$2k-$dimensional $N = 8$ supersymmetric quantum mechanics

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We demonstrate that two-dimensional $N = 8$ supersymmetric quantum mechanics which inherits the most interesting properties of $N = 2, d = 4$ SYM can be constructed if the reduction to one dimension is performed in terms of the basic object – $N = 2, d = 4$ vector multiplet. In such a reduction only complex scalar fields from the $N = 2, d = 4$ vector multiplet become physical bosons in $d = 1$, while the rest of the bosonic components are reduced to auxiliary fields thus giving rise to **(2, 8, 6)** supermultiplet in $d = 1$. We construct the most general action for this supermultiplet with all possible FI terms included and explicitly demonstrate that the action possesses duality symmetry extended to the fermionic sector of theory. To deal with the second–class constraints presented in the system, we introduce the Dirac brackets for the canonical variables and find supercharges and Hamiltonian which form the $N = 8$ super Poincarè algebra with central charges. Finally, we explicitly present the generalization of the two-dimensional $N = 8$ SQM to the $2k$-dimensional case with a special Kähler geometry in the target space.

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1 Introduction

Among extended supersymmetric theories in diverse dimensions those which have eight real supercharges are most interesting. Mainly, this interest is motivated by the existence of off-shell superfield formulations. In the $N = 2$, $d = 4$ case the invention of the harmonic superspace [1] and projective superspace [2] opened a way for a detailed description of these theories. Another motivation comes from a possibility of obtaining exact quantum results for $N = 2$, $d = 4$ theories in the famous Seiberg-Witten approach [3, 4]. Finally, let us mention that supersymmetry severely restricts possible target-space geometries. When the number of supercharges exceeds eight, the target spaces are restricted to be symmetric spaces, while beyond sixteen supercharges there is no freedom left. Thus, the theories with eight supercharges are the last case of theories with extended supersymmetries which have a rich geometric structure of the target space (see e.g. [5]).

One of the most investigated theories with eight supercharges is $N = 2$, $d = 4$ SYM theory. It has been much explored and many exciting results have been obtained. The heart of the $N = 2$, $d = 4$ SYM theory is formed by a vector supermultiplet, which describes spin-1 particles, accompanied by complex scalar fields and doublets of spinor fields. The geometry of the scalar fields is restricted to be a Kähler one [6] of special type. The restriction that the metric is defined by a holomorphic function is crucial for the Seiberg-Witten approach. Other interesting properties of the $N = 2$, $d = 4$ SYM theory are duality in the scalar sector [3, 4] and possibility of spontaneous partial breaking of the $N = 2$ supersymmetry by adding two types of Fayet-Iliopoulos (FI) terms [7, 8].

In [9] it has been shown that the theories with eight supercharges can be similarly formulated in diverse dimensions still sharing the common properties. In this respect, the one-dimensional case has a special status, because the standard reduction from the $N = 2$, $d = 4$ SYM to $d = 1$ gives rise to the $N = 8$ supersymmetric theory with five bosons, i.e., the **(5, 8, 3)** supermultiplet [9, 10]. Of course, after such a reduction almost all nice features of $N = 2$ SYM mentioned above disappear. Naturally, an obvious question arises whether it is possible to construct an $N = 8$, $d = 1$ theory which

- may be obtained by reduction from the $N = 2$, $d = 4$ SYM,
- contains $2k$ (in the simplest case only two) bosonic fields with a special Kähler geometry in the target-space,
- possesses the duality transformations, properly extended to the fermionic sector,
- has a proper place for FI terms.

The goal of the present paper is to demonstrate that $N = 8$ supersymmetric quantum mechanics with all such properties may indeed be constructed. Our main idea is to perform the reduction to one dimension in terms of the basic object – $N = 2$ vector multiplet $\mathcal{A}$ instead of the reduction in terms of a prepotential [9, 10]. In this approach only a complex scalar from the $N = 2$, $d = 4$ vector multiplet becomes a physical boson in $d = 1$, while the rest of the bosonic components are reduced to auxiliary fields. Thus, we finish with the **(2, 8, 6)** supermultiplet. In Subsection 2.1 we construct the most general action for this supermultiplet with all possible FI terms included. We also explicitly demonstrate that the action possesses duality symmetry extended to the fermionic sector of the theory. In Subsection 2.2 we demonstrate that our SQM contains the the second-class constraints. We then introduce the Dirac brackets for the canonical variables, and construct the supercharges and Hamiltonian which form $N = 8$ super Poincarè algebra with central charges. Finally, in Subsection 2.3 we explicitly present the generalization of our two-dimensional $N = 8$ SQM to a $2k$-dimensional case with a special Kähler geometry in the target space.

2 2k-dimensional N=8 supersymmetric quantum mechanics

In this section we describe a general superfield formalism of $2k$-dimensional $N=8$ supersymmetric quantum mechanics [11]. We start with the formulation of SQM in $N=8$ superspace and conclude with the component form of the Lagrangian and Hamiltonian. Due to an almost evident generalization of the 2-dimensional case to the $2k$-dimensional one and to avoid unneeded complications, we will describe in detail only the former case, explicitly presenting just the final results for the $2k$-dimensional SQM.

