov Yu. M., Kulikov K., El Samman H., Nashaat M.* Locking of magnetization and Josephson oscillations at ferromagnetic resonance in φ_0 junction under external radiation // Phys. Rev. B. 2022. V. 106. P. 014505.
23. *Shukrinov Yu. M.* Anomalous Josephson effect // Phys. Usp. 2022. V. 65, No. 4 (in press).
24. *Žutić I., Fabian J., Das Sarma S.* Spintronics: Fundamentals and applications // Rev. Mod. Phys. 2004. V. 76, No. 2. P. 323.
25. *Linder J., Robinson J. W. A.* Superconducting spintronics // Nat. Phys. 2015. V. 11. P. 307–315.
26. *Golubov A. A., Kupriyanov M. Yu., Il'ichev E.* The current–phase relation in Josephson junctions // Rev. Mod. Phys. 2004. V. 76. P.411.
27. *Mai S., Kandelaki E., Volkov A. F., Efetov K. B.* Interaction of Josephson and magnetic oscillations in Josephson tunnel junctions with a ferromagnetic layer // Phys. Rev. B. 2011. V. 84. P.144519.
28. *Buzdin A.* Direct Coupling between Magnetism and Superconducting Current in the Josephson φ_0 Junction // Phys. Rev. Lett. 2008 V. 101. P. 107005.
29. *Shukrinov Yu. M., Rahmonov I. R., Sengupta K., Buzdin A.* Magnetization reversal by superconducting current in φ_0 Josephson junctions // Appl. Phys. Lett. 2017. V. 110. P. 182407.
30. *Lifshitz E. M., Pitaevskii L. P.* Course of Theoretical Physics, Theory of the Condensed State. V. 9. Oxford: Butterworth Heinemann, 1991;
Hillebrands B., Ounadjela K. Spin Dynamics of Confined Magnetic Structures II. Berlin: Springer-Verlag, 2003. P. 1–26.
31. *Weides M., Kemmler M., Kohlstedt H., Waser R., Koelle D., Kleiner R., Goldobin E.* 0–π Josephson Tunnel Junctions with Ferromagnetic Barrier // Phys. Rev. Lett. 2006. V. 97. P. 247001.
32. *Pfeiffer J., Kemmler M., Koelle D., Kleiner R., Goldobin E., Weides M., Feofanov A. K., Lisenfeld J., Ustinov A. V.* Static and dynamic properties of 0, π, and 0–π ferromagnetic Josephson tunnel junctions // Phys. Rev. B. 2008. V. 77. P. 214506.
33. *Hikino S., Mori M., Takahashi S., Maekawa S.* Microwave-induced supercurrent in a ferromagnetic Josephson junction // Supercond. Sci. Technol. 2011. V. 24. P. 024008.
34. *Wild G., Probst C., Marx A., Gross R.* Josephson coupling and Fiske dynamics in ferromagnetic tunnel junctions // Eur. Phys. J. B. 2010 V.78. P. 509–523.
35. *Kemmler M., Weides M., Weiler M., Opel M., Goennenwein S. T. B., Vasenko A. S., Golubov A. A., Kohlstedt H., Koelle D., Kleiner R., Goldobin E.* Magnetic interference patterns in 0–π superconductor/insulator/ferromagnet/

superconductor Josephson junctions: Effects of asymmetry between 0 and π regions // Phys. Rev. B. 2010. V. 81. P. 054522.