2.1 Two-dimensional N=8 SQM: Lagrangian

The convenient point of departure is the $N=8$, $d=1$ superspace $\mathbb{R}^{(1|8)}$

$$\mathbb{R}^{(1|8)} = (t, \theta^{ia}, \vartheta^{i\alpha})\,, \qquad \left(\theta^{ia}\right)^\dagger = \theta_{ia}\,, \qquad \left(\vartheta^{i\alpha}\right)^\dagger = \vartheta_{i\alpha}\,,$$

where i, a, $\alpha = 1, 2$ are doublet indices of three $SU(2)$ subgroups of the automorphism group of $N=8$ superspace[1]). In this superspace we define the covariant spinor derivatives

$$\begin{array}{ll} D^{ia} = \dfrac{\partial}{\partial\theta_{ia}} + \mathrm{i}\theta^{ia}\partial_t\,, & \nabla^{i\alpha} = \dfrac{\partial}{\partial\vartheta_{ia}} + \mathrm{i}\theta^{ia}\partial_t\,, \\ \left\{D^{ia}, D^{jb}\right\} = 2\mathrm{i}\epsilon^{ij}\epsilon^{ab}\partial_t\,, & \left\{\nabla^{i\alpha}, D^{j\beta}\right\} = 2\mathrm{i}\epsilon^{ij}\epsilon^{\alpha\beta}\partial_t\,, \\ \left\{D^{ia}, \nabla^{j\alpha}\right\} = 0\,. & \end{array} \qquad (1)$$

To construct a supermultiplet with two physical bosonic, eight fermionic and six auxiliary bosonic components, i.e., the **(2, 8, 6)** supermultiplet, we, adhering to [11], introduce a complex $N=8$ superfield $\mathcal{Z}$, $\overline{\mathcal{Z}}$ subjected to the following constraints

$$\begin{array}{lr} D^{1a}\mathcal{Z} = \nabla^{1\alpha}\mathcal{Z} = 0, \qquad D^{2a}\overline{\mathcal{Z}} = \nabla^{2\alpha}\overline{\mathcal{Z}} = 0, & (a) \\ \nabla^{2\alpha}D^{2a}\mathcal{Z} + \nabla^{1\alpha}D^{1a}\overline{\mathcal{Z}} = \mathrm{i}M^{a\alpha}, & (b) \end{array} \qquad (2)$$

where $M^{a\alpha}$ are arbitrary constants obeying the reality condition $\left(M^{a\alpha}\right)^\dagger = M_{a\alpha}$. The constraints (2a) represent the twisted version of the standard chirality conditions, while (2b) are recognized as modified reality constraints [8]. As we will see below, the presence of these arbitrary parameters $M^{a\alpha}$ gives rise to potential terms in the component action and opens a possibility for a partial breaking of $N=8$ supersymmetry.

The constraints (2) leave the following components in the $N=8$ superfields $\mathcal{Z}$, $\overline{\mathcal{Z}}$:

$$\begin{array}{l} z = \mathcal{Z}|, \quad \bar{z} = \overline{\mathcal{Z}}|, \quad \psi^a = D^{2a}\mathcal{Z}|, \quad \bar{\psi}_a = -D^1_a\overline{\mathcal{Z}}|, \\ \xi^\alpha = \nabla^{2\alpha}\mathcal{Z}|, \quad \bar{\xi}_\alpha = -\nabla^1_\alpha\overline{\mathcal{Z}}|, \qquad A = -\mathrm{i}D^{2a}D^2_a\mathcal{Z}|, \quad \bar{A} = -\mathrm{i}D^{1a}D^1_a\overline{\mathcal{Z}}, \\ B = -\mathrm{i}\nabla^{2\alpha}\nabla^2_\alpha\mathcal{Z}|, \quad \bar{B} = -\mathrm{i}\nabla^{1\alpha}\nabla^1_\alpha\overline{\mathcal{Z}}|, \qquad Y^{a\alpha} = D^{2a}\nabla^{2\alpha}\mathcal{Z}|, \\ \overline{Y}^{a\alpha} = -D^{1a}\nabla^{1\alpha}\overline{\mathcal{Z}}| = Y^{a\alpha} + \mathrm{i}M^{a\alpha}, \end{array} \qquad (3)$$

[1]) We use the following convention for the skew-symmetric tensor ϵ: $\epsilon_{ij}\epsilon^{jk} = \delta_i^k$, $\epsilon_{12} = \epsilon^{21} = 1$.

where | means restriction to $\theta^{ia} = \vartheta^{i\alpha} = 0$. The bosonic fields A and B are subjected, in virtue of (2), to the additional constraints

$$\frac{\partial}{\partial t}\left(A - \bar{B}\right) = 0, \quad \frac{\partial}{\partial t}\left(\bar{A} - B\right) = 0. \tag{4}$$

To deal with these constraints we have two options

- to solve these constraints as

$$A = C + \frac{m}{2}, \qquad B = \overline{C} - \frac{\bar{m}}{2}, \tag{5}$$

  where C is a new independent complex auxiliary field and m is a complex constant parameter. The resulting supermultiplet will be just **(2, 8, 6)** one.

- to insert the constraints (4) with Lagrangian multipliers in the proper action. This option gives rise to a **(4, 8, 4)** supermultiplet and will be considered in the forthcoming paper [18].

Now one can write down the most general $N = 8$ supersymmetric Lagrangian in the $N = 8$ superspace[2]):

$$\begin{aligned} S = -\int dtd^2\theta_2 d^2\vartheta_2 &\left[\mathcal{F}\left(\mathcal{Z}\right) - \frac{1}{2}\theta_{2a}\vartheta_{2\alpha}N^{a\alpha}\mathcal{Z} - \frac{\mathrm{i}}{8}\left(\bar{n}\,\theta_2^a\theta_{2a} + n\,\vartheta_2^\alpha\vartheta_{2\alpha}\right)\,\mathcal{Z}\right] - \\ \int dtd^2\theta_1 d^2\vartheta_1 &\left[\overline{\mathcal{F}}\left(\overline{\mathcal{Z}}\right) + \frac{1}{2}\theta_{1a}\vartheta_{1\alpha}N^{a\alpha}\overline{\mathcal{Z}} - \frac{\mathrm{i}}{8}\left(n\,\theta_1^a\theta_{1a} + \bar{n}\,\vartheta_1^\alpha\vartheta_{1\alpha}\right)\,\overline{\mathcal{Z}}\right]. \end{aligned} \tag{6}$$

Here $\mathcal{F}(\mathcal{Z})$ and $\overline{\mathcal{F}}(\overline{\mathcal{Z}})$ are arbitrary holomorphic functions of the superfields $\mathcal{Z}$ and $\overline{\mathcal{Z}}$, respectively, and two terms with a constant real matrix parameter $N^{a\alpha}$ $((N^{a\alpha})^\dagger = N_{a\alpha})$ and a complex constant parameter n represent one-dimensional versions of two Fayet-Iliopoulos terms [8].

After integration over the Grassmann variables one obtains the component form of the action (6)[3]):

$$\begin{aligned} S = \int \mathrm{d}t \Big\{ &\left(F'' + \overline{F}''\right)\left[\dot{z}\dot{\bar{z}} + \tfrac{\mathrm{i}}{4}\left(\psi\dot{\bar{\psi}} - \dot{\psi}\bar{\psi} + \xi\dot{\bar{\xi}} - \dot{\xi}\bar{\xi}\right)\right] - \\ &-\tfrac{\mathrm{i}}{4}\left(F^{(3)}\dot{z} - \overline{F}^{(3)}\dot{\bar{z}}\right)\left(\psi\bar{\psi} + \xi\bar{\xi}\right) + \\ &+\tfrac{1}{16}\left[F''\left(2Y^2 + AB\right) + \overline{F}''\left(2\overline{Y}^2 + \bar{A}\bar{B}\right) - F^{(4)}\psi^2\xi^2 - \overline{F}^{(4)}\bar{\psi}^2\bar{\xi}^2\right] - \\ &-\tfrac{1}{16}\left[F^{(3)}\left(\mathrm{i}A\xi^2 + \mathrm{i}B\psi^2 - 4\psi^a\xi^\alpha Y_{a\alpha}\right) + \overline{F}^{(3)}\left(\mathrm{i}\bar{A}\bar{\xi}^2 + \mathrm{i}\bar{B}\bar{\psi}^2 + 4\bar{\psi}^a\bar{\xi}^\alpha\overline{Y}_{a\alpha}\right)\right] + \\ &+\tfrac{1}{32}\,n\left(A + \bar{B}\right) + \tfrac{1}{32}\bar{n}\left(\bar{A} + B\right) + \tfrac{1}{8}\,NY + \tfrac{1}{8}\,N\overline{Y}\Big\}. \end{aligned} \tag{7}$$

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[2]) We use the convention $\int dtd^2\theta_2 d^2\vartheta_2 \equiv \frac{1}{16}\int dt\; D^{2a}D_a^2\,\nabla^{2\alpha}\nabla_\alpha^2$.

[3]) All implicit summations go from "up-left" to "down-right", e.g., $\psi\bar{\psi} \equiv \psi^a\bar{\psi}_a$, $\psi^2 \equiv \psi^a\psi_a$, $M^2 \equiv M^{a\alpha}M_{a\alpha}$, etc.

Here the holomorphic function $F(z)$ is defined as a bosonic limit of $\mathcal{F}(\mathcal{Z})$

$$F(z) \equiv \mathcal{F}(\mathcal{Z})| \,.$$

Now following the first option (5), one may express the auxiliary fields in terms of the physical ones using their equations of motion

$$\begin{aligned}
C &= \frac{\mathrm{i}\left(F^{(3)}\psi^2 + \overline{F}^{(3)}\bar\xi^2\right) + \frac{m}{2}\left(\overline{F}'' - F''\right) - \bar n}{F'' + \overline{F}''}\,,\\
\bar C &= \frac{\mathrm{i}\left(F^{(3)}\xi^2 + \overline{F}^{(3)}\bar\psi^2\right) - \frac{\overline{m}}{2}\left(\overline{F}'' - F''\right) - n}{F'' + \overline{F}''}\,,\\
Y_{a\alpha} &= \frac{\overline{F}^{(3)}\bar\psi_a\bar\xi_\alpha - F^{(3)}\psi_a\xi_\alpha - \mathrm{i}\overline{F}'' M_{a\alpha} - N_{a\alpha}}{F'' + \overline{F}''}\,, \qquad \overline{Y}_{a\alpha} = Y_{a\alpha} + \mathrm{i}M_{a\alpha}\,.
\end{aligned} \tag{8}$$

Substituting these expressions back into Eq.(7) we will get the action in terms of the physical components

$$S = \int dt\,[\mathcal{K} - \mathcal{V}]\,, \tag{9}$$

where the kinetic $\mathcal{K}$ and potential $\mathcal{V}$ terms read as

$$\mathcal{K} = \left(F'' + \overline{F}''\right)\left[\dot z\dot{\bar z} + \tfrac{\mathrm{i}}{4}\left(\psi\dot{\bar\psi} - \dot\psi\bar\psi + \xi\dot{\bar\xi} - \dot\xi\bar\xi\right)\right] - \tfrac{\mathrm{i}}{4}\left(F'''\dot z - \overline{F}'''\dot{\bar z}\right)\left(\psi\bar\psi + \xi\bar\xi\right) \tag{10}$$

and

$$\begin{aligned}
\mathcal{V} = \frac{1}{16}\Bigg[&\left(F^{(4)} - \frac{3F'''F'''}{F'' + \overline{F}''}\right)\psi^2\xi^2 + \left(\overline{F}^{(4)} - \frac{3\overline{F}'''\overline{F}'''}{F'' + \overline{F}''}\right)\bar\psi^2\bar\xi^2 - \\
&- \frac{F'''\overline{F}'''}{F'' + \overline{F}''}\left(\psi^2\bar\psi^2 + \xi^2\bar\xi^2 - 4\psi\bar\psi\xi\bar\xi\right) + \\
&+ 2\,\frac{N^2 - \mathrm{i}\left(F'' - \overline{F}''\right)NM + F''\overline{F}''M^2}{F'' + \overline{F}''} + \\
&+ 4\mathrm{i}\,\frac{F'''\psi^a\xi^\alpha\left(\overline{F}''M_{a\alpha} - \mathrm{i}N_{a\alpha}\right) + \overline{F}'''\bar\psi^a\bar\xi^\alpha\left(F''M_{a\alpha} + \mathrm{i}N_{a\alpha}\right)}{F'' + \overline{F}''} + \\
&+ \frac{n\bar n + \frac{1}{2}\left(mn - \overline{m}\bar n\right)\left(F'' - \overline{F}''\right) + m\overline{m}F''\overline{F}''}{F'' + \overline{F}''} + \\
&+ \mathrm{i}\,\frac{F^{(3)}\xi^2\left(\overline{F}''m - \bar n\right) - F^{(3)}\psi^2\left(\overline{F}''\overline{m} + n\right)}{F'' + \overline{F}''} + \\
&+ \mathrm{i}\,\frac{\overline{F}^{(3)}\bar\xi^2\left(F''\overline{m} - n\right) - \overline{F}^{(3)}\bar\psi^2\left(F''m + \bar n\right)}{F'' + \overline{F}''}\Bigg].
\end{aligned} \tag{11}$$