36. *Volkov A. F., Efetov K. B.* Hybridization of spin and plasma waves in Josephson tunnel junctions containing a ferromagnetic layer // Phys. Rev. Lett. 2009. V. 103. P. 037003.
37. *Kapitza P. L.* Thermal conductance and diffusion in liquid medium in the case of periodic flow: I. Determination of coefficient of wave transfer in a pipe, slit, and channel // Sov. Phys. JETP. 1951. V. 21. P. 588.
38. *Bukov M., D'Alessio L., Polkovnikov A.* Universal high-frequency behavior of periodically driven systems: From dynamical stabilization to Floquet engineering // Adv. Phys. 2015. V. 64. P. 139.
39. *Borromeo M., Marchesoni F.* Artificial Sieves for quasimassless particles // Phys. Rev. Lett. 2007. V. 99. P. 150605.
40. *Aidelsburger M., Lohse M., Schweizer C., Atala M., Barreiro J. T., Nascimbène S., Cooper N. R., Bloch I., Goldman N.* Measuring the Chern number of Hofstadter bands with ultracold bosonic atoms // Nat. Phys. 2015. V. 11. P. 162.
41. *Wickenbrock A., Holz P. C., Abdul Wahab N. A., Phoonthong P., Cubero D., Renzoni F.* Vibrational mechanics in an optical lattice: Controlling transport via potential renormalization // Phys. Rev. Lett. 2012. V. 108. P. 020603.
42. *Chizhevsky V. N.* Experimental evidence of vibrational resonance in a multistable system // Phys. Rev. E. 2014. V. 89. P. 062914.
43. *Uzuntarla M., Yilmaz E., Wagemakers A., Ozer M.* Vibrational resonance in a heterogeneous scale free network of neurons // Commun. Nonlinear Sci. Numer. Simul. 2015. V. 22. P. 367.
44. *Boukobza E., Moore M. G., Cohen D., Vardi A.* Nonlinear Phase Dynamics in a Driven Bosonic Josephson Junction // Phys. Rev. Lett. 2010. V. 104. P. 240402.
45. *Citro R., Dalla Torre E. G., D'Alessio L., Polkovnikov A., Babadi M., Oka T., Demler E.* Dynamical stability of a many-body Kapitza pendulum // Ann. Phys. 2015. V. 360. P. 694.
46. *Fialko O., Opanchuk B., Sidorov A. I., Drummond P. D., Brand J.* Fate of the false vacuum: Towards realization with ultra-cold atoms // EPL. 2015. V. 110. P. 56001.
47. *Longhi S.* Rapidly oscillating scatteringless non-Hermitian potentials and the absence of Kapitza stabilization // EPL. 2017. V. 118. P. 20004.
48. *Shayak B.* A mechanism for electromagnetic trapping of extended objects // EPL. 2017. V. 118. P. 45002.
49. *Martin J., Georgeot B., Guéry-Odelin D., Shepelyansky D. L.* Kapitza stabilization of a repulsive Bose–Einstein condensate in an oscillating optical lattice // Phys. Rev. A. 2018. V. 97. P. 023607.
50. *Yurgens A. A.* Intrinsic Josephson junctions: Recent developments // Supercond. Sci. Technol. 2000. V. 13. P. R85.
51. *Kleiner R., Steinmeyer F., Kunkel G., Muller P.* Intrinsic Josephson effects in $Bi_2Sr_2CaCu_2O_8$ single crystals // Phys. Rev. Lett. 1992. V. 68. P. 2394.
52. *Tsujimoto M., Yamamoto T., Delfanazari K., Nakayama R., Kitamura T., Sawamura M., Kashiwagi T., Minami H., Tachiki M., Kadowaki K., Klemm R. A.* Broadly tunable subterahertz emission from internal branches of the current–voltage characteristics of superconducting $Bi_2Sr_2CaCu_2O_{8+\delta}$ single crystal // Phys. Rev. Lett. 2012. V. 108. P. 107006.