The action (9) is invariant with respect to the $N = 8$ supersymmetry which is realized on the physical component fields as follows:

$$
\begin{aligned}
&\delta z = \epsilon_{2a}\psi^a + \varepsilon_{2\alpha}\xi^\alpha, \qquad \delta\bar z = -\epsilon_{1a}\bar\psi^a - \varepsilon_{1\alpha}\bar\xi^\alpha,\\
&\delta\psi_a = \frac{\mathrm{i}}{2}\epsilon_{2a}\left(C + \frac{m}{2}\right) + \varepsilon_2^\alpha Y_{a\alpha} + 2\mathrm{i}\epsilon_{1a}\dot z,\\
&\delta\xi_\alpha = \frac{\mathrm{i}}{2}\varepsilon_{2\alpha}\left(\overline{C} - \frac{\overline{m}}{2}\right) - \epsilon_2^a Y_{a\alpha} + 2\mathrm{i}\varepsilon_{1\alpha}\dot z,
\end{aligned}
\tag{12}
$$

with ϵ_{ia}, $\varepsilon_{i\alpha}$ being the parameters of two $N = 4$ supersymmetries acting on θ^{ia} and $\vartheta^{i\alpha}$, respectively, and with the auxiliary fields C and $Y^{a\alpha}$ defined in (8).

Using the Noether theorem one can find classical expressions for the conserved supercharges corresponding to the supersymmetry transformations (12)

$$
\begin{aligned}
&Q_1^a = \left(F'' + \overline{F}''\right)\psi^a\dot{\bar z} - \tfrac{\mathrm{i}}{4}\overline{F}^{(3)}\bar\psi^a\bar\xi^2 + \tfrac{\mathrm{i}}{2}\left(\mathrm{i}\overline{F}''M^{a\alpha} + N^{a\alpha}\right)\bar\xi_\alpha - \tfrac{1}{4}\left(m\overline{F}'' - \overline{n}\right)\bar\psi^a\,,\\
&S_1^\alpha = \left(F'' + \overline{F}''\right)\xi^\alpha\dot{\bar z} - \tfrac{\mathrm{i}}{4}\overline{F}^{(3)}\bar\xi^\alpha\bar\psi^2 - \tfrac{\mathrm{i}}{2}\left(\mathrm{i}\overline{F}''M^{a\alpha} + N^{a\alpha}\right)\bar\psi_a + \tfrac{1}{4}\left(\overline{m}\overline{F}'' + n\right)\bar\xi^\alpha\,,\\
&Q_{2a} = \left(Q_1^a\right)^\dagger\,, \quad S_{2a} = \left(S_1^a\right)^\dagger\,.
\end{aligned}
\tag{13}
$$

Let us note that our variant of $N = 8$ SQM is a direct reduction of the $N = 2$, $d = 4$ SYM. So it is not unexpected that the metric of bosonic manifold is restricted to be the *special Kähler* one (of rigid type) (see, e.g., [15])

$$
g(z, \bar z) = F''(z) + \overline{F}''(\bar z). \tag{14}
$$

Secondly, one may immediately check that the action (9) exhibits the famous Seiberg-Witten duality [3]. Indeed, after passing to new variables defined as

$$
\begin{aligned}
&\tilde z = F'(z), \quad \tilde\psi^a = F''\psi^a, \quad \tilde{\bar\psi}_a = \overline{F}''\bar\psi_a, \quad \tilde\xi^\alpha = \mathrm{i}F''\xi^\alpha, \quad \tilde{\bar\xi}_\alpha = -\mathrm{i}\overline{F}''\bar\xi_\alpha,\\
&\tilde F''(\tilde z) = \frac{1}{F''(z)}, \quad \tilde N^{a\alpha} = M^{a\alpha}, \quad \tilde M^{a\alpha} = -N^{a\alpha}, \quad \tilde m = \overline{n}, \quad \tilde n = \overline{m},
\end{aligned}
\tag{15}
$$

the action (9) keeps its form being rewritten in the new tilded variables. Let us note that in the dual formulation the constants $M^{a\alpha}$ and m, which have appeared in the constraints (2) and (5), are interchanged with the constants $N^{a\alpha}$ and n, which have shown up in the FI-terms. This is just a simplified version of the electric-magnetic duality [3] for our $N = 8$ SQM case. Thus, our $N = 8$ SQM possesses the most interesting peculiarities of the $N = 2$, $d = 4$ SYM theory and can be used for a simplified analysis of some subtle properties of its ancestor.

2.2 Two−dimensional $N = 8$ SQM: Hamiltonian

To find the classical Hamiltonian, we follow the standard procedure of quantizing a system with bosonic and fermionic degrees of freedom [16]. From the action (9) we define the momenta p, $\bar p$, $\pi_a^{(\psi)}$, $\bar\pi^{(\psi)a}$, $\pi_\alpha^{(\xi)}$, $\bar\pi^{(\xi)\alpha}$ conjugated to z, $\bar z$, ψ^a, $\bar\psi_a$, ξ^α

and $\bar\xi_\alpha$, respectively, as

$$
\begin{aligned}
&p = g\dot{\bar z} - \frac{\mathrm{i}}{4}\,\partial_z g\left(\psi\bar\psi + \xi\bar\xi\right), \quad \bar p = g\dot z + \frac{\mathrm{i}}{4}\,\bar\partial_z g\left(\psi\bar\psi + \xi\bar\xi\right), && (a)\\
&\pi^{(\psi)}_a = -\frac{\mathrm{i}}{4}\,g\bar\psi_a,\ \bar\pi^{(\psi)a} = -\frac{\mathrm{i}}{4}\,g\psi^a, \quad \pi^{(\xi)}_\alpha = -\frac{\mathrm{i}}{4}\,g\bar\xi_\alpha,\ \bar\pi^{(\xi)\alpha} = -\frac{\mathrm{i}}{4} g\,\xi^\alpha, && (b)
\end{aligned}
\tag{16}
$$

with the metric $g(z,\bar z)$ defined in (14) and introduce the canonical Poisson brackets

$$
\begin{aligned}
&\{z,p\} = 1, \quad \{\bar z,\bar p\} = 1, \qquad \left\{\psi^a, \pi^{(\psi)}_b\right\} = -\delta^a_b, \quad \left\{\xi^\alpha, \pi^{(\psi)}_\beta\right\} = -\delta^\alpha_\beta,\\
&\left\{\bar\psi_a, \bar\pi^{(\psi)b}\right\} = -\delta^b_a, \quad \left\{\bar\xi_\alpha, \bar\pi^{(\xi)\beta}\right\} = -\delta^\beta_\alpha.
\end{aligned}
\tag{17}
$$

From the explicit form of the fermionic momenta (16 b,c) it follows that the system possesses the second–class constraints

$$
\begin{aligned}
\chi^{(\psi)}_a &= \pi^{(\psi)}_a + \frac{\mathrm{i}}{4}\,g\bar\psi_a, \quad & \bar\chi^{(\psi)a} &= \bar\pi^{(\psi)a} + \frac{\mathrm{i}}{4}\,g\psi^a,\\
\chi^{(\xi)}_\alpha &= \pi^{(\xi)}_\alpha + \frac{\mathrm{i}}{4}\,g\bar\xi_\alpha, \quad & \bar\chi^{(\xi)\alpha} &= \bar\pi^{(\xi)\alpha} + \frac{\mathrm{i}}{4}\,g\xi^\alpha,
\end{aligned}
\tag{18}
$$

since

$$
\left\{\chi^{(\psi)}_a, \bar\chi^{(\psi)b}\right\} = -\frac{\mathrm{i}}{2}\,g\delta^b_a, \quad \left\{\chi^{(\xi)}_\alpha, \bar\chi^{(\xi)\beta}\right\} = -\frac{\mathrm{i}}{2}\,g\delta^\beta_\alpha.
\tag{19}
$$

Therefore, we should pass to the Dirac brackets defined for arbitrary functions $\mathcal{V}$ and $\mathcal{W}$ as

$$
\begin{aligned}
\{\mathcal{V},\mathcal{W}\}_D = \{\mathcal{V},\mathcal{W}\} - \Bigg[&\left\{\mathcal{V},\chi^{(\psi)}_a\right\} \frac{1}{\left\{\chi^{(\psi)}_a, \bar\chi^{(\psi)b}\right\}} \left\{\bar\chi^{(\psi)b},\mathcal{W}\right\} + \\
&\left\{\mathcal{V},\bar\chi^{(\psi)a}\right\} \frac{1}{\left\{\bar\chi^{(\psi)a}, \chi^{(\psi)}_b\right\}} \left\{\chi^{(\psi)}_b,\mathcal{W}\right\} + \left(\chi^{(\psi)} \to \chi^{(\xi)}\right)\Bigg].
\end{aligned}
\tag{20}
$$

As a result, we get the following Dirac[4] brackets for the canonical variables:

$$
\begin{aligned}
&\{z,\,\hat p\} = 1, \quad \{\bar z,\hat{\bar p}\} = 1, \quad \{\hat p,\,\hat{\bar p}\} = -\frac{\mathrm{i}}{2}\frac{\partial_z g\,\bar\partial_z g}{g}\left(\psi\bar\psi + \xi\bar\xi\right),\\
&\left\{\psi^a,\,\bar\psi_b\right\} = -\frac{2\mathrm{i}}{g}\,\delta^a_b, \quad \left\{\xi^\alpha,\,\bar\xi_\beta\right\} = -\frac{2\mathrm{i}}{g}\,\delta^\alpha_\beta,\\
&\{\hat p,\,\psi_a\} = \frac{\partial_z g}{g}\,\psi_a, \quad \{\hat p,\,\xi_\alpha\} = \frac{\partial_z g}{g}\,\xi_\alpha,\\
&\left\{\hat{\bar p},\,\bar\psi_a\right\} = \frac{\bar\partial_z g}{g}\,\bar\psi_a, \quad \left\{\hat{\bar p},\,\bar\xi_\alpha\right\} = \frac{\bar\partial_z g}{g}\,\bar\xi_\alpha,
\end{aligned}
\tag{21}
$$

[4]) From now on the symbol $\{\,,\}$ stands for the Dirac bracket.

where the "improved" bosonic momenta have been defined as

$$\hat{p} \equiv p + \frac{\mathrm{i}}{4}\,\partial_z g\left(\psi\bar{\psi} + \xi\bar{\xi}\right), \quad \hat{\bar{p}} \equiv \bar{p} - \frac{\mathrm{i}}{4}\,\bar{\partial}_z g\left(\psi\bar{\psi} + \xi\bar{\xi}\right). \tag{22}$$