53. *Benseman T. M., Koshelev A. E., Gray K. E., Kwok W.-K., Welp U., Kadowaki K., Tachiki M., Yamamoto T.* Tunable terahertz emission from $Bi_2Sr_2CaCu_2O_{8+\delta}$ mesa devices // Phys. Rev. B. 2011. V. 84. P. 064523.
54. *Krasnov V. M.* Terahertz electromagnetic radiation from intrinsic Josephson junctions at zero magnetic field via breather-type self-oscillations // Phys. Rev. B. 2011. V.83. P. 174517.
55. *Wang H. B., Guènon S., Gross B., Yuan J., Jiang Z. G., Zhong Y. Y., Grünzweig M., Iishi A., Wu P. H., Hatano T., Koelle D., Kleiner R.* Coherent terahertz emission of intrinsic Josephson junction stacks in the hot spot regime // Phys. Rev. Lett. 2010. V. 105. P. 057002.
56. *Tachiki M., Fukuya S., Koyama T.* Mechanism of terahertz electromagnetic wave emission from intrinsic Josephson junctions // Phys. Rev. Lett. 2009. V. 102. P. 127002.
57. *Ozyuzer L., Koshelev A. E., Kurter C., Gopalsami N., Li Q., Tachiki M., Kadowaki K., Yamamoto T., Minami H., Yamaguchi H., Tachiki T., Gray K. E., Kwok W.-K., Welp U.* Emission of coherent THz radiation from superconductors // Science. 2007. V. 318. P. 1291.
58. *Shapiro S.* Josephson currents in superconducting tunneling: The effect of microwaves and other observations // Phys. Rev. Lett. 1963. V. 11. P. 80.
59. *Tinkham M.* Introduction to Superconductivity. 2nd ed. New York: McGraw-Hill, 1996.
60. *Pfeiffer J., Schuster M., Abdumalikov A. A., Jr., Ustinov A. V.* Observation of soliton fusion in a Josephson array // Phys. Rev. Lett. 2006. V. 96. P. 034103.
61. *Pfeiffer J., Abdumalikov A. A., Jr., Schuster M., Ustinov A. V.* Resonances between fluxons and plasma waves in underdamped Josephson transmission lines of stripline geometry // Phys. Rev. B. 2008. V. 77. P. 024511.
62. *Watanabe S., Strogatz S. H., van der Zant H. S., Orlando T. P.* Whirling modes and parametric instabilities in the discrete sine-Gordon equation: Experimental tests in Josephson rings // Phys. Rev. Lett. 1995. V. 74. P. 379.
63. *Koyama T., Tachiki M.* I–V characteristics of Josephson-coupled layered superconductors with longitudinal plasma excitations // Phys. Rev. B. 1996. V. 54. P. 16183.
64. *Kleiner R., Gaber T., Hechtfischer G.* Stacked long Josephson junctions in zero magnetic field: A numerical study of coupled one-dimensional sine-Gordon equations // Phys. Rev. B. 2000. V. 62. P. 4086.
65. *Shukrinov Yu. M., Mahfouzi F.* Branching in current–voltage characteristics of intrinsic Josephson junctions // Supercond. Sci. Technol. 2007. V. 19. P. S38–S42.
66. *Shukrinov Yu. M., Mahfouzi F.* Influence of coupling between junctions on breakpoint current in intrinsic Josephson junctions // Phys. Rev. Lett. 2007. V. 98. P. 157001.
67. *Shukrinov Yu. M., Mahfouzi F., Pedersen N.* Investigation of the breakpoint region in stacks with a finite number of intrinsic Josephson junctions // Phys. Rev. B. 2007. V. 75. P. 104508.
68. *Artemenko S., Kobelkov A.* Intrinsic Josephson effect and violation of the Josephson relation in layered superconductors // Phys. Rev. Lett. 1997. V. 78. P. 3551.

69. *Preis C., Helm C., Keller J., Sergeev A., Kleiner R.* Coupling of intrinsic Josephson oscillations in layered superconductors by charge fluctuations // Superconducting Superlattices II: Native and Artificial. 1998. V. 3480. P. 236.
70. *Shafranjuk S. E., Tachiki M.* Emission of plasmons caused by quasiparticle injection to a high-T_c superconductor // Phys. Rev. B. 1999. V. 59. P. 14087.
71. *Helm C., Preis C., Walter C., Keller J.* Theory for the coupling between longitudinal phonons and intrinsic Josephson oscillations in layered superconductors // Phys. Rev. B. 2000. V. 62. P. 6002.
72. *Helm C., Keller J., Preis C., Sergeev A.* Static charge coupling of intrinsic Josephson junctions // Physica C. 2001. V. 362. P. 43.
73. *Helm C., Bulaevskii L. N., Chudnovsky E. M., Maley M. P.* Reflectivity and Microwave Absorption in Crystals with Alternating Intrinsic Josephson Junctions // Phys. Rev. Lett. 2002. V. 89. P. 057003.
74. *Keller J., Ryndyk D. A.* Static charge-imbalance effects in intrinsic Josephson systems // Phys. Rev. B. 2005. V. 71. P. 054507.
75. *Rother S., Koval Y., Müller P., Kleiner R., Ryndyk D. A., Keller J., Helm C.* Charge-imbalance effects in intrinsic Josephson systems // Phys. Rev. B. 2003. V. 67. P. 024510.
76. *Maiti M., Kulikov K. V., Sengupta K., Shukrinov Yu. M.* Josephson junction detectors for Majorana modes and Dirac fermions // Phys. Rev. B. 2015. V. 92. P. 224501.
77. *Linder J., Halterman K.* Superconducting spintronics with magnetic domain walls // Phys. Rev. B. 2014. V. 90. P. 104502.
78. *Shukrinov Yu. M., Mazanik A., Rahmonov I., Botha A., Buzdin A.* Re-orientation of the easy axis in φ_0-junction // EPL. 2018. V. 122. P. 37001.
79. *Shukrinov Yu. M., Rahmonov I., Sengupta K., Buzdin A.* Magnetization reversal by superconducting current in φ_0-Josephson junctions // Appl. Phys. Lett. 2017. V. 110. P. 182407.
80. *Buzdin A.* Direct coupling between magnetism and superconducting current in the Josephson φ_0 junction // Phys. Rev. Lett. 2008. V. 101. P. 107005.
81. *Buzdin A. I.* Proximity effects in superconductor–ferromagnet heterostructures // Rev. Mod. Phys. 2005. V. 77. P. 935.
82. *Bergeret F., Volkov A. F., Efetov K. B.* Odd triplet superconductivity and related phenomena in superconductor–ferromagnet structures // Rev. Mod. Phys. 2005. V. 77. P. 1321.
83. *Golubov A. A., Kupriyanov M. Y., Il'chev E.* The current–phase relation in Josephson junctions // Rev. Mod. Phys. 2004. V. 76. P. 411.
84. *Silaev M. A., Tokatly I. V., Bergeret F. S.* Anomalous current in diffusive ferromagnetic Josephson junctions // Phys. Rev. B. 2017. V. 95. P. 184508.
85. *Bobkova I., Bobkov A., Silaev M.* Gauge theory of the long-range proximity effect and spontaneous currents in superconducting heterostructures with strong ferromagnets // Phys. Rev. B. 2017. V. 96. P. 094506.
86. *Ben-Jacob E., Braiman Y., Shainsky R., Imry Y.* Microwave induced "Devil's Staircase" structure and "chaotic" behavior in current fed Josephson junctions // Appl. Phys. Lett. 1981. V. 38. P. 822.
87. *Shukrinov Yu. M., Medvedeva S. Yu., Botha A. E., Kolahchi M. R., Irie A.* Devil's staircases and continued fractions in Josephson junctions // Phys. Rev. B. 2013. V. 88. P. 214515.