Now one can check that the supercharges Q_{ia}, $S_{i\alpha}$ (13), being rewritten through the momenta as

$$\begin{aligned} Q_1^a &= \hat{p}\,\psi^a - \tfrac{\mathrm{i}}{4}\,\bar{\partial}_z g\,\bar{\psi}^a\bar{\xi}^2 + \tfrac{\mathrm{i}}{2}\left(\mathrm{i}\overline{F}''M^{a\alpha} + N^{a\alpha}\right)\bar{\xi}_\alpha - \tfrac{1}{4}\left(m\overline{F}'' - \overline{n}\right)\bar{\psi}^a\,, \\ S_1^\alpha &= \hat{p}\,\xi^\alpha - \tfrac{\mathrm{i}}{4}\,\bar{\partial}_z g\,\bar{\xi}^\alpha\bar{\psi}^2 - \tfrac{\mathrm{i}}{2}\left(\mathrm{i}\overline{F}''M^{a\alpha} + N^{a\alpha}\right)\bar{\psi}_a + \tfrac{1}{4}\left(\overline{m}\overline{F}'' + n\right)\bar{\xi}^\alpha\,, \\ Q_{2a} &= \left(Q_1^a\right)^\dagger\,, \quad S_{2a} = \left(S_1^a\right)^\dagger\,, \end{aligned} \tag{23}$$

and the Hamiltonian

$$H = g^{-1}\,\hat{p}\hat{\overline{p}} + \mathcal{V} \tag{24}$$

form the following $N=8$ superalgebra

$$\begin{aligned} \{Q_{ia},\, Q_{jb}\} &= -2\mathrm{i}\epsilon_{ij}\epsilon_{ab}\left(H - \tfrac{1}{16}\left(nm + \bar{n}\overline{m}\right)\right) - \tfrac{1}{8}\,\epsilon_{ij}\left(N_a^\alpha M_{\alpha b} + N_b^\alpha M_{\alpha a}\right), \\ \{S_{i\alpha},\, S_{j\beta}\} &= -2\mathrm{i}\epsilon_{ij}\epsilon_{\alpha\beta}\left(H + \tfrac{1}{16}\left(nm + \bar{n}\overline{m}\right)\right) - \tfrac{1}{8}\,\epsilon_{ij}\left(N_\alpha^a M_{a\beta} + N_\beta^a M_{a\alpha}\right), \\ \{Q_{1a},\, S_{2\alpha}\} &= -mN_{a\alpha} - \mathrm{i}\bar{n}M_{a\alpha}\,, \quad \{Q_{2a},\, S_{1\alpha}\} = -\overline{m}N_{a\alpha} + \mathrm{i}nM_{a\alpha}\,. \end{aligned} \tag{25}$$

By these we complete the classical description of the two dimensional $N=8$ SQM. Before closing this subsection and going on to generalize our SQM to the $2k$-dimensional case, let us briefly discuss the main peculiarity of the model.

Firstly, as has already been mentioned, the $N=8$ supersymmetry strictly fixes the metric of the target space to be the *special Kähler* one.

Secondly, one can see that the Dirac brackets between the canonical variables are defined in terms of the metric $g(z,\bar{z})$ only. This fact has already been noted in [17]. However, if we are going to include the potential terms in the Hamiltonian (by adding the Fayet-Iliopoulos terms together with admitting the constant parts in the auxiliary fields) the modified supercharges will contain nonmetric pieces (the terms with explicit $F''(z)$ or $\overline{F}''(\bar{z})$ in (13), (23)). The way to keep the metric structure of the supercharges and to have the potential terms in the Hamiltonian is to add only the Fayet-Iliopoulos terms keeping $M_{a\alpha} = m = 0$. Alternatively, one may put all constant parts of the auxiliary fields equal to zero $N_{a\alpha} = n = 0$. However, in these cases our $N=8$ superalgebra (25) does not contain any central charges and the dual symmetry (15) is broken.

The presence of the central charges in the superalgebra (25), like in the $N=4$ SQM case [13], is the most exciting feature of the model. The central charges appear only when the Fayet-Iliopoulos terms are added (with the constants $N_{a\alpha}$ or n) and the auxiliary fields contain the constant parts ($M_{a\alpha}$ or m). The existence of the nonzero central charges in the superalgebra (25) opens up a possibility of realizing a partial spontaneous breaking of $N=8$ supersymmetry.

Finally, it is amusing that the bosonic potential terms which appear in the Hamiltonian explicitly break at least one of the $SU(2)$ automorphism groups. This is again very similar to the $N=4$ SQM [13].

2.3 $2k-$dimensional $N=8$ SQM

The generalization of the $N=8$ two-dimensional SQM to the $2k$-dimensional case is straightforward. The simplest is the superfield generalization. It goes as follows:

- We introduce k complex $N=8$ superfields $\mathcal{Z}^A$, $\overline{\mathcal{Z}}^B$ $(A,B=1,\ldots,k)$ each of them obeying the same constraints (2) with different constants $M^{A\,a\alpha}$:

$$
\begin{array}{lr}
D^{1a}\mathcal{Z}^A=\nabla^{1\alpha}\mathcal{Z}^A=0, \qquad D^{2a}\overline{\mathcal{Z}}^A=\nabla^{2\alpha}\overline{\mathcal{Z}}^A=0, & (a) \\
\nabla^{2\alpha}D^{2a}\mathcal{Z}^A+\nabla^{1\alpha}D^{1a}\overline{\mathcal{Z}}^A=\mathrm{i}M^{A\,a\alpha}. & (b)
\end{array}
\qquad (26)
$$

- The components of each superfield can be defined as in (3) and k different constants m^A may be introduced similarly to (5)

$$
A^A=C^A+\frac{m^A}{2}, \qquad B^A=\overline{C}^A-\frac{\bar{m}^A}{2}. \qquad (27)
$$

- The most general $N=8$ supersymmetric action reads

$$
S_{2k}=-\int dt d^2\theta_2 d^2\vartheta_2\left[\mathcal{F}(\mathcal{Z}^1,\ldots,\mathcal{Z}^k)-\frac{1}{2}\theta_{2a}\vartheta_{2\alpha}\sum_A N^{a\alpha}_A\mathcal{Z}^A-\right.
$$
$$
\left.\frac{\mathrm{i}}{8}\sum_A\left(\bar{n}_A\,\theta^a_2\theta_{2a}+n_A\,\vartheta^\alpha_2\vartheta_{2\alpha}\right)\,\mathcal{Z}^A\right]+c.c. \qquad (28)
$$

  where $\mathcal{F}(\mathcal{Z}^1,\ldots,\mathcal{Z}^k)$, $\overline{\mathcal{F}}(\overline{\mathcal{Z}}^1,\ldots,\overline{\mathcal{Z}}^k)$ are arbitrary holomorphic functions of the k-superfields $\mathcal{Z}^A$ and $\overline{\mathcal{Z}}^A$, respectively, and all possible Fayet-Iliopoulos terms with the constants $N^{a\alpha}_A$ and n_A were included.

The rest of the calculations goes in the same way as it is done in the previous subsections. For completeness, we present here the explicit structure of the Dirac brackets between the canonical variables

$$
\begin{array}{l}
\left\{z^A,\,\hat{p}_B\right\}=\delta^A_B, \quad \left\{\bar{z}^A,\,\hat{\bar{p}}_B\right\}=\delta^A_B, \\
\left\{\hat{p}_A,\,\hat{\bar{p}}_B\right\}=-\frac{\mathrm{i}}{2}g^{EE'}\partial^3_{ACE}F\,\bar{\partial}^3_{BC'E'}\overline{F}\left(\psi^{a\,C}\bar{\psi}^{C'}_a+\xi^{\alpha\,C}\bar{\xi}^{C'}_\alpha\right), \\
\left\{\psi^{Aa},\,\bar{\psi}^B_b\right\}=-2\mathrm{i}g^{AB}\delta^a_b, \quad \left\{\xi^{A\alpha},\,\bar{\xi}^B_\beta\right\}=-2\mathrm{i}g^{AB}\delta^\alpha_\beta, \\
\left\{\hat{p}_A,\,\psi^B_a\right\}=g^{BC}\partial^3_{ACE}F\,\psi^E_a, \quad \left\{\hat{p}_A,\,\xi^B_\alpha\right\}=g^{BC}\partial^3_{ACE}F\,\xi^E_\alpha, \\
\left\{\hat{\bar{p}}_A,\,\bar{\psi}^B_a\right\}=g^{BC}\bar{\partial}^3_{ACE}\overline{F}\,\bar{\psi}^E_a, \quad \left\{\hat{\bar{p}}_A,\,\bar{\xi}^B_\alpha\right\}=g^{BC}\bar{\partial}^3_{ACE}\overline{F}\,\bar{\xi}^E_\alpha,
\end{array}
\qquad (29)
$$

where the metric g_{AB} is defined as

$$
g_{AB}=\frac{\partial^2}{\partial z^A\partial z^B}F(z^1,\ldots,z^k)+\frac{\partial^2}{\partial\bar{z}^A\partial\bar{z}^B}\overline{F}(\bar{z}^1,\ldots,\bar{z}^k), \qquad g^{AB}g_{BC}=\delta^A_C. \qquad (30)
$$

Finally, the supercharges

$$
\begin{aligned}
Q^a_1 &= \hat{p}_A\psi^{Aa} - \tfrac{\mathrm{i}}{4}\bar\partial^3_{ABC}\overline{F}\bar\psi^{Aa}\bar\xi^{B\alpha}\bar\xi^{C}_{\alpha} + \tfrac{\mathrm{i}}{2}\left(\mathrm{i}\bar\partial^2_{AB}\overline{F}M^{Aa\alpha} + N^{a\alpha}_{B}\right)\bar\xi^{B}_{\alpha}\\
&\quad -\tfrac{1}{4}\left(\bar\partial^2_{AB}\overline{F}m^A - \bar n_B\right)\bar\psi^{Ba},\\
S^\alpha_1 &= \hat{p}_A\xi^{A\alpha} - \tfrac{\mathrm{i}}{4}\bar\partial^3_{ABC}\overline{F}\bar\xi^{A\alpha}\bar\psi^{Ba}\bar\psi^{C}_{a} - \tfrac{\mathrm{i}}{2}\left(\mathrm{i}\bar\partial^2_{AB}\overline{F}M^{Aa\alpha} + N^{a\alpha}_{B}\right)\bar\psi^{B}_{a}\\
&\quad +\tfrac{1}{4}\left(\bar\partial^2_{AB}\overline{F}m^A + \bar n_B\right)\bar\xi^{B\alpha},\\
Q_{2a} &= \left(Q^a_1\right)^\dagger, \quad S_{2a} = \left(S^a_1\right)^\dagger
\end{aligned}
\tag{31}
$$