88. *Shukrinov Yu. M., Botha A. E., Medvedeva S. Yu., Kolahchi M. R., Irie A.* Structured chaos in a devil's staircase of the Josephson junction // Chaos. 2014. V. 24. P. 033115.
89. *Sokolović I., Mali P., Odavić J., Radošević S., Medvedeva S. Yu., Botha A. E., Shukrinov Yu. M., Tekić J.* Devil's staircase and the absence of chaos in the dc- and ac-driven overdamped Frenkel–Kontorova model // Phys. Rev. E. 2017. V. 96. P. 022210.
90. *Nebendahl V., Dür W.* Improved numerical methods for infinite spin chains with long-range interactions // Phys. Rev. B. 2013. V. 87. P. 075413..
91. *Takigawa M., Horvatić M., Waki T., Krämer S., Berthier C., Lévy-Bertrand F., Sheikin I., Kageyama H., Ueda Y., Mila F.* Incomplete Devil's Staircase in the magnetization curve of $SrCu_2(BO_3)_2$ // Phys. Rev. Lett. 2013. V. 110. P. 067210.
92. *Hriscu A. M., Nazarov Yu. V.* Quantum synchronization of conjugated variables in a superconducting device leads to the fundamental resistance quantization // Phys. Rev. Lett. 2013. V. 110. P. 097002.
93. *Yao Y., Cai R., Yang S.-H., Xing W., Ma Y., Mori M., Ji Y., Maekawa S., Xie X.-C., Han W.* Half-integer Shapiro steps in strong ferromagnetic Josephson junctions // Phys. Rev. B. 2021. V. 104. P. 104414.
94. *Cai L., Chudnovsky E. M.* Interaction of a nanomagnet with a weak superconducting link // Phys. Rev. B. 2010. V. 82. P. 104429.
95. *Cai L., Garanin D. A., Chudnovsky E. M.* Reversal of magnetization of a single-domain magnetic particle by the ac field of time-dependent frequency // Phys. Rev. B. 2013. V. 87. P. 024418.
96. *Ghosh R., Maiti M., Shukrinov Yu. M., Sengupta K.* Magnetization-induced dynamics of a Josephson junction coupled to a nanomagnet // Phys. Rev. B. 2017. V. 96. P. 174517.
97. *Richards C. J., Smart T. J., Jones P. H., Cubero D.* A microscopic Kapitza pendulum // Sci. Rep. 2018. V. 8. P. 13107.
98. *Golovinski P. A., Dubinkin V. A.* Quantum states of the Kapitza pendulum. arXiv:2102.12711. 2021.

Received on July 6, 2022.

Редактор *Е. И. Кравченко*

Подписано в печать 28.07.2022.
Формат 60 × 90/16. Бумага офсетная. Печать офсетная.
Усл. печ. л. 2,0. Уч.-изд. л. 2,81. Тираж 150 экз. Заказ № 60477.

Издательский отдел Объединенного института ядерных исследований
141980, г. Дубна, Московская обл., ул. Жолио-Кюри, 6.
E-mail: publish@jinr.ru
www.jinr.ru/publish/