and the Hamiltinian

$$
\begin{aligned}
&H_{2k} = g^{AB}\hat{p}_A\hat{\bar{p}}_B +\\
&+\tfrac{1}{16}\Big(\partial^4_{ABCD}F - g^{EE'}\partial^3_{ABE}F\,\partial^3_{CDE'}F - 2g^{EE'}\partial^3_{ACE}F\,\partial^3_{BDE'}F\Big)\psi^{Aa}\psi^B_a\xi^{C\alpha}\xi^D_\alpha +\\
&+\tfrac{1}{16}\Big(\bar\partial^4_{ABCD}\overline{F} - g^{EE'}\bar\partial^3_{ABE}\,\overline{F}\bar\partial^3_{CDE'}\overline{F} - 2g^{EE'}\bar\partial^3_{ACE}\overline{F}\,\bar\partial^3_{BDE'}\overline{F}\Big)\bar\psi^{Aa}\bar\psi^B_a\bar\xi^{C\alpha}\bar\xi^D_\alpha -\\
&-\tfrac{1}{16}\,g^{EE'}\partial^3_{ABE}F\bar\partial^3_{CDE'}\overline{F}\left(\psi^{Aa}\psi^B_a\bar\psi^{Cb}\bar\psi^D_b + \xi^{A\alpha}\xi^B_\alpha\bar\xi^{C\beta}\bar\xi^D_\beta - 4\psi^{Aa}\bar\psi^C_a\xi^{B\alpha}\bar\xi^D_\alpha\right) +\\
&+\tfrac{1}{8}\,g^{AB}\left[N^{a\alpha}_A N_{B\,a\alpha} - \mathrm{i}\left(\partial^2_{BC}F - \bar\partial^2_{BC}\overline{F}\right)N^{a\alpha}_A M^C_{a\alpha} + \partial^2_{AC}F\,\bar\partial^2_{BD}\overline{F}\,M^{C\,a\alpha}M^D_{a\alpha}\right] +\\
&+\tfrac{\mathrm{i}}{4}\,g^{AB}\big[\partial^3_{ACD}F\psi^{Ca}\xi^{D\alpha}\left(\bar\partial^2_{BE}\overline{F}\,M^E_{a\alpha} + \mathrm{i}N_{a\alpha\,B}\right) +\\
&\qquad +\bar\partial^3_{ACD}\overline{F}\,\bar\psi^{Ca}\bar\xi^{D\alpha}\left(\partial^2_{BE}F\,M^E_{a\alpha} - \mathrm{i}N_{a\alpha\,B}\right)\big] +\\
&+\tfrac{1}{16}\,g^{AB}\Big[n_A\bar n_B + \tfrac{1}{2}\left(n_A m^C - \bar n_A\overline{m}^C\right)\left(\partial^2_{BC}F - \bar\partial^2_{BC}\overline{F}\right) + m^C\overline{m}^D\partial^2_{AC}F\,\bar\partial^2_{BD}\overline{F} +\\
&+\mathrm{i}\partial^3_{ACD}F\xi^{C\alpha}\xi^D_\alpha\left(\bar\partial^2_{BE}\overline{F}m^E - \bar n_B\right) - \mathrm{i}\partial^3_{ACD}F\psi^{Ca}\psi^D_a\left(\bar\partial^2_{BE}\overline{F}\overline{m}^E + n_B\right) +\\
&\mathrm{i}\bar\partial^3_{ACD}\overline{F}\bar\xi^{C\alpha}\bar\xi^D_\alpha\left(\partial^2_{BE}F\overline{m}^E - n_B\right) - \mathrm{i}\bar\partial^3_{ACD}\overline{F}\bar\psi^{Ca}\bar\psi^D_a\left(\partial^2_{BE}Fm^E + \bar n_B\right)\Big]
\end{aligned}
\tag{32}
$$

form the superalgebra

$$
\begin{aligned}
&\{Q_{ia},\,Q_{jb}\} = -2\mathrm{i}\epsilon_{ij}\epsilon_{ab}\left(H - \tfrac{1}{16}\left(n_A m^A + \bar n_A\overline{m}^A\right)\right) - \tfrac{1}{8}\,\epsilon_{ij}\left(N^\alpha_{A\,a}M^A_{\alpha b} + N^\alpha_{A\,b}M^A_{\alpha a}\right),\\
&\{S_{i\alpha},\,S_{j\beta}\} = -2\mathrm{i}\epsilon_{ij}\epsilon_{\alpha\beta}\left(H + \tfrac{1}{16}\left(n_A m^A + \bar n_A\overline{m}^A\right)\right) - \tfrac{1}{8}\,\epsilon_{ij}\left(N^a_{A\,\alpha}M^A_{a\beta} + N^a_{A\,\beta}M^A_{a\alpha}\right),\\
&\{Q_{1a},\,S_{2\alpha}\} = -m^A N_{A\,a\alpha} - \mathrm{i}\bar n_A M^A_{a\alpha}, \quad \{Q_{2a},\,S_{1\alpha}\} = -\overline{m}^A N_{A\,a\alpha} + \mathrm{i}n_A M^A_{a\alpha}.
\end{aligned}
\tag{33}
$$

3 Summary and conclusions

In this paper we presented a new version of $N = 8$ SQM with **(2, 8, 6)** components. This supermultiplet is obtained by a direct reduction from the $N = 2$, $d = 4$ vector supermultiplet. We constructed the most general action with all possible FI terms and explicitly showed that the geometry of the target space is restricted to be the special Kähler one. Apart from the $N = 8$ superfield formulation, we presented the component action with all auxiliary fields, as well as with the physical fields only. As a nice feature, the constructed action possesses duality which acts not only in the bosonic sector, but also in the fermionic one. We performed the

Hamiltonian analysis and found the Dirac brackets between the canonical variables. The supercharges and Hamiltonian form the $N = 8$ super Poincarè algebra with the central charges. These central charges are proportional to a product of two constants — one that comes from the FI terms, and the other that appears in the superfield constraints (or in their solution) and coincides with the constant part of the auxiliary fields. These constants are directly related to the appearance of the potential terms in the Hamiltonian. Finally, we presented the extension of the $N = 8$ two-dimensional SQM to the $2k$-dimensional case.

These results should be regarded as preparatory for more detailed study of $2k$-dimensional SQM with $N = 8$ supersymmetry. In particular, it would be interesting to construct the full quantum version with some specific Kähler potential. Generally speaking, we believe that just this version of SQM could be rather useful for a simplified analysis of subtle problems which appear in the $N = 2$, $d = 4$ SYM. For example, one may try to fully analyse the effects of non-anti-commutativity in superspace [20], including the modifications of spectra, etc.

Another obvious project for a future study is to construct a one-dimensional analog of the *c-map* [19]. It should relate the $2k$-dimensional $N = 8$ SQM to the $4k$-dimensional one with some special restriction on the geometry of the latter. The preliminary results in this direction will appear in a forthcoming paper [18].

Finally, due to the appearance of the central charges in the $N = 8$ Poincarè superalgebra one may expect the existence of different patterns of partial supersymmetry breaking, like in the $N = 4$ SQM case [13, 14].

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